

1 Interpolação Polinomial

Fórmula interpoladora de Lagrange:

$$p_n(x) = \sum_{i=0}^n f_i l_i(x)$$

$$l_i(x) = \prod_{j=0, j \neq i}^n \left(\frac{x - x_j}{x_i - x_j} \right)$$

Fórmula interpoladora de Newton:

$$p_n(x) = f(x_0) + \sum_{i=1}^n f[x_0, \dots, x_i](x - x_0) \cdots (x - x_{i-1})$$

$$e_n(x) = f[x_0, \dots, x_n, x] \prod_{i=0}^n (x - x_i)$$

$$f[x_0, \dots, x_k] = \frac{f^{(k)}(\xi)}{k!} \quad \xi \in \text{int}(x_0, \dots, x_k)$$

2 Mínimos Quadrados

Equações normais:

$$\begin{bmatrix} (\phi_0, \phi_0) & \cdots & (\phi_0, \phi_m) \\ \vdots & \ddots & \vdots \\ (\phi_m, \phi_0) & \cdots & (\phi_m, \phi_m) \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} (\phi_0, f) \\ \vdots \\ (\phi_m, f) \end{bmatrix}$$

3 Integração Numérica

Regra dos trapézios:

$$\begin{aligned} T_N(f) &= h \left[(f_0 + f_N)/2 + \sum_{i=1}^{N-1} f_i \right], \\ T_1(f) &= T(f) = \left(\frac{b-a}{2} \right) [f(a) + f(b)] \end{aligned}$$

$$\begin{aligned} E_N^T(f) &= -\frac{h^3}{12} \sum_{i=1}^n f''(\xi_i) \\ &= -\frac{Nh^3}{12} f''(\xi) \quad \xi \in (a, b) \end{aligned}$$

Regra de Simpson:

$$\begin{aligned} S_N(f) &= \frac{h}{3} \left[(f_0 + f_N) + 4 \sum_{i=1}^{N/2} f_{2i-1} + 2 \sum_{i=1}^{N/2-1} f_{2i} \right], \\ S_2(f) &= S(f) = \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \end{aligned}$$

$$\begin{aligned} E_N^S(f) &= -\frac{h^5}{90} \sum_{i=1}^{n/2} f^{(4)}(\xi_i) \\ &= -\frac{Nh^5}{180} f^{(4)}(\xi) \quad \xi \in (a, b) \end{aligned}$$

4 Equações Diferenciais

$$y'(x) = f(x, y(x)) \quad y(x_0) = y_0$$

Método de Euler:

$$\begin{aligned} y_{n+1} &= y_n + h f(x_n, y_n) \\ T_{n+1} &= \frac{h^2}{2} y''(\xi_n) \quad \xi \in (x_n, x_{n+1}) \end{aligned}$$

$$\begin{aligned} |e_n| &= |y(x_n) - y_n| \leq \frac{hM}{2K} \left(e^{K(x_n-x_0)} - 1 \right). \\ M &= \max_{x \in [a,b]} |y''(x)|, \quad K = \max \left| \frac{\partial f(x,y)}{\partial y} \right| \end{aligned}$$

Método baseado na série de Taylor:

$$\begin{aligned} y_{n+1} &= y_n + hy'_n + \frac{h^2}{2} y''_n \\ &= y_n + h f(x_n, y_n) + \frac{h^2}{2} [f'_x(x_n, y_n) + f'_y(x_n, y_n) f(x_n, y_n)], \end{aligned}$$

Alguns Métodos de Runge-Kutta:

Método do ponto médio:

$$y_{n+1} = y_n + hf(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n))$$

Método de Heun:

$$\begin{aligned} y_{n+1} &= y_n + \\ &+ \frac{h}{2} [f(x_n, y_n) + f(x_{n+1}, y_n + hf(x_n, y_n))] \end{aligned}$$