WAGP 2004 Workshop on Algebraic Geometry and Physics
Matrix Models and Algebraic Geometry
Lisbon 7-12 Sept. 2004

MINICOURSES

S. Katz, ADE geometry and dualities. In these lectures, I will explain the geometry of ADE surface singularities, their resolutions, and their deformations, interpret this geometry in gauge theory via quiver representations, and apply these results to $N = 1$ dualities.

Lecture 1. Geometry of ADE singularities

Lecture 2. ADE Quiver representations and branes

Lecture 3. Geometric transitions and dualities

References:

S. Katz, D. R. Morrison, *Gorenstein Threefold Singularities with Small Resolutions via Invariant Theory for Weyl Groups*, J. Alg. Geom. 1 (1992) 449-530, arXiv:alg-geom/9202002.

F. Cachazo, S. Katz, C. Vafa, *Geometric Transitions and N=1 Quiver Theories*, arXiv:hep-th/0108120.

F. Cachazo, B. Fiol, K. Intriligator, S. Katz, C. Vafa, *A Geometric Unification of Dualities*, Nucl.Phys. B628 (2002) 3-78, arXiv:hep-th/0108028.

M. Mariño, Topological strings on Calabi-Yau geometries, integrable hierarchies and matrix models. In this minicourse I provide an overview of the relations between Gromov-Witten theory on noncompact Calabi-Yau manifolds, integrable systems, and matrix models.

References:

M. Aganagic, A. Klemm, M. Mariño and C. Vafa, *Matrix model as a mirror of Chern-Simons theory*, JHEP 0402, 010 (2004) arXiv:hep-th/0211098.

M. Aganagic, A. Klemm, M. Mariño and C. Vafa, *The topological vertex*, arXiv:hep-th/0305132.

M. Aganagic, R. Dijkgraaf, A. Klemm, M. Mariño and C. Vafa, *Topological strings and integrable hierarchies*, arXiv:hep-th/0312085.

R. Pandharipande, Gromov-Witten theory in low dimensions. My lectures will cover topic in the study of Gromov-Witten theory in dimensions up to 3.

1. Kontsevich's combinatorial model for the GW theory of a point will be explained in the first lecture.
2. The second lecture will focus on the Toda hierarchy in dimension 1.
3. The GW/DT correspondence in dimension 3 will be discussed in the last lecture.

P. Van Moerbeke, Random permutations, random matrices and integrable systems. This mini-course will cover the following subjects:

1. Longest increasing sequences in random permutations and words
2. The spectrum of random matrices
3. Large random permutations and large random matrices: asymptotics

References:

M. Adler and P. van Moerbeke, *Hermitian, symmetric and symplectic random ensembles: PDE's for the distribution of the spectrum*, Annals of Mathematics, 153 (2001) 149-189; math-ph/0009001.

A. Borodin, A. Okounkov and G. Olshanski, *Asymptotics of Plancherel measures for symmetric groups*, J. Amer. Math. Soc. 13 (2000) 481–515; math.CO/9905032.

P. van Moerbeke, *Integrable lattices: random matrices and random permutations*, in Random matrices and their applications, Mathematical Sciences research Institute Publications #40, Cambridge University Press, pp. 321-406, (2001). (www.msri.org/publications/books/Book40/contents.html)

C.A. Tracy and H. Widom, *Level-Spacings distribution and the Airy kernel*, Comm. Math. Phys., 159 (1994) 151-174; hep-th/9211141.

SEMINARS

E. Aldrovandi, Hermitian-holomorphic 2-gerbes and tame symbols. We give a definition for a hermitian structure on a 2-gerbe with abelian band on a complex analytic manifold or algebraic variety. We also have the associated notion of type $(1,0)$ connective structure, with is the analog of the Griffiths or canonical connection for a line bundle with hermitian fiber metric. We show that hermitian 2-gerbes are classified in a suitable sense by Hermitian-holomorphic Deligne cohomology groups. Such groups refine the ordinary Deligne cohomology groups and have lifts of the Tame Symbol maps which, just as in K-theory and ordinary Deligne cohomology, are expressed as cup-products. One example is provided by the cup-product of two line bundles L and M with hermitian fiber metric: for an algebraic curve (or more generally for a family of curves) it agrees with the Deligne symbol $< L, M >$ for the determinant of cohomology. When both line bundles coincide with the tangent bundle equipped with a conformal metric, the symbol can be shown to agree with the Liouville action functional.

Reference:

E. Aldrovandi, *Hermitian-holomorphic (2)-Gerbes and tame symbols*, arXiv:math.CT/0310027.

B. Andreas, G-Flux and Calabi-Yau geometries. We discuss the requirement of G-flux in M-theory compactifications on Calabi-Yau fourfolds with ADE-singularities.

References:

E. Witten, *On flux quantization in M-theory and the effective action*, J.Geom.Phys. 22 (1997) 1-13, arXiv:hep-th/9609122.

B. Andreas, G. Curio, *On discrete twist and four-flux in N=1 heterotic/F-theory compactifications*, Adv. Theor. Math. Phys. 3 (1999) 1325-1413, arXiv:hep-th/9908193.

B. Acharya, S. Gukov, X. de la Ossa, *Supersymmetry, G-flux and Spin(7) manifolds*, JHEP 0209 (2002) 047, arXiv:hep-th/0201227.

C. Bartocci, Special Kaehler geometry of classical integrable systems. A classical integrable system over a symplectic manifold whose symplectic form is covariantly constant carries a natural hyper-symplectic structure. Moreover, a special Kaehler structure is induced on the base manifold.

References:

C. Bartocci, I. Mencattini, *Hyper-symplectic structures on integrable systems*, J. Geom. Phys. 50 (2004), 339-344, arXiv:math.DG/0308244.

D. Freed, *Special Kaehler manifolds*, Comm. Math. Phys. 203 (1999), 31-52, arXiv:hep-th/9712042.

A. Braverman, Instanton counting via affine Lie algebras. The main purpose of the talk is to explain how one can attack certain enumerative questions involving moduli spaces of G-bundles on the projective plane using the techniques developed in our joint paper with M.Finkelberg and D.Gaitsgory.

Reference:

A. Braverman, M. Finkelberg, D. Gaitsgory, *Uhlenbeck spaces via affine Lie algebras*, arXiv:math.AG/0301176.

R. Donagi, The Particle Spectrum of Heterotic Compactifications. We will start with the general setup for deriving both Grand Unified and Standard Model particle spectra from the geometry of a heterotic string (or M-theory) compactification. Determination of the particle content of a specific heterotic compactification boils down to calculation of the cohomology of various stable vector bundles over Calabi-Yau threefolds. We will outline some of the mathematical techniques involved in these calculations when the CY has either an elliptic fibration (leading to Grand Unified theories) or a genus one fibration (leading to Standard-Model-like theories). We will also discuss some of the physics applications of our results.

References:

R. Donagi, Y-H. He, B. Ovrut, R. Reinbacher, *The Particle Spectrum of Heterotic Compactifications*, arXiv:hep-th/0405014.

R. Donagi, Y-H. He, B. Ovrut, R. Reinbacher, *Moduli Dependent Spectra of Heterotic Compactifications*, arXiv:hep-th/0403291.

R. Donagi, B. Ovrut, T. Pantev, R. Reinbacher, *SU(4) Instantons on Calabi-Yau Threefolds with $\mathbb{Z}_2 \times \mathbb{Z}_2$ Fundamental Group*, arXiv:hep-th/0307273.

R. Donagi, T. Pantev, *Torus fibrations, gerbes, and duality*, arXiv:math.AG/0306213.

R. Donagi, B. Ovrut, T. Pantev, D. Waldram, *Spectral involutions on rational elliptic surfaces*, arXiv:math.AG/0008011.

R. Donagi, B. Ovrut, T. Pantev, D. Waldram, *Standard-model bundles*, arXiv:math.AG/0008010.

B. Dubrovin, Frobenius manifolds and matrix models. We will explain, following the recent joint work with T.Grava, how to apply Frobenius manifolds to computing the partition function in the Hermitean matrix model.

F. Fucito, Non perturbative computations in supersymmetric theories. I report on various computations in extended supersymmetric theories. We compute the partition function for N=2,4 theories and evaluate the Poincare polynomials for the relevant moduli space. We also discuss the case of ALE manifolds.

References:

R. Flume, F. Fucito, J. F. Morales and R. Poghossian, *Matone's relation in the presence of gravitational couplings*, JHEP 0404 (2004) 008; arXiv:hep-th/0403057.

U. Bruzzo and F. Fucito, *Superlocalization formulas and supersymmetric Yang-Mills theories*, Nucl. Phys. B 678 (2004) 638; arXiv:math-ph/0310036.

U. Bruzzo, F. Fucito, J. F. Morales and A. Tanzini, *Multi-instanton calculus and equivariant cohomology*, JHEP 0305 (2003) 054; arXiv:hep-th/0211108.

L. Goettsche, Instanton counting and Donaldson invariants. This is joint work with Nakajima and Yoshioka. Nekrasovs partition function can be viewed as the generating function for the Donaldson invariants of the affine plane $\mathbb{A}^2$. Our aim is to determine the Donaldson invariants of a compact smooth toric surface in terms of the Nekrasov partition function. We have carried this out in the case of rank 2. Using the Nekrasov conjecture, proven by Nekrasov-Okounkov and Nakajima-Yoshioka, this determines the Donaldson invariants of toric surfaces completely for rank 2. I will try to also mention work in progress on higher rank as well as on generalizations of Donaldson invariants.

References:

H. Nakajima, K. Yoshioka, *Instanton counting on blowup*, math.AG/0306198.

H. Nakajima, K. Yoshioka, *Lectures on Instanton Counting*, math.AG/0311058.

N. Nekrasov, A. Okounkov, *Seiberg-Witten Theory and Random Partitions*, hep-th/0306238.

D. Grunberg, Gromow-Witten potentials and automorphic forms. The GW potential is a free energy in string theory; its associated partition function can be written as an infinite product. We present examples of genus 0 potentials where such products have automorphic properties and try to motivate the search for similar properties in the general case.

References:

J.A. Harvey and G. Moore, *Algebras, BPS states, and strings*, Nucl. Phys. B463 (1996) 315-368; arXiv:hep-th/9510182.

R.E. Borcherds, *Automorphic Forms on* $O_{s+2,2}(\mathbb{R})$ *and Infinite Products*, Invent. Math. 120 (1995) 161-213

B. Pioline , A. Waldron, *Automorphic forms: a physicist's survey*, arXiv:hep-th/0312068.

My own work - in progress. or see my phd thesis Thesis.ps

B. Kreussler, Semi-stable sheaves on nodal cubics. Motivated by Kontsevich's homological mirror symmetry conjecture, in recent years, derived categories of coherent sheaves on smooth projective varieties and their groups of auto-equivalences attracted a lot of interest. In this talk, which presents joint work with Igor Burban, we study the first non-trivial singular example: nodal cubic curves. We use techniques of P. Seidel and R. Thomas to construct a Fourier-Mukai transform on such a curve. We apply this functor, which is an equivalence, to give an explicit description of all indecomposable semi-stable torsion free sheaves of degree zero on irreducible nodal cubic curves. For such sheaves, we are able to calculate the Fourier-Mukai transform explicitly. We apply results on the classification of modules of finite length over a nodal ring, which were obtained by Gelfand and Ponomarev in the 1960s.

References:

I.Burban, B.Kreussler, *Fourier-Mukai transforms and semi-stable sheaves on nodal Weierstrass cubics*, arXiv:math.AG/0401437.

Seidel, P.; Thomas, R. *Braid group actions on derived categories of coherent sheaves*, Duke Math. J. 108 (2001) 37–108, arXiv:math.AG/0001043.

Gelfand, I.M.; Ponomarev, V.A. *Indecomposable representations of the Lorentz group*, Russ. Math. Surv. 23 (1968) 1–58; translation from Usp. Mat. Nauk 23 (1968) no. 2 (140), 3–60.

A. Morozov, Matrix models partition functions and string theory.

M. Mulase, Symplectic geometry of the moduli space of pointed algebraic curves. This talk will be a survey of a recent brilliant work by Maryam Mirzakhani (Harvard) on the symplectic volume of the moduli spaces of bordered hyperbolic surfaces and a new purely geometric proof of the Witten-Kontsevich theory.

N. Nekrasov, Chasing topological M-theory. We review non-perturbative dualities in topological string theories and contemplate on their explanation.

T. Pantev, Heterotic compactifications with fluxes. I will describe the complex geometry underlying a compactification of both weakly and strongly coupled Heterotic theory in the presence of a background flux. I will discuss the classification of such geometries with an emphasis on the gauge fields and anomaly cancellation. I will describe explicit constructions of heterotic compactifications in the presence of fluxes as well as a no-go theorem establishing the sharpness of the existence results.

References:

K. Becker, M. Becker, K. Dasgupta, P. Green, *Compactifications of Heterotic Theory on Non-Kahler Complex Manifolds: I*, JHEP 0304 (2003) 007, arXiv:hep-th/0301161.

M. Lubke and A. Teleman, *The Kobayashi–Hitchin correspondence*, World Scientific, River Edge, NJ, 1995.

R. Donagi and T. Pantev, *Torus fibrations, gerbes, and duality*, arXiv:math.AG/0306213.

C. Pedrini, Finite dimensional motives and the conjectures of Bloch and Beilinson. We will relate the notion of finite dimensionality in the triangulated category $DM(k)$ of motivic complexes over a field k, constructed by Voevodsky, with the Conjectures by Beilinson and Bloch on the existence of a finite filtration on the Chow groups of smooth projective varieties. According to a recent result announced by J.Ayoub all objects in $DM(k)$ are Schur-finite dimensional. This result in particular implies the proof of a long-standing Conjecture in Algebraic Geometry: Bloch's Conjecture about the vanishing of the Albanese Kernel for complex surfaces of general type with geometric genus= 0.

R. Reinbacher, Moduli dependent spectra of heterotic compactifications. Explicit methods are presented for computing the cohomology of stable, holomorphic vector bundles on elliptically fibered Calabi-Yau threefolds. The complete particle spectrum of the low-energy, four-dimensional theory is specified by the dimensions of specific cohomology groups. The spectrum is shown to depend on the choice of vector bundle moduli, jumping up from a generic minimal result to attain many higher values on subspaces of co-dimension one or higher in the moduli space. An explicit example is presented within the context of a heterotic vacuum corresponding to an SU(5) GUT in four-dimensions.

Reference:

Ron Donagi, Yang-Hui He, Burt A. Ovrut, Rene Reinbacher, Moduli Dependent Spectra of Heterotic Compactifications, hep-th/0403291.

E. Scheidegger, Higher genus topological string amplitudes. We will show how to compute the higher genus instanton numbers in the topological string with a compact Calabi-Yau target space. We will explain the different methods involved and some of their relations to mathematics. In particular, we show that for K3 fibrations there exist generating functions for the Gopakumar-Vafa invariants which are automorphic forms. The knowledge of these automorphic forms at genus zero yields severe constraints for higher genus. However, additional information coming from the embedding of the Picard lattice of K3 into the one of the Calabi-Yau threefold plays a role.

References:

M. Bershadsky, S. Cecotti, H. Ooguri, C. Vafa, *Kodaira-Spencer Theory of Gravity and Exact Results for Quantum String Amplitudes*, Commun.Math.Phys. 165 (1994) 311-428, hep-th/9309140.

R. Gopakumar, C. Vafa, *M-Theory and Topological Strings - II*, hep-th/9812127.

M. Marino, G. Moore, *Counting higher genus curves in a Calabi-Yau manifold*, Nucl.Phys. B543 (1999) 592-614, hep-th/9808131.

S. Katz, A. Klemm, C. Vafa, *M-Theory, Topological Strings and Spinning Black Holes*, Adv. Theor. Math. Phys. 3 (1999) 1445-1537, hep-th/9910181.

A. Klemm, M. Kreuzer, E. Riegler, E. Scheidegger, to appear.

S. Shadrin, Getzler's relation in dGBV-algebras. There is a construction of solution to the WDVV equation starting from a dGBV algebra (Barannikov-Kontsevich, IMRN 1998). We have found a natural genus expansion of their construction and proved that Getzler's elliptic relation is valid in this case. This is a joint work with Andrei Losev, it should appear in arXiv soon.