MATEMTICA COMPUTACIONAL
FORMULÁRIO 2006-07

1 Teoria dos erros

$u = \beta^{1-n}$ *arredondamento por corte*
$u = \frac{1}{2}\beta^{1-n}$ *arredondamento simétrico*

2 Equações não-lineares

Método da secante:

$$x_{m+1} = x_m - f(x_m)\frac{x_m - x_{m-1}}{f(x_m) - f(x_{m-1})}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(\eta_m)}e_{m-1}e_m$$
$$\eta_m \in \text{int}(x_{m-1}, x_m) \quad \xi_m \in \text{int}(x_{m-1}, x_m, z)$$

Método de Newton:

$$x_{m+1} = x_m - \frac{f(x_m)}{f'(x_m)}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(x_m)}e_m^2 \qquad \xi_m \in \text{int}(z, x_m)$$

Método do ponto fixo:

$$x_{m+1} = g(x_m)$$

$$|e_{m+1}| \leq L|e_m|$$

$$|e_{m+1}| \leq \frac{L}{1-L}|x_{m+1} - x_m|$$

$$g'(z) \approx \frac{x_{m+1} - x_m}{x_m - x_{m-1}}$$

$$e_{m+1} = \frac{(-1)^{p-1}g^{(p)}(\xi_m)}{p!}e_m^p \qquad \xi_m \in \text{int}(z, x_m)$$
$$g^{(r)}(z) = 0 \qquad r = 0, \ldots, p-1 \qquad g^{(p)}(z) \neq 0$$

$$|e_{m+1}| \leq L_p|e_m|^p \qquad L_p = \max_{x \in I}|g^{(p)}(x)/p!|$$

3 Sistemas de equações

Método de Crout:

$$l_{ik} = a_{ik} - \sum_{r=1}^{k-1} l_{ir}u_{rk}$$

$$u_{kj} = (a_{kj} - \sum_{r=1}^{k-1} l_{kr}u_{rj})/l_{kk}$$

Método de Doolittle:

$$l_{ik} = (a_{ik} - \sum_{r=1}^{k-1} l_{ir}u_{rk})/u_{kk}$$

$$u_{kj} = a_{kj} - \sum_{r=1}^{k-1} l_{kr}u_{rj}$$

Método de Cholesky:

$$l_{kk} = \sqrt{a_{kk} - \sum_{r=1}^{k-1} l_{kr}^2}$$

$$l_{ik} = (a_{ik} - \sum_{r=1}^{k-1} l_{ir}l_{kr})/l_{kk}$$

Normas e condicionamento:

$$\|\mathbf{A}\|_\infty = \max_{1 \leq i \leq n} \sum_{j=1}^{n} |a_{ij}|$$

$$\|\mathbf{A}\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^{n} |a_{ij}|$$

$$\|\mathbf{A}\|_2 = (\rho(\mathbf{A}^T\mathbf{A}))^{1/2}$$

$$cond(\mathbf{A}) = \|\mathbf{A}\|\|\mathbf{A}^{-1}\|$$

$$\frac{\|\Delta\mathbf{x}\|}{\|\mathbf{x}\|} \leq \frac{cond(\mathbf{A})}{1 - cond(\mathbf{A})\frac{\|\Delta\mathbf{A}\|}{\|\mathbf{A}\|}}\left(\frac{\|\Delta\mathbf{A}\|}{\|\mathbf{A}\|} + \frac{\|\Delta\mathbf{b}\|}{\|\mathbf{b}\|}\right)$$

Métodos iterativos:

$$\mathbf{M}\mathbf{x}^{(k+1)} = \mathbf{b} - \mathbf{N}\mathbf{x}^{(k)}$$

$$\mathbf{x} - \mathbf{x}^{(k+1)} = \mathbf{C}(\mathbf{x} - \mathbf{x}^{(k)})$$
$$\mathbf{C} = -\mathbf{M}^{-1}\mathbf{N}$$

Método de Jacobi:

$$x_i^{(k+1)} = (b_i - \sum_{j=1, j \neq i}^{n} a_{ij}x_j^{(k)})/a_{ii}$$

Método de Gauss-Seidel:

$$x_i^{(k+1)} = (b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)})/a_{ii}$$

Método de relaxacão SOR:

$$z_i^{(k+1)} = (b_i - \textstyle\sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)})/a_{ii}$$
$$x_i^{(k+1)} = \omega z_i^{(k+1)} + (1-\omega)x_i^{(k)}$$

Método de Newton:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}^{(k)} \qquad \mathbf{J}(\mathbf{x}^{(k)})\Delta\mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)})$$

4 Interpolação Polinomial

Fórmula interpoladora de Lagrange:

$$p_n(x) = \sum_{i=0}^{n} f_i \, l_i(x)$$

$$l_i(x) = \prod_{j=0, j\neq i}^{n} \left(\frac{x - x_j}{x_i - x_j}\right)$$

Fórmula interpoladora de Newton:

$$p_n(x) = f(x_0) + \sum_{i=1}^{n} f[x_0, \ldots, x_i](x - x_0)\cdots(x - x_{i-1})$$

$$e_n(x) = f[x_0, \ldots, x_n, x] \quad \prod_{i=0}^{n}(x - x_i)$$

$$f[x_0, \ldots, x_k] = \frac{f^{(k)}(\xi)}{k!} \qquad \xi \in \text{int}(x_0, \ldots, x_k)$$

5 Mínimos Quadrados

Equações normais:

$$\begin{bmatrix} (\phi_0, \phi_0) & \cdot & (\phi_0, \phi_m) \\ \cdot & \cdot & \cdot \\ (\phi_m, \phi_0) & \cdot & (\phi_m, \phi_m) \end{bmatrix} \begin{bmatrix} a_0 \\ \cdot \\ a_m \end{bmatrix} = \begin{bmatrix} (\phi_0, f) \\ \cdot \\ (\phi_m, f) \end{bmatrix}$$

6 Integração Numérica

$$I_n(f) = \sum_{i=0}^{n} A_i f(x_i)$$

$$A_i = \int_a^b l_i(x)\, dx$$

$$\sum_{i=0}^{n} A_i x_i^k = \frac{b^{k+1} - a^{k+1}}{k+1} \qquad k = 0, \ldots, n$$

Regra dos trapézios:

$$I_1(f) = \left(\frac{h}{2}\right)[f(a) + f(b)]$$

$$E_1(f) = -\frac{h^3}{12} f''(\xi) \qquad \xi \in (a, b)$$

$$I_n(f) = h\left[(f_0 + f_n)/2 + \sum_{i=1}^{n-1} f_i\right]$$

$$\begin{aligned} E_n(f) &= -\frac{h^3}{12}\textstyle\sum_{i=1}^{n} f''(\xi_i) \\ &= -\frac{h^2(b-a)}{12} f''(\xi) \qquad \xi \in (a, b) \end{aligned}$$

Regra de Simpson:

$$I_2(f) = \frac{h}{3}\left[f(a) + 4f(a+h) + f(b)\right]$$

$$E_2(f) = -\frac{h^5}{90} f^{(4)}(\xi) \qquad \xi \in (a, b)$$

$$I_n(f) = \frac{h}{3}\left[(f_0 + f_n) + 4\sum_{i=1}^{n/2} f_{2i-1} + 2\sum_{i=1}^{n/2-1} f_{2i}\right]$$

$$\begin{aligned} E_n(f) &= -\frac{h^5}{90}\textstyle\sum_{i=1}^{n/2} f^{(4)}(\xi_i) \\ &= -\frac{h^4(b-a)}{180} f^{(4)}(\xi) \qquad \xi \in (a, b) \end{aligned}$$

Regra do ponto médio:

$$I_1 = hf\left(\frac{a+b}{2}\right)$$

$$E_1 = \frac{h^3}{24} f''(\xi) \qquad \xi \in (a, b)$$

$$I_n(f) = h\sum_{i=1}^{n} f(a + (i - 1/2)h)$$

$$E_n(f) = \frac{h^2(b-a)}{24} f''(\xi) \qquad \xi \in (a, b)$$

7 Equações Diferenciais

$$y' = f(x, y) \qquad y(x_0) = y_0$$

Método de Euler:

$$\begin{aligned} &y_{n+1} = y_n + h\, f(x_n, y_n) \\ &T_n = \tfrac{h^2}{2} y''(\xi_n) \qquad \xi \in (x_n, x_{n+1}) \end{aligned}$$

$$\begin{aligned} |y(x_n) - \tilde{y}_n| \leq \; & |y_0 - \tilde{y}_0|\, e^{(x_n - x_0)K} + \\ & + \left(\tfrac{hY_2}{2} + \tfrac{\epsilon}{h}\right) \frac{e^{(x_n - x_0)K} - 1}{K} \end{aligned}$$

$$K = \max_{x \in [x_0, b]} |f'_y(x, y)|$$

$$Y_2 = \max_{x \in [x_0, b]} |y''(x)|$$

Método baseado na série de Taylor:

$$y_{n+1} = y_n + hf_n + \frac{h^2}{2}[f'_x(x_n, y_n) + f'_y(x_n, y_n) f_n]$$

Métodos de Runge-Kutta:

$$\begin{aligned} y_{n+1} = \; & y_n + h(1 - \tfrac{1}{2a_2}) f_n + \\ & + \tfrac{h}{2a_2} f(x_n + a_2 h\,,\, y_n + a_2 h f_n) \end{aligned}$$

$$\begin{aligned} &y_{n+1} = y_n + \tfrac{h}{6}(V_1 + 2V_2 + 2V_3 + V_4) \\ &V_1 = f(x_n, y_n) \qquad V_2 = f(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hV_1) \\ &V_3 = f(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hV_2) \\ &V_4 = f(x_n + h, y_n + hV_3) \end{aligned}$$