ANÁLISE NUMÉRICA

FORMULÁRIO

2 Resolução de Equações não lineares

Métodos Iterativos

Método da secante:

$$x_{m+1} = x_m - f(x_m)\frac{x_m - x_{m-1}}{f(x_m) - f(x_{m-1})}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(\eta_m)}e_{m-1}e_m$$

$\eta_m \in \text{int}(x_{m-1}, x_m) \quad \xi_m \in \text{int}(x_{m-1}, x_m, z)$

Método de Newton:

$$x_{m+1} = x_m - \frac{f(x_m)}{f'(x_m)}$$

$$e_{m+1} = -\frac{f''(\xi_m)}{2f'(x_m)}e_m^2 \qquad \xi_m \in \text{int}(z, x_m)$$

Método do ponto fixo:

$$x_{m+1} = g(x_m)$$

$$|e_{m+1}| \le L|e_m|$$

$$|e_{m+1}| \le \tfrac{L}{1-L}|x_{m+1} - x_m|$$

$$g'(z) \simeq \frac{x_{m+1} - x_m}{x_m - x_{m-1}}$$

$$e_{m+1} = \frac{(-1)^{p-1}g^{(p)}(\xi_m)}{p!}e_m^p \qquad \xi_m \in \text{int}(z, x_m)$$

$$g^{(r)}(z) = 0 \qquad r = 0, \dots, p-1 \qquad g^{(p)}(z) \neq 0$$

3 Resolução de Sistemas

Sistemas Lineares

Normas e Condicionamento:

$$\|\mathbf{A}\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}|$$

$$\|\mathbf{A}\|_1 = \max_{1\le j\le n} \sum_{i=1}^{n} |a_{ij}|$$

$$\|\mathbf{A}\|_2 = (\rho(\mathbf{A}^T\mathbf{A}))^{1/2}$$

$$cond(\mathbf{A}) = \|\mathbf{A}\|\,\|\mathbf{A}^{-1}\|$$

$$||\delta_{\mathbf{x}}|| \le cond(\mathbf{A})\ ||\delta_{\mathbf{b}}||$$

Métodos Iterativos para Sist. Lineares

$$\mathbf{x}^{(k+1)} = \mathbf{C}\mathbf{x}^{(k)} + \mathbf{d}$$

$$\mathbf{x} - \mathbf{x}^{(k+1)} = \mathbf{C}(\mathbf{x} - \mathbf{x}^{(k)})$$

$$\mathbf{M}\mathbf{x}^{(k+1)} = \mathbf{b} - \mathbf{N}\mathbf{x}^{(k)}, \quad \mathbf{C} = -\mathbf{M}^{-1}\mathbf{N}$$

Método de Jacobi:

$$x_i^{(k+1)} = (b_i - \sum_{j=1, j\neq i}^{n} a_{ij}x_j^{(k)})/a_{ii}$$

Método de Gauss-Seidel:

$$x_i^{(k+1)} = (b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)})/a_{ii}$$

Métodos Iterativos para Sist. Não-Lineares

Método de Newton:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}^{(k)} \qquad \mathbf{J}(\mathbf{x}^{(k)})\Delta\mathbf{x}^{(k)} = -\mathbf{f}(\mathbf{x}^{(k)})$$

4 Aproximação de funções

4.1 Interpolação Polinomial

Fórmula de Lagrange:

$$p_n(x) = \sum_{i=0}^{n} f_i\ l_i(x)$$

$$l_i(x) = \prod_{j=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

Fórmula de Newton com dif. divididas:

$$p_n(x) = f(x_0) + \sum_{i=1}^{n} f[x_0, \dots, x_i](x - x_0)\cdots(x - x_{i-1})$$

$$e_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}\prod_{i=0}^{n}(x - x_i), \quad \xi \in \text{int}(x_0, \dots, x_n, x)$$

$$f[x_0, \dots, x_k] = \frac{f^{(k)}(\eta)}{k!}$$

4.2 Mínimos Quadrados

$$\begin{bmatrix} (\phi_0,\phi_0) & \dots & (\phi_0,\phi_m) \\ \dots & \dots & \dots \\ (\phi_m,\phi_0) & \dots & (\phi_m,\phi_m) \end{bmatrix} \begin{bmatrix} a_0 \\ \dots \\ a_m \end{bmatrix} = \begin{bmatrix} (\phi_0, f) \\ \dots \\ (\phi_m, f) \end{bmatrix}$$

5 Integração Numérica

Regra dos trapézios:

$$T_N(f) = h\left[(f_0 + f_N)/2 + \sum_{i=1}^{N-1} f_i\right], \qquad T_1(f) = T(f) = (\frac{b-a}{2})[f(a) + f(b)]$$

$$E_N^T(f) = -\frac{Nh^3}{12}f''(\xi) \qquad \xi \in (a,b)$$

Regra de Simpson:

$$S_N(f) = \frac{h}{3}\left[(f_0 + f_N) + 4\sum_{i=1}^{N/2} f_{2i-1} + 2\sum_{i=1}^{N/2-1} f_{2i}\right], \qquad S_2(f) = S(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$E_N^S(f) = -\frac{Nh^5}{180}f^{(4)}(\xi) \qquad \xi \in (a,b)$$

6 Métodos numéricos para equações diferenciais

Métodos de Taylor:

1. **Método de Euler**:

$$y_{n+1} = y_n + h\,f(x_n, y_n)$$

$$|y''(x)| \le M, \quad \left|\frac{\partial f(x,y)}{\partial y}\right| \le K, \quad |e_n| = |y(x_n) - y_n| \le \frac{h\,M}{2K}\left(e^{K(x_n - x_0)} - 1\right).$$

2. **Método de Taylor de ordem 2**:

$$y_{n+1} = y_n + h\,y_n' + \frac{h^2}{2}\,y_n''$$

Alguns métodos de Runge-Kutta de ordem 2:

1.

$$y_{n+1} = y_n + \frac{h}{4}[f(x_n, y_n) + 3f(x_n + \frac{2}{3}h, y_n + \frac{2}{3}hf(x_n, y_n))]$$

2. **Método do ponto médio (ou Euler modificado)**:

$$y_{n+1} = y_n + hf(x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n))$$

3. **Método de Heun**:

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n, y_n) + f(x_{n+1}, y_n + hf(x_n, y_n))]$$

Um método de Runge-Kutta de ordem 4:

$$y_{n+1} = y_n + \frac{h}{6}(V_1 + 2V_2 + 2V_3 + V_4)$$

$$V_1 = f(x_n, y_n) \quad V_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}V_1)$$

$$V_3 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}V_2) \quad V_4 = f(x_n + h, y_n + hV_3)$$