

FORMULÁRIO

Métodos iterativos para equações não-lineares

Método da bissecção: $x_{k+1} = \frac{a_k + b_k}{2}$, $f(a_k)f(b_k) < 0$

$$|x - x_k| \leq \frac{b - a}{2^k}$$

$$|x - x_{k+1}| \leq |x_{k+1} - x_k|$$

Método da secante: $x_{k+1} = x_k - f(x_k) \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

$$|x - x_{k+1}| \leq \mathbb{K} |x - x_k| |x - x_{k-1}|, \quad \mathbb{K} = \frac{\max |f''|}{2 \min |f'|}$$

$$|x - x_k| \leq \frac{1}{\mathbb{K}} \left(\frac{(\mathbb{K}|z - x_0|)^\varrho}{\mathbb{K}|z - x_{-1}|} \right)^{\varrho^n}, \quad \varrho = \frac{1 + \sqrt{5}}{2}$$

Método de Newton: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$$|x - x_{k+1}| \leq \frac{\max |f''|}{2|f'(x_k)|} |x - x_k|^2$$

$$|x - x_k| \leq \frac{1}{\mathbb{K}} (\mathbb{K}|z - x_0|)^{2^n}$$

Método do ponto fixo: $x_{k+1} = g(x_k)$

$$|x - x_k| \leq L^k |x - x_0|, \quad |x - x_k| \leq \frac{L^k}{1 - L} |x_1 - x_0|$$

$$|x - x_{k+1}| \leq \frac{L}{1 - L} |x_{k+1} - x_k|$$

Normas e Condicionamento

$$\|\mathbf{A}\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}|$$

$$\|\mathbf{A}\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|$$

$$\|\mathbf{A}\|_2 = (\rho(\mathbf{A}^T \mathbf{A}))^{1/2}$$

$$\text{cond}(\mathbf{A}) = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|$$

$$\|\delta_{\mathbf{x}}\| \leq \text{cond}(\mathbf{A}) \|\delta_{\mathbf{b}}\|, \text{ para } \mathbf{A}\mathbf{x} = \mathbf{b}$$

Métodos iterativos para sistemas lineares

$$\mathbf{Ax} = \mathbf{b} \Leftrightarrow \mathbf{x} = \mathbf{Cx} + \mathbf{d} \quad \rightarrow \quad \mathbf{x}^{(k+1)} = \mathbf{Cx}^{(k)} + \mathbf{d}$$

$$\|\mathbf{x} - \mathbf{x}^{(k)}\| \leq \|\mathbf{C}\|^k \|\mathbf{x} - \mathbf{x}^{(0)}\|, \quad \|\mathbf{x} - \mathbf{x}^{(k)}\| \leq \frac{\|\mathbf{C}\|^k}{1 - \|\mathbf{C}\|} \|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\|$$

$$\|\mathbf{x} - \mathbf{x}^{(k+1)}\| \leq \frac{\|\mathbf{C}\|}{1 - \|\mathbf{C}\|} \|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\|$$

Método de Jacobi: $\mathbf{C} = -\mathbf{D}^{-1}(\mathbf{L} + \mathbf{U})$;

Método de Gauss-Seidel: $\mathbf{C} = -(\mathbf{L} + \mathbf{D})^{-1}\mathbf{U}$

Método de Newton para sistemas não-lineares

$$\mathbf{J}(\mathbf{x}^{(k)})\Delta\mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)}) \quad \mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}^{(k)}$$

Aproximação de funções

1. Interpolação Polinomial

Fórmula de Lagrange:

$$l_i(x) = \prod_{j=0, j \neq i}^n \left(\frac{x - x_j}{x_i - x_j} \right) \quad p_n(x) = \sum_{i=0}^n y_i l_i(x)$$

Fórmula de Newton com dif. divididas:

$$\begin{cases} D_j^0 = y_j, & j = 0, \dots, n \\ D_j^k = \frac{D_{j+1}^{k-1} - D_j^{k-1}}{x_{j+k} - x_j}, & j = 0, \dots, n-k, \quad k = 1, \dots, n \end{cases} \quad p_n(x) = y_0 + \sum_{i=1}^n D_i^0 (x - x_0) \cdots (x - x_{i-1})$$

Fórmula de erro: $e_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^n (x - x_i)$

2. Mínimos Quadrados

$$\begin{bmatrix} (\phi_0, \phi_0) & \dots & (\phi_0, \phi_m) \\ \dots & \dots & \dots \\ (\phi_m, \phi_0) & \dots & (\phi_m, \phi_m) \end{bmatrix} \begin{bmatrix} a_0 \\ \dots \\ a_m \end{bmatrix} = \begin{bmatrix} (\phi_0, f) \\ \dots \\ (\phi_m, f) \end{bmatrix}$$

$$(\phi_i, \phi_j) = \sum_{k=0}^n \phi_i(x_k) \phi_j(x_k), \quad (\phi_i, f) = \sum_{k=0}^n \phi_i(x_k) f_k$$

Integração Numérica

Regra dos trapézios:

$$T_N(f) = h \left[(f(x_0) + f(x_N))/2 + \sum_{i=1}^{N-1} f(x_i) \right]$$
$$E_N^T(f) = -\frac{(b-a)h^2}{12} f''(\xi) \quad \xi \in (a, b)$$

Regra de Simpson:

$$S_N(f) = \frac{h}{3} \left[f(x_0) + f(x_N) + 4 \sum_{i=1}^{N/2} f(x_{2i-1}) + 2 \sum_{i=1}^{N/2-1} f(x_{2i}) \right]$$
$$E_N^S(f) = -\frac{(b-a)h^4}{180} f^{(4)}(\xi) \quad \xi \in (a, b)$$

Métodos numéricos para equações diferenciais

$$y_{i+1} = y_i + hf(t_i, y_i) \quad \text{Método de Euler}$$