

# NONLOCAL DIFFUSION, A MITTAG-LEFFLER FUNCTION AND A TWO-DIMENSIONAL VOLTERRA INTEGRAL EQUATION

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In this paper we consider a particular class of two-dimensional singular Volterra integral equations. Firstly we show that these kind of equations can indeed arise in practice by considering a diffusion problem with an output flux which is nonlocal in time. This problem is shown to admit an analytic solution in the form of an integral. More crucially this problem can be re-characterized as an integral equation of this particular class. This example then provides motivation for a more general study: an analytic solution is obtained for the case when the kernel and the forcing function are both unity. This analytic solution, in the form of a series solution, is a variant of the Mittag-Leffler function. As a consequence it is an entire function. A Gronwall lemma is obtained. This then permits a general existence and uniqueness theorem to be proved.