Non adapted transformations of the Wiener measure

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1 Introduction

Since the introduction by Feynman of his famous path integral approach to Quantum Mechanics many mathematiciens have tried through different ways to provide a rigorous and general framework for this approach. The Feynman original integral is associated with an Hamiltonian of the form $H = -\frac{1}{2}\Delta + V$, for V a scalar potential, and it should look like an integral with respect to a "probability measure" on the space of paths $x : [0,t] \to \mathbb{R}^d$:

$$d\nu(x) = \frac{1}{Z} exp\ i(\frac{1}{2}\int_0^t |\dot{x}|^2(s)ds - \int_0^t V(x(s))ds)dx$$

where Z is a normalization constant and "$dx = \Pi_{s\in[0,t]}dx(s)$" should account for a kind of Lebesgue measure, which does not exist on an infinite dimensional space.

It is known since [3] that such a measure is not well defined, even in the free case ($V = 0$), although some rigorous notion of Feynman integral can be formulated (cf.[1],[16]). Nevertheless another object, the Wiener measure, which is a perfectly well defined probability measure on the space of continuous paths, looks almost the same as ν except for the imaginary parameter i. To use a physical language, let's say that working with the Wiener measure is working "in imaginary time". This measure, and the random motion it describes (the Brownian motion) are at the basis of the field called Stochastic Analysis, which had an enormous development in the last century, specially after the work of Itô, with many applications not only in Physics, but also within Mathematics, and came out to have a great impact in various other areas, like, more recently, in Mathematical Finance.

One fundamental problem concerning the study of the Wiener measure is the way it behaves under transformations, that is, its change of variables transformation rules. The need for that study was already stressed by Feynman himself. The first works concerning these problems are due to Cameron and Martin ([4],[5]). They have considered transformations under deterministic shifts, as well as under certain linear and a few non-linear transformations.

Let $X = \{x : [0,1] \to \mathbb{R}^d,$ x continuous, $x(0) = 0\}$ and μ the Wiener measure on X. As a probability space X is the so-called (classical) Wiener space. Let $H = \{h \in X : \dot{h}$ exists a.e. and satisfies $\int_0^1 |\dot{h}|^2 ds < \infty\}$. H is an Hilbert space densely embedded in X (with respect to the usual topologies); it satisfies $\mu(H) = 0$.

The Cameron-Martin theorem for deterministic shifts states that if $h \in H$ then the image of the Wiener measure under $\tau_h(x) = x + h$ is absolutly continuous with respect to μ with Radon-Nikodym density explicitly given by:

$$\frac{(d\tau_h)_*\mu}{d\mu}(x) = exp(\int_0^1 \dot{h}.dx - \frac{1}{2}|\dot{h}|^2 ds)$$

where $.dx$ stands for (Itô's) stochastic integral with respect to the Brownian motion x.

With the development of Itô calculus, this theorem was later extended to shifts by adapted H-valued random variables, the expression for the density being analogous to (1.1) (this is the well known Girsanov theorem). Itô calculus is a powerfull machinery requiring adapteness to the underlying filtration (usually taken to be the filtration of past events) in order to work out. One can nevertheless legitimely ask what this notion of adaptness has to do with transformations of the Wiener measure. A priori, nothing, of course, except that Itô integral is no longer defined. On the other hand another notion of stochastic integral, due to Skorohod ([23]), exists: it is a generalization of Itô's in the sense that it does not require adapteness of the integrand. With the development of Malliavin calculus the Skorohod integral was identified with a basic notion (essential for this calculus), namely the divergence on the Wiener space. This allowed to consider much more general change of variable formulae of the Wiener measure.

In this work we review in the next section some results on (non adapted) transformations $T : X \to X$ verifying $T(x) - x \in H$. In section 3 we describe a more general class of maps (the tangent processes) where a rotation of the space is allowed and which, more recently, became a central object in order to study path spaces of Riemannian manifolds - integration by parts on such spaces is described in paragraph 4. Finally, in the last paragraph, we refer to an application of tangent processes in geometry.

2 Non linear transformations of the Wiener measure

The classical Jacobi change of variables theorem for diffeomorphisms T in $\mathbb{R}^d$ states that

$$\int_{\mathbb{R}^d} f(T(x))|J(x)|dx = \int_{\mathbb{R}^d} f(x)dx$$

where J is the Jacobian determinant of T.

Let us now write $T(x) = x + h(x)$. Replacing the Lebesgue measure dx by the Gaussian measure $d\nu(x) = \frac{1}{(2\pi)^{\frac{d}{2}}} exp(-\frac{|x|^2}{2})dx$ we obtain

$$\int_{\mathbb{R}^d} f(T(x))|\tilde{J}(x)|d\nu(x) = \int_{\mathbb{R}^d} f(x)d\nu(x)$$

where

$$\tilde{J}(x) = J(x)exp(-h(x).x - \frac{1}{2}|h(x)|^2)$$

Let us recall some notions of Malliavin calculus on the Wiener space that will allow us to write down an infinite dimensional "analogue" of the Jacobi formula.

2.1. Differentiation on the Wiener space.

We shall follow the Nualart-Pardoux-Zakai terminology (c.f.[20]) in the framework of Malliavin calculus ([19]).

For a cylindrical functional on the Wiener space, of the form $F(x) = f(x(\tau_1), ..., x(\tau_m))$, with f smooth in $\mathbb{R}^m$ and $0 \leq \tau_1 < ...\tau_m < 1$ a partition of the interval $[0,1]$, one defines the following derivative operators:

$$D_\tau F(x) = \sum_{k=0}^{m} 1_{\tau<\tau_k} \partial_k f(x(\tau_1), ..., x(\tau_m))$$

For $h \in H$, the derivative of F in the direction of h is defined as

$$D_h F(x) = \int_0^1 D_\tau F. \frac{d}{d\tau} h d\tau$$

The cylindrical functionals are dense in L^p_μ and derivation operators can be extended by closure to a corresponding domain.

As a linear operator on the (Hilbert) Cameron-Martin space, the derivation gives rise to the gradient operator, $\nabla F(x)(h) = D_h F(x)$, and we can define the Sobolev spaces

$$W^p_1(X) = \{F \in L^p_\mu : \nabla F \in L^p_\mu(X; H)\}$$

Higher order Sobolev spaces W^p_r (r derivatives in L^p_μ) can be defined when we consider a norm on the space of k-linear operators, $k \leq p$.

2.2. The divergence operator.

Let $A : X \rightarrow H$ be a L^2_μ "vector field". Its dual with respect to the Wiener measure, when it exists, is called the divergence of A and will be denoted by $\delta(A)$. We have then, by definition, the following integration by parts formula:

$$E_\mu(D_h F) = E_\mu(F\delta(A)) \quad \forall F \in W^2_1$$

If A belongs to the Sobolev space W^2_1 then the divergence exists - this was proved in [17]. Gaveau and Trauber ([14]) showed that the divergence coincides with the integral previously defined by Skorohod. In particular if A is an adapted random variable this integral coincides with Itô's one and we have

$$\delta(A) = \int_0^1 \frac{d}{d\tau} A(x)(\tau) dx(\tau)$$

.

In this paper we take this characterization as the definition of Skorohod integral itself, namely, for a process $u(x)(\tau)$ we write $\int_0^1 u(x)(\tau)dx(\tau) = \delta(\int_0^\cdot u(x)(\tau)d\tau)$ when the divergence is well defined.

On the other hand if we can associate a flow $U_t(x)$ to the vector field A, namely a solution of the ordinary differential equation

$$\frac{d}{dt}U_t(x) = A(U_t(x)), \qquad U_0(x) = x$$

(in the sense that the corresponding integral equation holds μ almost everywhere in x), then the divergence accounts for the infinitesimal action of the flow on the measure, i.e., we have:

$$\frac{d}{dt}_{|t=0} E_\mu(U_t^* F) = E_\mu(F\delta(A))$$

for every test function f.

We remark also that the dual of the derivative with respect to the Gaussian measure on $\mathbb{R}^d$ can be obtained explicitely by the integration by parts formula:

$$\int_{\mathbb{R}^d} (\nabla f|A)(x)d\nu(x) = \int_{\mathbb{R}^d} f[(A(x)|x)_H - trace\nabla(A)(x)]d\nu(x)$$

It is possible to derive the Skorohod integral expression for the divergence by using a suitable finite dimensional approximation argument on the Wiener space.

2.3. Non linear transformations.

Under suitable hypothesis, the change of variables formula for transformations $T(x) : X \to X$ on the Wiener space of the form $T(x) = x + A(x), \quad A(x) \in H$ a.e., can be written

$$E_\mu(|\mathcal{J}|(x)F(T(x))) = E_\mu(F(x))$$

where

$$\mathcal{J}(x) = det_2(I_H + \nabla A)exp(-\delta(A) - \frac{1}{2}||A||_H^2),$$

det_2 being the Carleman-Fredholm determinant and I_H denoting the identity operator on H. Various authors have contributed to the discovery of this formula. The version we have written is due to Ramer ([22], c.f. also [25]); it requires some non trivial assumptions on the Sobolev norms of the transformation T.

For non-random vector fields A the formula gives precisely Cameron-Martin initial theorem. The fact that it is also a generalization of Girsanov theorem is true but much more delicate: for adapted shifts ∇A turns out to be quasi-nilpotent (i.e. $trace(\nabla A)^k = 0 \; \forall k \geq 2$) and this is equivalent to the condition $det_2(I_h + \nabla A) = 1$ ([26]). In the adapted case the classical approach via Itô calculus requires much weaker hypothesis.

We can look at a (deterministic) shift by $h \in H$ as the value at time $t = 1$ of the flow associated to the vector h, namely $U_t^h(x) = x + th$. We know that

$$\frac{d}{dt}_{|t=0} (U_t^h)_* \mu = \delta(h)\mu$$

and that $\delta(h) = \int_0^1 \frac{d}{d\tau}(\tau)dx(\tau)$. By using the group property of the flow we can integrate this infinitesimal formula and obtain

$$\frac{d(U_t^h)_*\mu}{d\mu}(x) = exp(\int_0^t \delta(h)(U_{-s}^h(x))ds$$

which is precisely Girsanov theorem.

Under suitable Sobolev assumptions on the vector field A these ideas can be extended (c.f.[7]) in order to define the corresponding flow U_t and derive the following transformation formula

$$E_\mu(FoU_t) = E_\mu(F\rho_t)$$

where

$$\rho_t(x) = exp(\int_0^t \delta(A)(U_{-s}(x)ds)$$

A fairly complete reference concerning these ideas can be found in [26] (c.f. also [19]).

3 Tangent processes

With the development of analysis and geometry on the path space of a Riemannian manifold the need to extend the tangent space and include rotations of the Wiener space became clear (c.f.[8],[12]).

We call a *tangent process* a process ξ on the Wiener space of the form:

$$d\xi^\alpha(\tau) = a_\beta^\alpha dx^\beta(\tau) + c^\beta d\tau$$

where $a_\beta^\alpha + a_\alpha^\beta = 0,\ a_\beta^\alpha(0) = 0,\ c^\alpha(0) = 0$ with $\alpha, \beta = 1, ..., d$, and $E\int_0^1 |c|^2 d\tau < \infty$.

When a is adapted the process ξ is a martingale with an antisymmetric diffusion coefficient and we know, by Levy's theorem, that the martingale part keeps the Wiener measure invariant. When a is not adapted dx is to be interpreted in the Skorohod sense and assumptions on a for this integral to be well defined are required (c.f.[20]). In both cases we also assume that, besides the Skorohod (resp. Itô) representation, ξ also has a Stratonovich-Skorohod (resp. Stratonovich) representation.

Given a cylindrical functional $F(x) = f(x(\tau_1), ..., x(\tau_m))$ on X we define the derivative of F with respect to ξ as

$$D_\xi F = \sum_{k=1}^m (\partial_k f . \xi(\tau_k))$$

In [10] we have considered the question of wether Wiener measure is still invariant under non adapted rotations, that is under transformations by anticipative tangent processes with bounded variation part (c) equal to zero. The answer is yes and was also treated independently in [15]. It is a consequence of the representation formula of next theorem together with the fact that a Skorohod integral is a random variable with divergence zero.

Theorem 1. *([10]) Let* $d\xi^\alpha = a^\alpha_\beta dx^\beta(\tau)$ *with* $\int_0^1 ||a(\tau)||_{W_1^p} d\tau < \infty$ *for all* p. *Then* $W_2^q \subset$ *Dom* (D_ξ) *forall* $q > 1$ *and the following formula holds:*

$$D_\xi F = \int_0^1 (\sum_\alpha a^\alpha_\beta D_{\tau,\alpha} F) dx^\beta(\tau)$$

for every $F \in W_2^q$.

Tangent processes give rise, as Cameron-Martin vector fields do, to flows defined μ almost everywhere on the Wiener space under suitable (Sobolev type) assumptions on the coefficients. The adapted case has been studied in [6] but the methods extend to the anticipative situation.

4 Integration by parts on the path space of a Riemannian manifold

Let M be a d-dimensional compact Riemannian manifold. For $m_0 \in M$ we consider the path space:

$$P_{m_0}(M) = \{p : [0,1] \to M, \quad p(0) = m_0\}$$

endowed with the Wiener measure σ, i.e. the law of the Brownian motion associated to the Laplace-Beltrami operator on M.

Let $O(M)$ denote the orthonormal frame bundle over M and consider its canonical parallelization, which is given by a differential form (θ, ω) with values in $\mathbb{R}^d \times so(d)$. If $(A_\alpha)_{\alpha=1,...,d}$ are the horizontal vector fields on $O(M)$ ($< A_\alpha, \omega >= 0$ and $< A_\alpha, \theta >= \epsilon_\alpha$ where ϵ_α denotes the canonical basis on $\mathbb{R}^d$), then the Stratonovich stochastic differential equation

$$dr_x(\tau) = \sum_\alpha A_\alpha(r_x) dx^\alpha, \quad r_x(0) = r_0$$

defines a horizontal flow of diffeomorphisms.

The Itô map $I : X \to P_{m_0}(M)$, $I(x)(.) = \pi(r_x(.))$, where π is the canonical projection, was constructed in [18] and shown to be a a.s. bijection that preserves the measure, namely such that $(I)_*(\mu) = \sigma$.

For a cylindrical functional on the path space, $F(p) = f(p(\tau_1), ..., p(\tau_m))$,with f smooth, we consider the operators

$$D_{\tau,\alpha} F(p) = \sum_k 1_{\tau < \tau_k} (t^p_{0 \leftarrow \tau_k} (\partial_{\tau_k} F) | \epsilon_\alpha))$$

where $t^p_{0\leftarrow\tau}$ is the Levi-Civita parallel transport over Brownian paths, which was defined by Itô.

With respect to the norms $||DF||^q_{L^q_\sigma} = E_\sigma(||DF||^q)$, where

$$||DF||^2(p) = \sum_\alpha \int_0^1 [D_{\tau,\alpha} F]^2 d\tau$$

this operator is closable, the domain being the Sobolev space $W_1^q(P_{m_0}(M))$.

A Cameron-Martin vector field on the path space is a map $Z_p(\tau) \in T_{p(\tau)}(M)$ whose parallel transport to the origin, that we denote by $z(\tau) = t^p_{0\leftarrow\tau} Z(\tau)$, belongs to the Cameron-Martin space. Then one defines

$$D_Z F = \sum_\alpha \int_0^1 \frac{d}{d\tau} z^\alpha D_{\tau,\alpha} F d\tau$$

In the last decade analysis and geometry on path spaces has been considerably developed. We mostly refer to the pioneering works [2] and [12] and to [8] that follows the development of the Markovian stochastic calculus on the path space established in [13]. Other sources of information on this subject are [19], [24] and references within.

One could expect that the differential structure would be preserved as well by the map I, but differentiation of the Itô map makes things much more complicated and it turns out that a derivation with respect to a Cameron-Martin vector field on $P_{m_0}(M)$ corresponds to a derivation on the Wiener space in the direction of a tangent process. This is why, as already mentionned, such processes became of crucial importance for the study of the path space. More precisely we have:

Theorem 2. *A functional F on $P_{m_0}(M)$ is differentiable along a Cameron-Martin vector field Z if and only if FoI is differentiable on the Wiener space in the direction of the tangent process*

$$d\xi = \frac{d}{d\tau} z d\tau + \rho o dx(\tau)$$

where

$$d\rho(\tau) = \Omega(odx, z)$$

and Ω denotes the curvature tensor on the manifold M.

This theorem was proved in [12] and [13], generalized to tangent processes on the path space in [8] and to non adapted vector fields Z in [10](in this case stochastic integration must be interpreted in the Stratonovich-Skorohod sense). One of its consequences, in the adapted case, is Bismut's integration by parts formula which may be deduced from the expression of the Itô contraction term $d\rho.dx = \frac{1}{2} Ricci^M d\tau$.

Theorem 3. *([2]) If $F \in W_1^2(P_{m_0}(M))$ and Z is a L^2_σ adapted Cameron-Martin vector field on the path space, we have:*

$$E_\sigma(D_Z F) = E_\mu((FoI) \int_0^1 [\frac{d}{d\tau} z + \frac{1}{2} R^M z] dx)$$

where $R^M_\tau = t^p_{0\leftarrow\tau} o Ricci^M_{p(\tau)} o t^p_{\tau\leftarrow 0}$.

An extension of the integration by parts formula for non adapted vector fields also holds (c.f.[9], [10] and [21]).

Theorem 4. *Let Z be a Cameron-Martin vector field in the path space such that $E\int_0^1 |\frac{d}{d\tau}z|^2 d\tau < \infty$, where $z(\tau) = t^p_{0\leftarrow\tau}Z(\tau)$ and such that $d\rho^\alpha_\beta(\tau) = \Omega^\alpha_{\gamma,\delta,\beta} z^\delta odx^\gamma(\tau)$ is well defined and satisfies $E||\rho||_{W^p_1} < \infty$. Assume also that*

$$E\int_0^1 |\sum_\beta \int_0^\tau \Omega(odx, \bar{D}_{\tau,\beta}z)|^2 d\tau < \infty,$$

where $\bar{D}_\tau$ denotes the sum of D^+_τ and D^-_τ defined by $D^{+(-)}_\tau.u(\tau) = lim_{\eta\to\tau^{+(-)}} D_\tau.u(\eta)$. Then we have

$$E_\sigma(D_Z F) = E_\mu((FoI)\int_0^1 [\frac{d}{d\tau}z + \frac{1}{2}(R^M z)(\tau) - \frac{1}{2}\sum_\beta \int_0^\tau \Omega^\beta(odx, \bar{D}_{\tau,\beta}z)].dx(\tau))$$

The supplementary term with respect to the adapted case is essentially due to the more complex form of the contraction that gives the difference between Skorohod and Stratonovich-Skorohod integrals.

5 An assymptotic estimate for the vertical derivatives of the horizontal Laplacian

In this paragraph we describe an application of the non adapted integration by parts on the path space which has been derived in [11].

Let us consider the horizontal Laplacian on the frame bundle $O(M)$, namely $\mathcal{L} = \sum_\alpha A^2_\alpha$. We assume that the curvature tensor of the manifold M satisfies

$$(H) \quad c(\Omega) = sup_{r\in O(M)}||[(\Omega)_r]^{-1}|| < \infty$$

Since the brackets $[A_\alpha, A_\beta]$ are vertical vectors whose vertical component in the canonical parallelism of $O(M)$ is precisely $\Omega_{\beta,\alpha}$, assumption (H) implies that $\mathcal{L}$ satisfies Hörmander condition and therefore the heat kernel associated to $\frac{1}{2}\mathcal{L}$ exists. We denote it by π_t.

If $q \in so(d)$ is a vertical vector, we have $\mathcal{L}\partial_q = \partial_q\mathcal{L}$, which implies that the vertical derivatives commute with the semigroup associated to $\mathcal{L}$ and, in particular, that

$$\partial^1_q \pi_t(r_0, r) = -\partial^2_q \pi_t(r_0, r)$$

Theorem 5. **([11])** *Under hypothesis (H) and assuming the Ricci curvature of the manifold M is bounded, we have, for every $q \in so(d)$,*

$$\limsup_{t\to 0} \frac{t}{||q||}\int_{O(M)} |\partial_q \pi_t(r_o, r)|\gamma_{O(M)}(dr) \le c(d)c(\Omega)$$

where $\gamma_{O(M)}$ denotes the volume element of $O(M)$ and $c(d)$ is a constant depending only of the dimension d of M.

The rest of this section is devoted to give an idea of the proof.

We consider the functional

$$\Phi^{\beta}(x,\tau) = \sum_{\alpha} \int_0^{\tau} \Omega_{\alpha,\beta} o dx^{\alpha}$$

where here Ω denotes the scalarization of the curvature tensor at $r_x(\tau)$, the Brownian flow starting at r_0 at time 0. This functional takes values at $so(d)$. For $t \in [0,1]$ we define $\tilde{\Phi}^{\beta} = \Phi^{\beta} - \frac{1}{t}\int_0^t \Phi^{\beta}(\tau)d\tau$ and the covariance matrix

$$\sigma_{\theta}^{\theta'} = \sum_{\beta} \int_0^t \tilde{\Phi}_{\theta}^{\beta}(\tau)\tilde{\Phi}_{\theta'}^{\beta}(\tau)d\tau$$

where θ, θ' denote double indeces.

We consider the variation process

$$z_q^{\beta}(\tau) = \int_0^{\tau} (\sum_{\theta} a_{\theta} \tilde{\Phi}_{\theta}^{\beta}(\eta))d\eta$$

where

$$a_{\theta} = \sum_{\theta'} [\sigma^{-1}]_{\theta}^{\theta'} q_{\theta'}$$

Direct verification shows that $z_q(t) = 0$ and that

$$\rho(t) = -\sum_{\beta} \int_0^t \Phi^{\beta}(\tau) dz_q^{\beta}(\tau)d\tau = q$$

Therefore, by Theorem 2 the infinitesimal variation of the horizontal flow is due to the corresponding variation ξ of the Brownian motion (on the Wiener space) defined by

$$d\xi = dz_q + \rho o dx$$

Notice that, because of the expressions of $\tilde{\Phi}$ and σ, this process is an anticipative tangent process. In terms of Skorohod integration, it reads:

$$d\xi(\tau) = dz_q(\tau) + \rho dx(\tau) + \frac{1}{2}R^M(z_q)d\tau - \frac{1}{2}\sum_{\beta}\int_0^{\tau} \Omega^{\beta}(odx, \bar{D}_{\tau,\beta} z_q)dx(\tau)$$

Now we have

$$\frac{\partial_q^1 \pi_t(r_0, r)}{\pi_t(r_0, r)} = E^{r_x(t,r_0)=r}[\delta(z_q)]$$

and the theorem follows from the L^2 estimates of the divergence of z_q, which is given by the Skorohod integrals of the bounded variation part of the tangent process ξ defined above.

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