Floating bodies subject to capillary attractions

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Abstract

In a 1992 paper, Raphael, diMeglio, Berger and Calabi showed that an infinite convex cylinder placed horizontally on the surface of an infinite liquid bath admits at least four orientations of its cross-section, such that for a prescribed contact angle γ, in zero gravity, it will float horizontally (i.e., be in energy equilibrium). For example, an elliptical cylinder can in general be made to float in exactly four orientations; it will however float in every orientation if the ellipse is a circle. The present work addresses the question as to the most general section for which the cylinder will float horizontally in every orientation. For the particular choice $\gamma = \pi/2$ it is shown that in addition to the circle there is a continuum of distinct such sections. For general contact angle γ, non-trivial sections are constructed such that the cylinder will float in a range π of distinct orientations. On the other hand, it is shown that if a compact three-dimensional body will float in every orientation in a horizontal bath with $\gamma = \pi/2$, then it must be a round metric ball. Several related theorems are also established. If a smooth convex body is endowed with a range of contact angles such that it will float in a horizontal bath following any vertical translation that leaves it partially immersed, then the body is rotationally symmetric about a vertical axis. If it has that property with respect to two distinct such axes, then it is a round ball.

Keywords: Capillarity, floating criteria, contact angle.

References

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