ANALYSIS AND NUMERICAL APPROXIMATION OF FORWARD-BACKWARD DIFFERENTIAL EQUATIONS

M.F. TEODORO[1,2], P.M. LIMA[1]

[1] *CEMAT, Departamento de Matemática, Instituto Superior Técnico,* [2] *Escola Superior de Tecnologia de Setúbal, Instituto Politécnico de Setúbal*

[1] Av. Rovisco Pais, 1049-001 Lisboa, Portugal, [2] Campus do IPS, Estefanilha 2910-761 Setúbal, Portugal

E-mail: `mteodoro@est.ips.pt, plima@math.ist.utl.pt`

N.J. FORD, P.M. LUMB

Mathematics Department, University of Chester

Parkgate Road, Chester, CH1 4BJ, UK

E-mail: `njford@chester.ac.uk, plumb@chester.ac.uk`

This talk is concerned with the approximate solution of a forward-backward differential equation of the form:

$$x'(t) = \alpha(t)x(t) + \beta(t)x(t-1) + \gamma(t)x(t+1). \qquad (1)$$

We search for a solution x, defined for $t \in [-1, k]$,($k \in \mathbf{N}$), which takes given values on the intervals $[-1, 0]$ and $(k-1, k]$. This problem has been studied both analytically and numerically [1]. Continuing the work started in [2], we introduce and analyse some new computational methods for the solution of this problem which are applicable both in the case of constant and variable coefficients. Here we search for a solution in the form

$$x(t) = x_0(t) + \bar{x}(t), \qquad t \in [-1, k] \qquad (2)$$

where x_0 is a given function, such that a) x_0 coincides with ϕ_1 on $[-1, 0]$; b)x_0 is a $2k$-degree polynomial on $[0, 1)$; c) $x_0 \in C^k([-1, 1[)$; d) for $t \in [1, k]$, x_0 is extended as a solution of equation (1), by recurrence formulae. According to this approach, in order to have x a solution of the problem (1), the function $\bar{x}$ on the right-hand side of (2) must satisfy a $(k-1)$-th order ODE with boundary conditions on the interval $[0, 1]$. By analysing the obtained boundary value problem, we can obtain existence results about the original problem (1), and in the case where it is solvable, we can obtain the numerical solution, by applying standard computational methods for ODEs. Numerical results are presented and compared with the results obtained by other methods.

REFERENCES

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