Chain Sequences and Location of Continuous Spectrum in Self-Adjoint Difference Operators

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A closed solution in product form for the self-adjoint second order difference equation, $x(n+1) = b_n(\epsilon)x(n) - a_n x(n-1)$, relates oscillation properties [1] of the solution with Chain Sequences [2]. The solution $\{x(n)\}_{n=n_0}^{\infty}$ is non oscillatory if and only if the sequence $\{\alpha(n+1) = \frac{a_{n+1}}{b_n(\epsilon)b_{n+1}(\epsilon)}\}$, $n > n_0$, is a chain sequence.
Here, the difference operator associated to the previous equation has ϵ as principal parameter, an example is the energy in difference Schrödinger operators. Applying the previous result in case of linear dependency, conditions are obtained for the rank of values of ϵ where the solution can generate continuous spectrum. The equations of Harper and Fibonacci illustrate the results with numerical examples. The achievement of similar conditions seems admissible in other cases of dependency.

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Predictor-Corrector Method for Fuzzy Initial Value Problem

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In this paper three numerical methods to solve " fuzzy initial value problem" are discussed. These methods are explicit three-step, implicit two-step and Predictor-Corrector. Predictor-Corrector is obtained by combining explicit three-step method and implicit two-step methods. Convergence and stability of the proposed methods are also proved in detail. In addition, these methods are illustrated by solving a Fuzzy Cauchy Problem.

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A Generalized Upper and Lower Method for Singular Boundary Value Problems for Quasilinear Dynamic Equations

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We obtain some existence results of a singular boundary value problem for quasilinear dynamic equations on time scales. In particular, our nonlinearity may be singular in its dependent variable and is allowed to change sign.

Newton's Problem of Minimal Resistance in the Class of Solids of Revolution

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As is well known, Newton found the solution for the problem of the body of minimal resistance in the class of convex solids of revolution. We consider this problem in the wider class of (generally nonconvex) solids of revolution. It appears that omitting the convexity assumption allows one to find bodies of smaller resistance than Newton's body of minimal resistance. Here we present the solution for the problem and construct the minimizing sequence of bodies.

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A Harmonic Relation Between Second-Order Linear Difference Equations and Tridiagonal Matrices with Applications

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We show that second-order linear difference equations and tridiagonal matrices are closely related. We also present the mutual service each one of them renders to the other. We demonstrate this harmony by applications, especially in the representation of orthogonal polynomials.

Golden Tilings of the Real Line

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Pinto-Sullivan developed a theory relating Pinto-Sullivan tilings of the real line with smooth conjugacy classes of doubling expanding circle maps.
Here we present the definition of golden sequences $\{r_i\}_{i\in\mathbb{N}}$. These golden sequences, in particular, have the propriety of being Fibonacci quasi-periodic and determine a tilling of the real line. We prove a one-to-one correspondence between:
(i) golden sequences;
(ii) Smooth conjugacy classes of Anosov diffeomorphisms in the topological conjugacy class of the toral automorphism $T(x, y) = (x + y, x)$;
(iii) Smooth conjugacy classes of diffeomorphisms of the circle with rotation number equal to the inverse of the golden number and that are smooth fixed points of the renormalization operator.

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Attractors for Unimodal Quasiperiodically Forced Maps

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We consider unimodal quasiperiodically forced maps, that is, skew products with irrational rotations of the circle in the base and unimodal interval maps in the fibers. The map in the fiber over x is a unimodal map f of the interval $[0, 1]$ onto itself multiplied by $g(x)$, where g is a continuous function from the circle to $[0, 1]$. The case when g does not take the value 0 has been extensively studied by various authors. Here we consider a more difficult case, the "pinched" one, when g attains value 0. This case is similar to the one considered by Gerhard Keller, except that the function f in his case is increasing. Since in our case f is unimodal, the basic tools from the Keller's paper do not work.
We prove that under some additional assumptions on the system there exists a strange nonchaotic attractor. It is a graph of a nontrivial function from the circle to $[0, 1]$, which attracts almost all trajectories. Both Lyapunov exponents on this attractor are nonpositive. There are also cases when the dynamics is completely different, because one can apply the results of Jerome Buzzi implying the existence of an invariant measure absolutely continuous with respect to the Lebesgue measure. Finally, there are cases when we can only guess what the behavior is by performing computer experiments.

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Computing the Periods of a Periodic Nonautonomous Difference Equation

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Consider the nonautonomous difference equation

$$x_{n+1} = f_n(x_n), \tag{1}$$

where $\{f_n : [a,b] \to [a,b]\}_{n\geq 0}$ is a sequence of continuous interval maps. As usual we say that a positive integer, q, is a period of (1) if there exists a q-periodic solution of (1). Recall that the nonautonomous equation (1) is called p-periodic if $p \geq 2$ is the smallest positive integer satisfying $f_{n+p} = f_n$ for all $n \geq 0$.

This talk concerns to the set of periods of (1) when this equation is p-periodic. Our main theorem establishes that, if the equation is p-periodic, and the sets

$$\{x \in [a,b] : f_i(x) = f_j(x)\} \tag{2}$$

are finite for all $i \neq j \pmod p$, then it is possible to compute explicitly all periods of the equation which are not a multiple of p.
Combining this result with a recent extension of Sharkovsky's theorem for periodic difference equations [1], we are able to describe the set of periods of any equation satisfying the generic condition (2).

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q-Hypergeometric Functions and Holonomic q-Difference Equations

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q-hypergeometric functions are q-analog of hypergeometric functions and defined by Jackson integrals of a q-multiplicative function. The associated cohomology is of finite dimension. As a result of it they sastisfy holonomic q-difference equations with respect to the parameters involved. There arise several basic problems such as (1) to formulate holonomic q-difference equations (type of regular or irregular singularities) and find them explicitly, (2) to find fundamental solutions having specific asymptptic behaviours, (3) to find connection relations among the fundamental solutions. In this talk I shall discuss, in case of one dimensional BC-type root system , how the above problems can be formulated and can be answered. As a corollary I shall give some applications to Askey-Wilson type hypergeometric functions.

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Numerical Detection of Finite-Time Explosions in Stochastic Differential Equations

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This paper studies the forward Euler discretisation of a scalar nonlinear stochastic differential equation whose solutions explode in finite time. The discrete time model mimics the explosion under an appropriate discretisation, and also distinguishes between solutions which tend to infinity without exploding, and exploding solutions. The paper is co-authored with Alexandra Rodkina and Cónall Kelly (UWI, Kingston).

Linear Response in the Absence of Structural Stability

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(Joint work with D. Smania.)
D. Ruelle proved 10 years ago that the average of a smooth observable with respect to the physical (or SRB) measure μ_t of a smooth one-parameter family f_t of smooth hyperbolic attractors depends differentiably on t, and he gave a formula for the derivative: the linear response formula. He conjectured that linear response should hold much more generally. We prove that if f_t is tangent to the topological class of a piecewise expanding interval map f_0, then μ_t is differentiable at zero, and the derivative coincides with a resummation of the (a priori divergent) series given by Ruelle's conjecture. It is the first time that a linear response formula is obtained in a setting where structural stability does not hold. The proof exploits the spectral gap of the transfer operators.

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Origin, Motivation and Results on two Scalar Non-Linear Difference Equations: Thue-Morse and Fibonacci

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Some types of 1D quasicrystals are described appealing to the sequences named after Fibonacci, Thue-Morse or Rudin-Shapiro; that is, structures which are disordered in an interesting, non-trivial way, for example by aperiodic (quasiperiodic) sequences.
In such setting we want to know whether a quasicrystal generated by any of the former sequences behaves as a conductor or as an insulator. After some mathematical preparation, it can be done taking $(|r_n|)_{n=0}^{\infty}$. When $|r_n|_{n\to\infty} \to 1$ we have the insulator case and if $|r_n|_{n\to\infty} \nrightarrow 1$ we have different degrees of conductor.

Magnitudes r_n for $n \geq 0$ are related to some matrix traces x_n which hold the following scalar non-linear difference equations

$$x_{n+2} = x_n^2(x_{n+1} - 2) + 2, \quad n \geq 0$$
$$x_{n+3} = x_{n+1}x_{n+2} - x_n, \quad n \geq 0$$

named respectively, as Thue-Morse and Fibonacci equations.

Unfolding them we obtain two discrete dynamical systems on dimension two and three, which allow to understand the asymptotic of the original equations. Results on boundeness of trajectories, periodicities, invariant curves, transitivity or stability of fixed and periodic points are considered.

Difference Equations on Matrix Algebras

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We study non-linear difference equations $x_{n+1} = f(x_n)$, where x_n belongs to a certain matrix algebra A, and f is a polynomial map defined on A. We are interested in analyzing the type of periodic orbits and their stability. We also will discuss the dependence of the dynamical behaviour on parameters in different situations, since we can consider the parameters to be also in the algebra A instead of just being scalars, or in some sub-algebra of A. We study the concrete cases when f is a quadratic map and A is $M_2(\mathbb{R})$, or some sub-algebra of $M_2(\mathbb{R})$.

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Topological Invariants for Lozi Maps

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From the kneading theory for Lozi mappings, presented by Ishii, we study its topological invariants.

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Nonoscillation and Stability for Linear Difference Equations with Several Delays

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We study the existence of positive solutions and positiveness of the fundamental function for a scalar linear difference equation with several delays:

$$x(n+1) - x(n) = -\sum_{l=1}^{m} a_l(n)x(h_l(n)), \;\; h_l(n) \le n, \;\; n > n_0,$$

where a_l can be either positive or negative.
Nonoscillation conditions are obtained, explicit tests are presented. In particular, the following result holds.
If $a_l(n) \ge b_l(n) \ge 0$, $h_l(n) \le g_l(n) \le n$, $\lim_{n\to\infty} h_l(n) = \infty, \; l = 1, 2, \dots, m$, *and*

$$\sum_{l=1}^{m} \sum_{k=h_l(n)}^{n-1} (a_l(k) - 0.5b_l(k)) \le \frac{1}{4}, \;\; n \ge n_0,$$

then the fundamental function $X(n,k)$ *of the equation*

$$x(n+1) - x(n) = -\sum_{l=1}^{m} [a_l(n)x(h_l(n)) - b_l(n)x(g_l(n))]$$

is positive for $n \ge k \ge n_0$.
We discuss also connection between positiveness and exponential stability for linear delay difference equations.

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Zero-Entropy Vertex Maps on Graphs

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Let G be a finite connected graph with v vertices. Vertex maps are maps $f : G \to G$ that are locally monotonic on edges and such that the vertices form a periodic orbit of period v. In this talk we will classify all zero-entropy vertex maps.

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Asymptotic Integration under Weak Dichotomies

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We are interested in the asymptotic integration of systems of linear difference equations of the form

$$y(n+1) = [\Lambda(n) + R(n)]\; y(n)\,, \qquad n \geq n_0, \tag{1}$$

where $\Lambda(n)$ is a diagonal matrix and $R(n)$ is a perturbation.
A classical result, first formulated by Levinson for differential equations and later by Benzaid and Lutz for difference equations, assumes that $R(n)$ is an absolutely summable perturbation and that $\Lambda(n)$ satisfies what has become known as "Levinson's dichotomy condition". It was then shown that (1) has a fundamental matrix of the form

$$Y(n) = [I + \mathrm{o}(1)] \prod_{k=n_0}^{n-1} \Lambda(k) \qquad \text{as } n \to \infty. \tag{2}$$

In this talk we do not assume that $\Lambda(n)$ in (1) satisfies Levinson's dichotomy condition. We show that (1) still has a fundamental matrix of the form (2) provided that the perturbation $R(n)$ is sufficiently small.
We will present results for linear systems of difference equations and indicate analogous results for linear systems of differential equations.
This is joint work with D.A. Lutz from San Diego State University.

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Asymptotic Integration of Dynamic Equations

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The purpose of this talk is to study the existence and asymptotic behavior of solutions to a class of second-order nonlinear dynamic equations on unbounded time scales. Four different results are obtained by using the Banach fixed point theorem, the Boyd and Wong fixed point theorem, the Schauder fixed point theorem, and the Leray–Schauder nonlinear alternative.

On Cushing Henson Conjecture and Attenuant Cycles

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The second part of the Cushing-Henson conjecture [1] claims that the cycle's average (once a periodic solution exists) is less than the average of carrying capacities. For the Beverton Holt equation and a nonconstant 2-cycle it was confirmed by Cushing and Henson [1], for any cycle by Elaydi, Sacker [2] and Kocic [3]. For the difference equation $x_{n+1} = g\left(\frac{x_n}{K_n}\right) x_n$, including Beverton Holt equation and some other models of population dynamics as special cases, Kon [4] deduced that for a concave g any cycle's average is less than the average of carrying capacities (a cycle is attenuant). We study the delay equation

$$x_{n+1} = f\left(K_n, x_{n-h_1(n)}, \dots, x_{n-h_r(n)}\right)$$

and present sufficient conditions on f and h_i, when the Cushing-Henson conjecture is valid. We demonstrate sharpness of these conditions by presenting several counterexamples. This is joint work with S.H. Saker.

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Jacobi Polynomials ($\alpha = \beta = -1$), their Sobolev Orthogonality, and Self-Adjoint Operators

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It is well known that, for $-\alpha, -\beta, -\alpha-\beta-1 \notin \mathbb{N}$, the Jacobi polynomials $\left\{P_n^{(\alpha,\beta)}\right\}_{n=0}^{\infty}$ are orthogonal on $\mathbb{R}$ with respect to a bilinear form of the type

$$(f, g)_\mu = \int_{\mathbb{R}} f\overline{g}d\mu,$$

for some measure μ [1]. However, for $\alpha = \beta = -1$, from Favard's theorem, the Jacobi polynomials cannot be orthogonal with respect to a bilinear form of this type for any measure. Are they orthogonal with respect to some "natural" inner product? Indeed, they are orthogonal with respect to a Sobolev inner product [3]. We discuss this Sobolev orthogonality and construct a self-adjoint operator in this Sobolev space that has the Jacobi polynomials as a complete set of eigenfunctions.

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Existence of Stable and Unstable Periodic Solutions of Non Autonomous First Order Difference Equations

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In this talk we give sufficient conditions to ensure the existence of at least one, two or three periodic solutions of first order difference equations. Moreover we study the stability character of such solutions. We assume the existence of lower and upper solutions that can be well ordered, to appear in reversed order or do not have no order between them. The sign of the related Green's functions coupled with the growth and/or the *a priori* bounds of the nonlinear part of the equation are the main hypotheses of the given results. Some of them are deduced from degree theory.

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The Limit of Linear Elliptic Problems with Variable Operators and Varying Boundary Conditions

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For a fixed domain $\Omega \subset \mathbb{R}^N$ and an arbitrary sequence $\Gamma_n \subset \partial\Omega$, the aim of this work is to study the asymptotic behavior of the solutions u_n of the problems

$$\begin{cases} -div\, A_n(x)\nabla u_n = g_n & \text{in } \mathcal{D}'(\Omega) \\ u_n = 0 & \text{on } \Gamma_n \\ A_n(x)\nabla u_n\, \nu = 0 & \text{on } \partial\Omega \setminus \Gamma_n, \end{cases}$$

where $A_n \in L^\infty(\Omega)^{N\times N}$ is a sequence of matrices which define linear operators uniformly bounded and elliptic and ν denotes the unitary outward normal to $\partial\Omega$. The sequence g_n is assumed to converge in $L^{p'}(\Omega)$ weakly to a function g.
Assuming that there exists a constant $C_P > 0$ independent of n such that the following Poincaré inequality holds $\|u_n\|_{L^2(\Omega)} \leq C_P\|\nabla u\|_{L^2(\Omega)^N}, \quad \forall u \in H^1(\Omega), \quad u = 0 \text{ on } \Gamma_n$, we obtain a representation of the limit problem which is stable by homogenization and where it appears a generalized Fourier boundary condition.

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Periodically Forced Rational Equations

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We study the boundedness nature and the periodic character of solutions, of non-autonomous rational difference equations including Pielou's equation.

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Chaotic Synchronization of Unimodal and Bimodal Maps

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We derive a threshold value for the coupling strength in terms of the growth number, to achieve synchronization of two coupled piecewise linear m-modal maps, with m=1 and m=2, for the unidirectional and for the bidirectional coupling. This gives us information about the synchronization of unimodal and bimodal maps. An application to the coupling of two identical chaotic Duffing equations will be given.

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Calculus for Sequences Arising from Seeking Analytic Solutions of Functional Equations

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Analytic functions can be regarded as functions generated by sequences. When seeking analytic solutions of functional equations, we are thus led to equations involving unknown sequences. These equations and their solutions may or may not be easy to obtain if we apply the standard method of undetermined coefficients. We may, however, establish a 'symbolic calculus' for sequences that may help us to derive the desired equations and solutions. In terms of the symbolic operations, the resulting equations involving unknown sequences take abstract forms and their qualitative and quantitative properties can be investigated by further operational means. This theory is not fully developed yet but we have already obtained many interesting consequences including an elementary proof of the Formula of Faa di Bruno for composite functions.

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Periodic Solution of Matrix Difference Nonlinear Equations with Bilinear Main Part

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In our paper we consider the difference matrix equation. It can appear after discretization of Riccaty type differential matrix equation and can be written as

$$X_{n+1} = A_n X_n - X_n B_n + F_n(X_n), \tag{1}$$

where $F_{n+p}(X_{n+p}) = F_n(X_n)$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times m}$, $X_n, F_n \in \mathbb{R}^{n\times m}$, $\mathbb{R}^{n\times m}$ is the space of $n \times m$ matrix-valued functions [1]. We consider the problem of existence of periodic solution which satisfy the condition $X_{n+p} = X_n$, where p is integer number, where $Z \in \mathbb{R}^{n\times m}$ is arbitrary matrix. The equation (1) can be transformed into equivalent equation which is analog of integral equation $X_n = \sum_{k=0}^{p} [G_k^n] F_k(X_n)$, where $[G_k^n]$ is discrete analog of Green's operator function [3]. It can be rewritten in the form [2]

$$X_n = X_0 + \sum_{k=1}^{n} [U_k^n](F_k(X_n) - \widehat{F}_n(X_n)). \tag{2}$$

From the condition of periodicity we obtain $\sum_{k=1}^{p} [U_k^p] F_k(X_n) = \sum_{k=1}^{p} [U_k^p] \widehat{F}_n(X_n)$, where $\widehat{F}_n(X_n) = (\sum_{k=1}^{p} [U_k^p])^{-1} \sum_{k=1}^{p} [U_k^p] F_k(X_k)$. The solution of equation (1) can be obtained by the Chezary's method [1]. The solution of the last equation can be obtained by applying the method of successive approximation, it is a limit of sequence of periodic matrix functions $\Psi_n(X_0) = \lim_{s\to\infty} X_n^{(s)}(X_0)$, where $X_n^{(s+1)}$ is determined by equality $X_n^{(s+1)} = X_0 + \sum_{k=1}^{n} [U_k^n](F_k^{(s)} - \widehat{F}_n^{(s)})$, where $F_k^{(s)} = F_k(X_k^{(s)})$, $\widehat{F}_n^{(s)} = \widehat{F}_n(X_n^{(s)})$. The exact solution $\Psi_n(X_0)$ should satisfy the determining equation $AX_0 - X_0 B - X_0 + \widehat{F}_n(X_0) = 0$, where $\widehat{F}_n(X_0) = \widehat{F}_n(\Psi_n(X_0))$.

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On the Iteration of Smooth Maps

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Iteration of smooth maps appears naturally in the study of continuous difference equations and boundary value problems, see [1]. Moreover, it is a subject that may be studied by its own interest, generalizing the iteration theory for interval maps. Our study is motivated by the works [1], [2], [3], namely those concerning ideal turbulence, and by the works [4], [5].
We study families of discrete dynamical systems of the type (Ω, f), where Ω is some class of smooth functions, *e.g.*, a sub-class of $C^r(I, J)$, where I, J are intervals, and f is a smooth map $f : J \to J$. The action is given by $\varphi \mapsto f \circ \varphi$. We analyze in particular the cases when f is a family of quadratic or cubic maps. For these families we analyze the topological behaviour of the system, and the parameter dependence of the spectral decomposition of the iterates.

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Planar Fronts in Bistable Coupled Map Lattices

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Planar fronts in multidimensional coupled map lattices can be studied by reduction to an one-dimensional extended dynamical system that generalises one-dimensional coupled map lattices. This methodology is fully investigated and developed. Continuity of fronts velocity with the coupling strength and with the propagation direction is proven. Examples are provided and illustrated by some numerical pictures.

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Difference Equation Models of Biological Semelparity

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I will give a bifurcation analysis of a class of discrete, nonlinear dynamic (matrix) models that have a non-generic bifurcation from a trivial equilibrium. In population dynamics this bifurcation occurs in models of semelparous populations as the inherent reproductive number increases through the critical value of one (and the population passes from asymptotic extinction to persistence). For models of dimension two, a complete local analysis of this bifurcation is known [2]. Although higher dimensional models are of significant importance in applications (e.g., see [1]), their analysis is mathematically more difficult and incomplete. In this talk, I will give a complete local bifurcation analysis of the three dimensional case for a general class of nonlinearities. The analysis involves bifurcation theory, perturbation methods, monotone maps [5], and average Liapunov functions [3,4].

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Nonautonomous Beverton-Holt Equations and the Cushing-Henson Conjectures

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"Difference equations" is relatively a new, but a hot, field because of its many applications in biology, chemistry, economics, dynamical systems, life and social sciences, and optimization.
Autonomous Beverton-Holt equation is given by

$$x(n+1) = \frac{rKx(n)}{K + (r-1)x(n)}, \qquad K > 0, r > 0.$$

This equation has been used to model populations of bottom-feeding fish, including the North Atlantic plaice and haddock. These species have very high fertility rates and very low survivorship to adulthood. Furthermore, recruitment is essentially unaffected by fishing.
In 2002 Cushing and Henson proposed two conjectures for the global asymptotic stability of the periodic solution of the previous equation.
During 2003-2006, several authors proved Cushing and Henson conjectures for different types of difference equations [1 , 2 , 3].
In this talk, we prove Cushing and Henson conjectures for the following nonautonomous Beverton-Holt equation

$$x(n+1) = \frac{\mu(n)K(n)x(n)}{K(n) + (\mu(n)-1)x(n)}, \qquad K(n) > 0, \mu(n) > 0.$$

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Sierpinski Galore

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In this talk we describe a number of different ways that Sierpinski curves arise in the complex dynamics of singularly perturbed rational maps, i.e., as Julia sets for maps of the form $z^n + \lambda/z^d$. We shall show that certain of these Julia sets converge to the unit disk as $\lambda \to 0$. We will also investigate singular perturbations of other quadratic polynomials, and show that many of these come "wrapped" in Sierpinski carpets.

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Stability of the Zero Solution of Difference Systems with Quadratic Right-hand Sides in the Critical Case

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Many important processes (e.g. in industry and biology) are mathematically simulated by systems of discrete equations with quadratic right-hand sides. Such systems can be written in a general matrix form

$$x(k+1) = Ax(k) + X^T(k)Bx(k), \quad k = 0, 1, 2, \dots,$$

where $x = (x_1, x_2, ..., x_n)^T$, A is an $n \times n$ constant square matrix, X^T is an $n \times n^2$ matrix, B is an $n^2 \times n$ constant matrix, $X^T = (X_1^T, X_2^T, ...X_n^T)$, $B^T = (B_1, B_2, ...B_n)$, elements of $n \times n$ matrices X_i, $i = 1, \dots, n$ are equal to zero except the i-th line that equals x^T and matrices B_i, $i = 1, \dots, n$ are $n \times n$ constant and symmetric. We deal with stability results of quadratic discrete systems in the critical case (in the presence of an eigenvalue $\lambda = 1$ of the matrix of linear terms, all remaining eigenvalues lie inside the unit circle). In addition to the stability investigation of a zero solution, we also estimate stability domains. These are defined by means of two second-order polynomial inequalities and, geometrically, they are described as the interior of the intersection of two ellipses. (This research was supported by the Grant 201/07/0145 of Czech Grant Agency (Prague) and by the Council of Czech Government MSM 0021630503.)

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An Extension of the Casoratian for Half-Linear Difference Equations

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This is a joint work with Mariella Cecchi and Mauro Marini from University of Florence, Italy.

In this talk we study asymptotic properties of nonoscillatory solutions of half-linear difference equations

$$\Delta(a_n\Phi(\Delta x_n)) + b_n\Phi(x_{n+1}) = 0 \tag{1}$$

where Δ is the forward difference operator $\Delta x_n = x_{n+1} - x_n$, $\Phi(u) = |u|^{p-2}u$ with $p > 1$ and $a = \{a_n\}, b = \{b_n\}$ are positive real sequences for $n \geq 1$. An extension of the casoratian for (1) and a complete analysis of possible types of nonoscillatory solutions will be given. The limit and integral characterizations of recessive and dominant solutions for (1) will be discussed as well.

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Linearized Riccati Technique and (Non) Oscillation Criteria for Half-Linear Difference Equations

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We consider the half-linear second order difference equation

$$\Delta(r_k\Phi(\Delta x_k)) + c_k\Phi(x_{k+1}) = 0, \quad \Phi(x) := |x|^{p-2}x, \ p > 1, \tag{1}$$

where r, c are real-valued sequences. We associate with (1) a linear second order difference equation and we show that oscillatory properties of (1) can be investigated using properties of this associated linear equation. The main tool we use is a linearization technique applied to a certain Riccati type difference equation corresponding to (1). The presented results were achieved in the joint research with S. Fišnarová, Mendel University, Brno.

A Discrete Competition Model with Periodically Oscillating Habitat Size

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We consider a discrete-time model of two competing species with a periodically oscillating habitat size. In a corresponding single-species model, varying habitat size has produced some interesting results which were then verified via laboratory experiments with flour beetles, such as an increase in overall average population size. In a corresponding competition model with constant habitat size, numerous results contradictory to the classical principle of competitive exclusion have been observed, most notably the existence of coexistence states that arise from increased inter-specific competition. The addition of oscillating habitat size to this model appears to strongly promote coexistence of two species even when in heavy competition with one another.

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Center Manifold, Bifurcation and Stability of two-dimensional Systems

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This talk is a tutorial and will cover some essential tools in difference equations and discrete dynamical systems. Among topics covered are center manifolds, a reduction principle, bifurcations, stability, and the trace-determinant analysis in two dimensions.

The Comparative Index and Riccati Difference Equations

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We investigate solution's properties of generalized Riccati difference equations associated with symplectic difference systems $Y_{i+1} = W_i Y_i$, $W_i^T J W_i = J$. The consideration is based on the concept of the comparative index introduced in [1].

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Homoclinic Solutions for 2nth Order Self-Adjoint Difference Equations

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We will show how to use the Mountain Pass Theorem to obtain nontrivial homoclinic solutions to the 2nth order, n $\in \mathbb{N}$ (formally) self-adjoint nonlinear difference equation:

$$\sum_{i=0}^{n} \Delta^i [r_i(t) \Delta^i u(t-i)] = f(t, u(t)), \quad t \in \mathbb{Z}.$$

No periodicity assumptions will be placed on $r_i, i = 0, 1, ..., n$ or f and it will be assumed that f grows superlinearly both at the origin and at infinity.

Conductance in Discrete Dynamical Systems

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We've introduced the notion of conductance in discrete dynamical systems using the known results from graph theory applied to systems arising from the iteration of continuous functions. The conductance allowed differentiating several systems with the same topological entropy, characterizing them from the point of view of the ability of the system to go out from a small subset of the state space. There are several other definitions of conductance and the results differ from one to another. Our goal is to understand the meaning of each one concerning the dynamical behaviour. Our results are supported by computational techniques using symbolic dynamics, and the tree-structure of the unimodal and bimodal maps.

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Stability through Migration

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As a sort of rescue of instability of tatonnement process in general equilibrium discovered by Scarf, and also as a non-tatonnement process, we present a generic spatial model of migration, and establish global stability (strong ergodicity). Our model is of a nonlinear Markov-chain-like structure. Dynamics is, however, not restricted to a compact simplex, and may be expanding or shrinking. There can be any finite number of regions among which agents migrate, and those regions can form any shape, a circle or a plane or whatever. The results depend crucially on the "primitivity" or "irreducibility" of a given nonlinear positive operator. Thus, we concentrate on finding out some sufficient conditions which guarantee primitivity. Also discussed is the case of inhomogeneous operators, i.e., the case where operators change through time. Various applications to other fields are also mentioned.

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Geometric Methods for Continuous and Discrete Dynamical Systems

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We establish the global qualitative analysis of planar polynomial continuous dynamical systems and suggest a new geometric approach to solving Hilbert's Sixteenth Problem on the maximum number and relative position of their limit cycles in two special cases of such systems [1, 2]. First, using geometric properties of four field rotation parameters of a new canonical system, we present a proof of our earlier conjecture that the maximum number of limit cycles in a quadratic system is equal to four and the only possible their distribution is (3 : 1). Then, by means of the same geometric approach, we solve the Problem for Liénard's polynomial system (in this special case, it is considered as Smale's Thirteenth Problem). Besides, generalizing the obtained results, we present a solution of Hilbert's Sixteenth Problem on the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and, applying the Wintner-Perko termination principle for multiple limit cycles, we develop an alternative approach to solving the Problem. By means of this approach, for example, we give another proof of the main theorem for a quadratic system and complete the global qualitative analysis of a generalized Liénard's cubic system with three finite singularities. Since the asymptotic behavior of the solutions of discrete dynamical systems is absolutely similar to the behavior of the trajectories of the corresponding continuous systems, we discuss also the possibilities of applications of the developed geometric methods to the corresponding planar polynomial discrete dynamical systems.

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Construction of Lyapunov Functions for Discrete Dynamical Systems using Radial Basis Functions

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Consider a discrete dynamical system given by the iteration $x_{n+1} = g(x_n)$ with exponentially asymptotically stable fixed point $\overline{x}$. One can study the basin of attraction of $\overline{x}$ using sublevel sets of a Lyapunov function. We introduce a construction method for Lyapunov functions by starting from the existence of a smooth Lyapunov function V with certain values of its discrete orbital derivative. This equation is used to approximate the Lyapunov function V by an approximation v using radial basis functions. Error estimates show that the approximation has negative discrete orbital derivative; however, this cannot be ensured locally near the fixed point.
To overcome this problem, two solutions are proposed: either a local Lyapunov function obtained by linearization at the fixed point is used, or a different approximation including the Taylor polynomial of V is performed. With both methods every connected and bounded subset of the basin of attraction can be determined.

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Unfulfilled Rational Expectations and the Stability of Monetary Policy

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In this paper, we use a standard New Keynesian model as developed by, e.g. Goodfriend and King (1997), Clarida et al. (1999), or Woodford (2003), to analyze the stability of monetary policy in the presence of unfulfilled forward looking or rational expectations.
We present a different approach to model by the introduction of expectations into the model. Instead of assuming by construction that forward looking expectations are always fulfilled (rational expectations), or that agents formulate their expectations following some complicated process supplied *only* with backward information (adaptive learning, like recursive least squares, mean forecasting, among others), it is assumed that the private agents follow an approach that somewhat resembles the theory of reinforcement learning. In this approach the agent tries to guess the true future value of the state variables (inflation and the output gap). If they have got it right in the past, in the next round there is a large probability that they have a go and get it right again; however, if they got it wrong in the past, then it is highly probable that the expected value will be wrong again. This procedure leads to two fundamental results in the standard monetary model. Firstly, the model shows no convergence to the rational expectations equilibrium (REE) if the initial guess turns out to be different from the true values of the state variables; that is, such equilibrium is achieved only in the particular case in which agents keep correctly guessing the true value of variables associated to the REE. Secondly, despite not converging to the rational expectations equilibrium, the dynamics move around this equilibrium, if the initial guess is not extremely distant from the true values. The dynamics of this model economy neither explode, nor implode, ending up with endogenous cycles caused by unfulfilled forward looking expectations.

Universal BHP Distribution and Nonlinear Prediction in Complex Systems

using the Ruelle-Takens Embedding

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We exploit ideas of nonlinear dynamics and statistical physics in a complex non-deterministic dynamical setting. Our object of study is the observed riverflow time series of the Portuguese Paiva river whose water is used for public supply. The Ruelle-Takens delay embedding of the daily riverflow time series revealed an intermittent dynamical behavior due to precipitation occurrence. The laminar phase occurs in the absence of rainfall. The nearest neighbor method of prediction revealed good predictability in the laminar regime, but we warn that this method is misleading in the presence of rain. We present some new insights between the quality of the prediction, the embedding dimension, and the number of nearest neighbors considered. After this careful study of the laminar phase, we find, unexpectedly, that the BHP distribution is an approximation of the empirical distribution of the relative decay given by the nearest neighbors predictor.

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Creative Properties of the 2D Lattice Distributed Interconnected Chaotic 0scillators:Colored Ornaments Generated by Discrete Logistic Equation

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Mathematical modeling of the brain's neurons networks by interconnected oscillators with chaotic regimes is one of the effective methods to simulate brain creativity in a form of artistic images within their relation to the "brain waves" (EEG) [1,2]. In this work we would like to present some results on creative patterning based on exploring dynamics of $L2$ chaotic oscillators distributes on 2D squared $L \times L$ lattice where each cell designated by the integer coordinates $i, j = 1, 2, \ldots, L$ and contain discrete dynamical system ("chaotic oscillator"):

$$X_{n+1}(i,j) = \lambda_n(\circledast) X_n(i,j)(1-) X_n(i,j)).$$

Here parameter $\lambda_n(\circledast) = F(\theta, X_{n-1}(\circledast))$ depends from the values $X_{n-1}(\circledast)$ distributed on the lattice and calculated at $(n-1)$. In the simplest case $X_{n-1}(\circledast)$ just the closet to considered cell with coordinates (i, j) eight neighbors with coordinates $(\circledast) = (i-1, j-1; i+1, j; \ldots, i+1, j+1)$, F is an arbitrary chosen analytic function with parameters θ. It will be shown that discrete logistic equation can be used for generating colored symmetrical patterns (ornaments) when $3.6 < \lambda_n(\circledast) < 4$. Evolution of $X_n(i,j)$ within the lattice cells will be presented by discrete "time series " and their chaotic character will be demonstrated and analyzed. Variety of the patterns resulted from the proposed approach can be drastically increased by using different functions F, adjusting parameters θ and $\circledast$ by using other types of basic difference equations. It will be shown that use of difference equations which are related to the physicochemical laws of nature will result to the patterns in a form of rings and spirals waves among the other patterns resembling naturally observed. Applications of the presented method for mathematical imaging and modeling of complex, living and thinking systems will be discussed.

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Graphs, Symbolics Dynamics and Synchronization of Networks

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The synchronization of coupled chaotic systems depends typically on a number of factors, including the strength of the coupling, the connection topology and the dynamical characteristics of the individual units. Methods from graph theory and symbolic dynamics will be used to study the synchronization of complex networks.

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Stability of an Equilibrium Point in Goodwin's Growth Cycle Model

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In theory of economics, most models describing economic growth use differential equations. However, when trying to use them by econometricians, many questions arise. First, economic data (especially macroeconomic) are discrete, what forces use of difference equations. Second, transformation from continuous to discrete form of a model is still controversial. The essence of above-mentioned problems and proposal of solving them will be presented on the basis of Goodwin's Growth Cycle Model.

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Exponential Representation of the Solutions of Nonlinear Volterra Difference Equations

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In this talk we investigate the exponential growth of solutions of a Volterra-type difference equation which frequently arises in applications. These results are applicable to some quasilinear delay difference equations with bounded and unbounded delays. Examples are given to illustrate the sharpness of the results.

On the Criteria for Irrational Number of Recursive Sequences

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In 1900, D. Hilbert gave his famous report on Mathematical Problems at the 2nd International Congress of Mathematicians. We can recall the title of first part on his seventh problem; "Irrationality (and transcendence) of certain numbers". On the other hand, for Fibonacci sequences $\{F_n\}$, D. Duverney has shown that $\sum_{n=1}^{\infty} \frac{1}{F_n}$ is an irrational number. In this talk, more generally, we consider the sufficient condition for which, $\sum_{n=1}^{\infty} \frac{1}{H_n}$ is an irrational number, where $\{H_n\}$ is the solution of a linear difference equation with constant coefficients.

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Multiscale Expansion of the Discrete KdV Equation

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The formalism multiscale expansion (cf. [1] for the case of differential equations) of a difference equation is developed, applying it to a difference version of the potential Korteweg-de Vries (KdV) equation [2]. This is an integrable lattice equation, and its first slow scale approximation is computed to be the nonlinear Schrödinger equation (NLS), an integrable PDE. A conjecture on the preservation of integrability by the multiscale expansion, similar to that of the differential, continuous case [3], is formulated.

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Category of Time Dependent Ladders

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After introducing the category of time dependent ladders, definition of homomorphisms, inversion, sum and product of ladders we will present various examples showing the power of this concept. It will be shown how basic equations of quantum mechanics fit nicely into the general framework. A crucial example will be the statement that the product of two Heisenberg–Dirac ladders provide representation ladders for the Lie algebra $\mathbf{sl}(2,\mathbb{C})$. Finally we will show how the ladder formalism provides the background for structural analysis of the deformations arising from the transition from the continuous ($\mathbb{R}$) to the discrete ($h\mathbb{Z}$) or q–difference ($q^{\mathbb{Z}}$) case.

Time Scale Embedding Theorem and Coercivity of Quadratic Functionals

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This is a joint work with Vera Zeidan from Michigan State University, Department of Mathematics, East Lansing, MI 48824-1028, USA, *zeidan@math.msu.edu*.

In this talk we study the relation between the coercivity and positivity of a time scale quadratic functional $\mathcal{J}$, which could be a second variation for a nonlinear time scale calculus of variations problem (P). We prove for the case of general jointly varying endpoints that $\mathcal{J}$ is coercive if and only if it is positive definite and the time scale version of the strengthened Legendre condition holds. In order to prove this, we establish a time scale embedding theorem and apply it to the Riccati matrix equation associated with the quadratic functional $\mathcal{J}$. Consequently, we obtain sufficiency criteria for the nonlinear problem (P) in terms of the positivity of $\mathcal{J}$ or in terms of the time scale Riccati equation. This result is new even for the continuous time case when the endpoints are *jointly varying*.

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Almost Totally Disconnected Minimal Systems

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I will present new results from a joint work [1].
A space X is said to be almost totally disconnected if the set of its degenerate components is dense in X. We prove that an almost totally disconnected compact metric space admits a minimal map if and only if either it is a finite set or it has no isolated point. As a consequence we obtain a characterization of minimal sets on dendrites and local dendrites.

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Coexistence of Plant Species -the Analysis of Revised Lottery Model-

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(Joint work with Y. Takeuchi, *Shizuoka University, Japan*)

We discuss the multiple species coexistence problem which promotes plant-endophyte interaction by using the difference equations based on a lottery model (Chesson and Warner 1981):

$$\begin{aligned} P_{i,t+1} &= g_i(x_t)(1-\delta_i)P_{i,t} + S(\mathbb{P},\mathbb{P}^e,x)R_i(\mathbb{P},\mathbb{P}^e,x) \\ P^e_{k,t+1} &= g^e_k(x_t)(1-\delta^e_k)P^e_{k,t} + S(\mathbb{P},\mathbb{P}^e,x)R^e_i(\mathbb{P},\mathbb{P}^e,x) \\ x_{t+1} &= f(x(t)), \end{aligned}$$

where $\mathbb{P} = (P_1,\cdots,P_n)$, $\mathbb{P}^e = (P^e_1,\cdots,P^e_n)$,

$$S(\mathbb{P},\mathbb{P}^e,x) = K - \sum_{j=1}^{n} g_i(x)(1-\delta_i)P_j - \sum_{j=1}^{l} g^e_i(x)(1-\delta^e_i)P^e_j,$$

$$R_i(\mathbb{P},\mathbb{P}^e,x) = (\beta_i P_{i,t} + \theta\beta_i P^e_{i,t}) / \sum_{j=1}^{n} \beta_j P_j + \sum_{j=1}^{l} \beta_j P^e_j,$$

and

$$R^e_i(\mathbb{P},\mathbb{P}^e,x) = (1-\theta)\beta_k P^e_k / \sum_{j=1}^{n} \beta_j P_j + \sum_{j=1}^{l} \beta_j P^e_j,$$

$i,\, k = 1,\cdots,n$. P_i (P^e_i) denotes the occupation rate of plant without (resp. with) endophyte. $g_i(x_t)$ ($g^e_i(x_t)$) denotes the survival rate of plant without (resp. with) by the predate.

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The Global Properties of a Two-Dimensional Competing Species Model Exhibiting Mixed Competition

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In this paper we demonstrate how the global dynamics of a biological model can be analysed. In particular, as an example, we consider a competing species population model based on the discretisation of the original Lotka-Volterra competition equations. We analyse the local and global dynamic properties of the resulting two-dimensional noninvertible dynamical system in the case where the interspecific competition is "mixed" - the intraspecific competition is structurally stronger than the interspecific competition for one of the species, while the converse is true for the other. The main results of this paper are derived from the study of some global bifurcations that change the structure of the attractors and their basins. These bifurcations are investigated by the method of Critical Curves, a powerful tool for the analysis of the global properties of noninvertible two-dimensional maps.

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Ordinal Pattern Distributions in Dynamical Systems

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Ordinal time series analysis is a new promising approach to the qualitative investigation of long and complex time series. The idea behind it is to consider the order relation between the values of a time series instead of the values themselves. Roughly speaking, a given time series is transformed into a series of so called ordinal patterns describing the up and down in the original series. Then the distribution of ordinal patterns obtained is the base of the analysis. Here we focus to the ergodic structure of ordinal pattern distributions obtained from dynamical systems. In particular, we discuss an ordinal analogue of the Feigenbaum diagram and measures quantifying the complexity of a dynamical system.

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Dynamical Consistency in Discrete and Continuous Equations with Finite-Time Explosions

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(Joint work with J. Appleby, *Dublin City University*, and A. Rodkina, *University of the West Indies*)

We consider a scalar nonlinear differential equation whose solutions explode in finite time. By choosing an appropriate nonuniform mesh, a discrete-time model consisting of a forward Euler discretisation over this mesh can be constructed that mimics the explosion.
On this mesh other properties of the continuous local solution emerge in the discrete model: the rate of growth in the neighbourhood of the explosion is reproduced, and in fact the explosion time itself can be approximated arbitrarily well.
We also discuss how the finite-time explosion of a nonlinear stochastic differential equation can be mimicked, when the perturbing noise source has a truncated left tail.

A Cardiac Reentry Loop Model with Thresholds

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We investigate the two-dimensional, multiple-threshold map, or bimodal system,

$$F(x\,,y) = \begin{cases} G(x\,,y)\,, & \text{if } (x\,,y) \in \mathcal{T}\,, \\ H(x\,,y)\,, & \text{if } (x\,,y) \notin \mathcal{T}\,, \end{cases}$$

where $G : \mathbf{R}^2 \to \mathbf{R}^2$ and $H : \mathbf{R}^2 \to \mathbf{R}^2$ are continuous and $\mathcal{T}$ is a region in $(0\,,\infty)^2$. We denote the region $(0\,,\infty)^2 - \mathcal{T}$ by $\mathcal{N}$, and we call $\mathcal{T}$ and $\mathcal{N}$ the *reentrant* and *regular pulse regions*, respectively. The boundary, $\partial\mathcal{T}$, between these two regions is referred to as the *threshold* of the system. Three different sets of sufficient conditions are placed on G and H, resulting in the following three behaviors of the bimodel system: **(1)** Every orbit under F that begins in $\mathcal{T}$ eventually ends up and remains in $\mathcal{N}$. **(2)** There exist orbits under F that begin in $\mathcal{T}$ and pass between $\mathcal{T}$ and $\mathcal{N}$ infinitely often. **(3)** There exist orbits that begin in $\mathcal{T}$; pass a finite number of times between $\mathcal{T}$ and $\mathcal{N}$; but then end up and remain in $\mathcal{N}$.
Our bimodal system is intended to serve as a simple discrete model of the dynamics of a circulating pulse of depolarization in a ring of two cardiac units, each unit, in turn, composed of an arbitrary number of cardiac cells. Such a ring of cardiac cells, called a *cardiac reentry loop* or *reentrant circuit*, in reality consists of hundreds of cells and is considered to be a major player in the etiology of certain forms of cardiac arrhythmias, or irregular heartbeat.

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Explicit Stability Conditions for Difference Equations

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We prove that if $a_i \geqslant 0$ $(1 \leqslant i \leqslant k)$ and

$$0 < \sum_{i=1}^{k} \frac{a_i}{2\sin\frac{\pi}{2(2i-1)}} < 1, \tag{1}$$

then the scalar equation

$$x(n) = x(n-1) - \sum_{i=1}^{k} a_i x(n-i) \tag{2}$$

is asymptotically stable (M.Kipnis, D.Komissarova, Difference Equ. and Appl., 2007, V.13, No 5, 457–461).
As a corollary we obtain sufficient asymptotic stability conditions for the latter equation: $a_i \geqslant 0$ $(1 \leqslant i \leqslant k)$ and $0 < \sum_{i=1}^{k} i a_i \leqslant \pi/2$. It improves the known result with $1 + 1/e$ instead of $\pi/2$ (see [1]). We consider the vector variant of (1) too (M.Kipnis, D.Komissarova, Advances in Difference Equ., 2006, ID 31409).
In the second part of the talk some results of Maligina and A. Kulikov will be discussed (with authors' permission), which have to do with the stability of a nonautonomous version of (2): $x(n) = x(n-1) - \sum_{i=1}^{k} a_i(n)x(n-h_i(n))$. They extend the results of [2,3].

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Semi-Hyperbolicity and Bi-Shadowing in Difference Equations with Lipschitz Mappings

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About a decade go, the speaker with Phil Diamond, Viktor Kozyakin, Mark Krasnossel'skii and Alexei Pokrovskii introduced a generalization of the concept of hyperbolicity which they called semi-hyperbolicity. In particular, smoothness and invertibility of the system mapping are not essential, nor are the continuity and equivariance of the tangent space splitting at each point of a hyperbolic set, nor in fact is the invariance of the hyperbolic set itself. This allowed for much wider applicability, in particular for when perturbations of the systems were not smooth. This and a related concept of bi-shadowing will be discussed in the talk and illustrated with examples and applications.

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On the Centre and the Set of ω-Limit Points of Dynamical Systems on Dendrites

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For dynamical systems generated by a continuous map of a compact interval, the centre of the dynamical system is a subset of the set of ω-limit points. This holds even for graphs. In this note we provide an example of a continuous selfmap f of a dendrite such that $\omega(f) \subsetneq \mathrm{C}(f)$.

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Functional Envelope of a Dynamical System

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If (X, f) is a dynamical system given by a compact metric space X and a continuous map $f : X \to X$ then by the functional envelope of (X, f) we mean the dynamical system $(S(X), F_f)$ whose phase space $S(X)$ is the space of all continuous selfmaps of X and the map $F_f : S(X) \to S(X)$ is defined by $F_f(\varphi) = f \circ \varphi$ for any $\varphi \in S(X)$. The functional envelope of a system always contains a copy of the original system.
Our motivation for the study of dynamics in functional envelopes comes from the semigroup theory, from the theory of functional difference equations and from dynamical systems theory. Mainly we will speak about the connection between the properties of a system and the properties of its functional envelope. A special attention will paid to orbit closures, ω-limit sets, (non)existence of dense orbits and topological entropy.

On Riccati Difference Equations with Complex Coefficients

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We give a detailed analysis of the Riccati difference equation

$$x_{n+1} = \frac{\alpha + \beta x_n}{A + B x_n} \quad , \quad n = 0, 1 \ldots$$

where the coefficients

$$\{\alpha\} \quad , \quad \{\beta\} \quad , \quad \{A\} \quad , \quad \{B\}$$

are complex numbers and where the initial condition x_0 is a complex number.

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On Asymptotics of Discretized Delay Differential Equation

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Delay differential equations are widely utilized in areas like optimal control, biology, chemistry, physics and many others. Most of the problems described by such differential equations are not solvable in an analytical way. Hence the numerical solution is usually taken in advance. It is necessary to deal with qualitative properties of difference equation the used numerical formula is based on to assure its convenience for solving the problem. The qualitative properties of the differential equation and its difference representation should coincide. The contribution is focused on asymptotic investigation of Euler's discretization of delay differential equation with proportional delay, where the delayed term is approximated by linear interpolation. Obtained qualitative results are compared with the corresponding properties in the continuous (differential) case and illustrated by some examples.

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Rational Difference Equations

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We study the global character of solutions of rational difference equations. We are concerned with the boundedness nature of their solutions, the global stability of the equilibrium points and with convergence to periodic solutions including the periodic trichotomies. Several open problems and conjectures will be presented.

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Linear Functional Equations in the Space of Strictly Monotonic Functions

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The classical theory of k-th order linear functional and difference equations is obtained as a special case of the theory developed here for the k-th order functional equation model which generalizes the first order model $f \circ \phi - Af = B$. The function ϕ is assumed to solve a given Abel functional equation and belong to a special space S of continuous strictly monotonic functions equipped with a group multiplication $\circ$.
Announced here are the main theorems on solution structure for k-th order homogeneous linear functional equations in S. These results apply to functional equations with non-constant coefficients.
Assume $a_j \in R, j = 0, 1, 2, \ldots, k$, are given and consider a k-th order linear homogeneous functional equation

$$a_k f \circ \Phi^k(x) + a_{k-1} f \circ \Phi^{k-1}(x) + \cdots + a_1 f \circ \Phi^1(x) + a_0 f \circ \Phi^0(x) = 0.$$

The theory developed applies to solve the equation using roots of the characteristic equation together with a continuous solution α of the associated Abel functional equation $\alpha \circ \Phi(x) = X(x+1) \circ \alpha(x)$. Details about the canonical function X and Abel functional equations appear in references [1], [2].
Illustrations of iterative solutions formulas are given for the first order linear equation

$$f \circ \Phi(x) = Af(x) + B.$$

The formulas produce approximate values for the solution $f(x)$ and aid in computer assisted visualization of solutions.

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Scattering by Obstacles and Wave Problem of Minimal Resistance

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We consider the wave analogue of the classical Newton problem of the body of minimal resistance. In wave setting, instead of a homogeneous parallel flow of particles and classical resistance, one should consider a plane wave and the transport cross section. Trying to solve the problem, we found some new natural effects of scattering theory unknown even for physicists.

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Discrete Mechanics and Optimal Control of Molecular Dynamics

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The discrete mechanics and optimal control (DMOC) method [1,5] is developed for applications in N-body molecular dynamics [2-4]. The Lagrange-d'Alembert principle, a variational principle equivalent to Lagrange's equations with external forces, plays a crucial role in enforcing the equality constraint for the dynamics of the mechanical system under an external force while optimizing a cost functional with variable control function and trajectory. The DMOC method applies to simulations of molecular dissociation to determine the driving forces and trajectories to minimize the cost of the driving force effort among other objectives. Including overall rotation in the center-of-mass frame due to internal motion [4], the DMOC method applies to the dissociation and isomerization of triatomic molecular species and to triatomic systems governed by chaotic potentials.

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Global Attractors for Difference Equations Dominated by One-dimensional Maps

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This is an account of the joint work with Hassan A. El-Morshedy (Damietta Faculty of Sciences, Egypt) [1].
The global attractivity character of nonlinear higher order difference equations of the form

$$x_{n+1} = g(x_n, x_{n-1}, \ldots, x_{n-k}), \quad n \geq 0$$

is investigated when g is dominated by an interval scalar map. Some basic properties of the interval map are obtained and used to prove new global attractivity criteria for the above equation with no monotonicity restrictions on g. Our results are applied to many models from mathematical biology and economy. The derived global attractivity criteria of these models are either new or improve substantially known ones.

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A Dynamical System for a Long Term Economical Model

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The Marx model for the profit rate r depending on the exploitation e rate and on the organic composition of the capital k is studied using a dynamical approach. Supposing both $e(t)$ and $k(t)$ continuous functions of the time we derive a law for $r(t)$ in the long term. Depending on the hypothesis set on the growth of $k(t)$ and $e(t)$ in the long term $r(t)$ can fall to zero or remain constant, this last case contradicts the classical hypothesis of Marx stating that the profit rate must decrease in the long term. Introducing a discrete dynamical system in the model, supposing that both k and e depend on the profit rate of the previous cycle we get a discrete dynamical system for r, $r_{n+1} = f_a(r_n)$ which is an family of unimodal maps depending on the parameter a, the exploitation rate when the profit is zero. In this map we can have a fixed point when a is small, when we increase a we get a cascade of period doubling bifurcations leading to chaos. When a is very big the system has again periodic stable orbits of period five and, finally, period three. This can justify the Kondratieff waves in long term economical systems.

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On Variants of Distributional Chaos in One-dimensional Spaces

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For continuous maps of a compact metric space there are three mutually non-equivalent versions of distributional chaos. In this talk we show that the two strongest ones are equivalent, for continuous maps of finite graphs. When the space is a finite tree then also the weakest version of distributional chaos is equivalent to other ones.

Singular State-dependent Delay Equations: Asymptotics and Stability

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We discuss recent results on differential-delay equations of the form

$$\varepsilon\dot{x}(t) = f(x(t), x(t-r)).$$

Here the delay $r \geq 0$ is either a constant, or else can vary in a state-dependent fashion $r = r(x(t))$. Results on stability, uniqueness, and asymptotic behavior of periodic solutions for small ε are obtained, and a new phenomenon of superstability is proved for the state-dependent case. Our approach, which uses degree theory, max-plus operators, and geometric singular perturbation theory, seems appropriate for multiple-delay problems as well. Indeed, numerical results suggest a very rich structure, particularly for multiple-delay problems.

Interval Maps and Cellular Automata

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We associate to cellular automata elementary rules a class of interval maps defined in $[0, 1]$. The considered configuration space is the space of the one-sided sequences in the set $\{0, 1\}$ and with the appropriate choice of the update procedure the interval maps do not depend on boundary conditions, obtaining different results from [1]. We study the rule 184 and obtain a family of transition matrices that characterizes the dynamics of the cellular automaton. We show that these matrices can be obtained recursively by an algorithm that depends on the local rule.

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Boundary Value Problems, Periodic and Bounded Solutions for some Nonlinear Difference Equations

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We survey some recent results on the existence and multiplicity of solutions of some boundary value for nonlinear difference equations or systems, as well as of periodic or bounded solutions.
We compare the results with similar ones for ordinary differential equations or systems.

Absolute Stability of Nonlinear Delay Difference Systems

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We investigate the absolute stability of nonlinear non-autonomous difference systems with delaying arguments and slowly varying coefficients. The method of Lyapunov functions is an efficient tool in stability analysis of discrete processes,and is widely used, but finding Lyapunov functions is usually a difficult task. By contrast, based on the "freezing" technique to discrete-time systems we derive explicit conditions for the absolute stability of a class of nonlinear non-autonomous delay difference systems.

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Numerical Range, Numerical Radii and the Dynamics of a Rational Function

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Sometimes we obtain attractive results when associating facts to simple elements. The goal of this work is to introduce a possible alternative in the study of the dynamics of rational maps. In this study we use the family of maps $f(x) = \frac{x^2+a}{x^2+b}$, making some associations with a matrix

$$A = \begin{pmatrix} 1 & a \\ 1 & b \end{pmatrix}$$

of its coefficients.
Calculating the numerical range, $W(A)$, the numerical radii, $r(A)$ and $\widehat{r}(A)$, the boundary of the numerical range $\delta W(A)$, powers and iterations, we found relations very interesting, specially with the entropy of this maps.

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Dynamics and Control in a Implicitly Defined Matching Labor Market Model

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The possibility that chaotic dynamics may arise in modern labor markets had been totally strange to economics until recently. Following an interesting paper of Bhattacharya and Bunzel [2] we have found that the discrete time version of the Pissarides-Mortensen matching model, as formulated in Ljungqvist and Sargent [3], can lead to chaotic dynamics under standard sets of parameter values, contrary to the conclusion of these authors. The difference equation has the unusual property that its dynamics is well-defined backward in time, but not forward in time. This fact brings some problems in the study of forward dynamics and question the relevance of knowledge gained from the backward dynamics. Some mathematical tools from inverse limits [1] and symbolic dynamics can be applied to this type of implicit difference equations.
This paper explores this version of the model with the following objectives in mind: to clarify some open questions raised by Bhattacharya and Bunzel in a previous working paper by providing a rigorous proof of the existence of chaotic dynamics in the model, by computing topological entropy and to apply an adapted version of the OGY chaos control technique to the implicitely defined labor market model that shows erratic behavior and large volatility in employment flows.

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Chaotic Interest Rates Rules and Stabilization

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In this paper we apply the techniques of chaos control to the analysis of an optimal monetary policy model which shows chaotic behavior and large volatility in output. In a series of papers, Benhabib, Schmitt-Grohé and Uribe (2001a, 2004) have shown that active interest rules may lead to very unexpected consequences: indeterminacy, deflation traps, large cyclical instability, and can even lead to chaotic dynamics under standard sets of parameter values. This paper explores this particular model and puts forward four basic points: (i) the model developed by Benhabib and associates seems to suffer from serious drawbacks to be used as a theoretical benchmark to guide optimal monetary policy, as the more aggressive the central bank becomes, the more unstable the economy will be; (ii) the time span required to achieve successful control is generally small, by linear feedback techniques — the OGY method — without producing modifications to the original model, apart from locally changing its type of stability; (iii) ignorance about the true state of initial conditions are not a serious impediment to obtain control of the chaotic dynamics in the model; (iv) we argue that the conventional view of economic policy in nonlinear general equilibrium models — when endogenous fluctuations exist in optimizing models, the associated policy advice is laissez–faire — seems to be based on a misconception of chaos in general, and on the control of chaos in particular.

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On a Class of First-Order Nonlinear Difference Equations of Neutral Type

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We study the behavior of solutions of the difference equations of neutral type of the form

$$\Delta(x_n - p_n x_{n-\tau}) = \delta q_n f(x_{n-\sigma}, x_{n+1-\sigma}, \ldots, x_n).$$

We give classification of possible nonoscillatory solutions. Conditions for the existence of nonoscillatory solutions and sufficient conditions under which all (or all bounded) solutions are oscillatory are also presented.

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Optimal Choice of Delay in Shrimp Population Models

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Studying two prediction models of the shrimp population dynamics in Sofala Bank region, where one is linear:

$$X(k+1) = aX(k) + \sum_{i=1}^{m} b_i X(k - c_i) \tag{1}$$

(where we assume $m = 2$ or $m = 3$), the other is nonlinear:

$$x(k) = ax(k-1)[1 - (bx(k-1) + dx(k-c) + gx(k-e))^{\beta}], \tag{2}$$

our aim is two-fold. On one hand, we want to justify significance of time-delay in these models modeling shrimp population dynamics, which also has a biological meaning (maturation period). On the other hand, we suggest numerical algorithms providing priori unknown values of delays (c_i in the first model, c and e in the second model). All delay parameters are discrete, as they describe time units (in our case months). The other parameters are allowed to change continuously. The challenge is to optimize the delay parameters, together with the the other coefficients. The data used in the model are mostly based on the catch records and on the regular cruises. We estimated the parameters for all species and for the total catch separately. An essential part of the proposed algorithm is the Levenberg-Marquardt gradient method [1] adjusted to difference-delay equations. We use MATLAB to implement the algorithm. As shrimp fishing is forbidden from January to February, the initial value of x_0 corresponds to March. In fact, however, it would be useful for practical needs to estimate x_0 as a result of the prehistory.

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Dynamics of Nash maps for 2×2 games

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We study the dynamics of 2×2 games associated with the *better response* map [3]. While it was used by Nash only to prove the existence of an equilibrium point, it can be interpreted as a real way the strategies are determined and the game is played. We concentrate on the mathematical aspects of maps obtained in such a way. For the Matching Pennies game the fixed (equilibrium) point is repelling, while all other trajectories are attracted to a period 8 orbit [1]. After introducing a parameter that can be interpreted as an *index of caution*, we get a family of maps with the fixed point still repelling. Apparently for each of them there is an invariant attracting closed curve, on which the map is a homeomorphism with the rotation number varying with the parameter. When this parameter c goes to 0 (the players make very small changes in their strategies), after rescaling by the factor $1/c$ this curve approaches the circle of radius $3\pi/32$. Another interesting one-parameter family, containing Coordination Game and the Game of Chicken, has two pure strategy attracting fixed points and a mixed strategy fixed point. As the parameter varies, the system undergoes interesting bifurcations [2]. We describe rigorously the dynamics of all maps from this family.

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Higher Order Difference Equations in Economic Theory

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We study the underlying structure of the two-dimensional dynamical system generated by a class of dynamic optimization models, which allow for intertemporal complementarity between adjacent periods. We obtain conditions for global convergence results, as well as persistent flactuations and relate them to the local analyses.

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Dynamics of Iterated Nonlinear Maps of Finite Dimensional Cones

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Over the past twenty years, a surprisingly detailed picture has emerged concerning the dynamics of iterates of certain natural classes of nonlinear maps of finite dimensional cones into themselves. The theory which has developed can be considered a generalization of the beautiful classical results of Perron and Frobenius concerning matrices with nonnegative entries. An underlying theme of the nonlinear theory is that the maps considered are nonexpansive with respect to appropriate metrics, e.g., the l_1 norm or Hilbert's projective metric or the so-called Thompson's metric. In this talk we shall begin with some basic definitions and then describe theorems and conjectures due to several authors. We shall conclude with some recent results of B. C. Lins and R. D. Nussbaum concerning what can be called Denjoy-Wolff theory for maps which are nonexpansive with respect to Hilbert's projective metric and the applications of that theory to generalizations of the class of reproduction-decimation operators from the theory of fractal diffusion.

Boundedness, Periodicity and Stability of the Difference Equation $x_{n+1} = A_n + \left(\frac{x_{n-1}}{x_n}\right)^p$

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This paper studies the boundedness, the persistence, the periodicity and the stability of the positive solutions of the nonautonomous difference equation:

$$x_{n+1} = A_n + \left(\frac{x_{n-1}}{x_n}\right)^p, \ \ n = 0, 1, ...,$$

where A_n, is a positive bounded sequence, $p \in (0,1) \cup (1,\infty)$ and $x_{-1}, x_0 \in (0,\infty)$.

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Oscillation Criteria for a Forced Second Order Nonlinear Dynamic Equation

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We are concerned with finding oscillation criteria for both forced second order linear and nonlinear dynamic equations on a time scale. Our results generalize known results for difference equations and for differential equations. In particular we generalize results due to J. S. W. Wong [1] and A. H. Nasr [2]. Examples will be given.

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A "Discretization" Technique for the Solution of ODEs.

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(Joint work with P. D. Siafarikas and E. E. Tzirtzilakis, *Department of Mathematics, University of Patras*)

A functional-analytic technique was developed in the past for the establishment of unique solutions of ODEs in $H_2(\mathbb{D})$ and $H_1(\mathbb{D})$ and of difference equations in ℓ_2 and ℓ_1. This technique is based on two isomorphisms between the involved spaces. These two isomorphisms can be combined in order to find discrete equivalent counterparts of ODEs, so as to obtain eventually the solution of the ODEs under consideration. As an application, the Duffing equation and the Lorenz system are studied. The results are compared with numerical ones obtained using the 4^{th} order Runge-Kutta method. The advantages of the present method are that, it is accurate, the only errors involved are the round-off errors, it does not depend on the grid used and the obtained solution is proved to be unique.

Oscillatory Mixed Differential Difference Equations

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In this work is studied the oscillatory behavior of the delay differential difference equation of mixed type

$$x'(t) = \sum_{i=1}^{\ell} p_i x(t - r_i) + \sum_{j=1}^{m} q_j x(t + \tau_j) \tag{1}$$

with $r_i > 0, i = 1, ..., \ell$ and $\tau_j > 0, j = 1, ..., m$. Some criteria are obtained in order to guarantee that all solutions of (1) are oscillatory.

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Hyperbolic Dynamics

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We are interested to classify the smooth conjugacy classes of unimodal maps and hyperbolic diffeomorphisms on surfaces with hyperbolic basic sets, for a given topological class, via the construction of a moduli space. We describe a construction of such a moduli space using *solenoid functions*. If the holonomies are sufficiently smooth then the diffeomorphism is *rigid* in the sense that it is C^{1+} conjugate to a hyperbolic affine model. The techniques developed in these works allow us to extend *Sinai's eigenvalue formula* for Anosov diffeomorphisms, that preserve an absolutely continuous measure, to hyperbolic basic sets on surfaces, that possess an invariant measure absolutely continuous with respect to Hausdorff measure. Sullivan has proved that the generalization of the doubling operator, called the renormalization operator, acts in an invariant set topologically as an horseshoe. Lyubich proved that the renormalization operator has, in fact, an hyperbolic splitting in a special analytic space made out of germs. De Faria, de Melo and Pinto extended this hyperbolic picture to the smooth class of unimodal maps showing that the unstable holonomies are smooth. De Melo and Pinto proved that infinitely renormalizable C^3 unimodal maps, with the same combinatorics of bounded type, are smooth conjugate. Alves, Pinheiro and Pinto have shown, for C^3 non-uniformly expanding unimodal maps, that if the topological conjugacy has a derivative at a point then it is smooth.

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Monotone Solutions of a Nonlinear Difference Equation with Continuous Time

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We give an asymptotic description of the monotone increasing solutions of a nonlinear scalar difference equation with continuous time arising during the study of traveling waves in spatially discrete dynamical systems.

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Dynamic Contact Problems in Linear Viscoelasticity

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The existence of a solution of the dynamic contact problem is proved for a particular case, when the material is considered viscoelastic with elastic behaviour and when in the bilinear form is included the displacement velocity, with Coulomb friction. The proof of the main result is based on the penalization and regularization methods. The dynamic contact problem will be discretized with finite element method and finite difference method. The concept of the a priori stability estimate for dynamic frictional contact, will be analyzed by using of the local split of the Coulomb model. The split will be implemented such that the global balance of energy will be preserved in the case of perfect stick, while in the case of slip an algorithmically consistent approximation, will be produced pointwise on the contact interface.

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Boundedness and Periodic Convergence in Rational Difference Equations

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We examine the boundedness and periodic character of difference equations of the form

$$x_{n+1} = \frac{\alpha + \beta x_n + \gamma x_{n-1} + \delta x_{n-2} + \epsilon x_{n-3}}{A + Bx_n + Cx_{n-1} + Dx_{n-2} + Ex_{n-3}}, \quad n = 0, 1, \ldots$$

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Dynamical Solution from the Confluence of Fibonacci-Horner Methods and Computation of the Matrix Exponential

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This talk concerns the *Fibonacci-Horner* decomposition of the matrix powers and the matrix exponential, which is derived from the combinatorial properties of some linear difference equations known in the literature as *generalized Fibonacci sequences*. More precisely, the matrix exponential is expressed in the so called Fibonacci-Horner basis, with the aid of the dynamical solution, of the associated ordinary differential equation. To this aim, two simple process for computing the dynamical solution and the fundamental system of solutions are given. Connection with the Verde-Star's approach is discussed. Moreover, an extension to the computation of $f(A)$, where f is an analytic function, is initiated. Finally, some significant examples are considered..

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Stability of Unimodal Maps under Small Perturbations

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A map $T : [0,1] \to [0,1]$ is called unimodal, if T is continuous, and if there is a $c \in (0,1)$ such that $T\big|_{(0,c)}$ is strictly increasing and $T\big|_{(c,1)}$ is strictly decreasing. Consider the collection $\mathcal{U}$ of all unimodal maps endowed with the topology of uniform convergence. Examples of unimodal maps are the well known family of quadratic maps, $x \mapsto ax(1-x)$ for $a \in [0,4]$, and the well known family of tent maps, $x \mapsto \frac{a}{2} - a\left|x - \frac{a}{2}\right|$ for $a \in [0,2]$.
It is known that for general piecewise monotonic maps the topological entropy is lower semi-continuous, but it is not upper semi-continuous in general. According to a result of Michał Misiurewicz the topological entropy as a map defined on $\mathcal{U}$ is continuous at T, if $h_{\text{top}}(T) > 0$ (it need not be upper semi-continuous, if $h_{\text{top}}(T) = 0$). Moreover, a result of Franz Hofbauer states that every unimodal map T with positive entropy has a unique measure of maximal entropy. This means that there exists a unique T-invariant Borel probability measure μ, such that the measure theoretic entropy $h_\mu(T)$ equals the topological entropy $h_{\text{top}}(T)$. If we assign to each $S \in \mathcal{U}$ with $h_{\text{top}}(S) > 0$ the unique measure μ_S of maximal entropy, and if we endow the collection of Borel probability measures with the w*-topology, then the map $S \mapsto \mu_S$ is continuous at T for every unimodal map T with positive entropy.
Define $\omega(T)$ as the collection of all ω-limit sets of T. This means that $\omega(T)$ consists of all x such that there is a y and an increasing sequence $(n_k)_{k\in\mathbb{N}}$ of natural numbers with $\lim_{k\to\infty} T^{n_k} y = x$. Assume that the collections of all subsets of $[0,1]$ is endowed with the Hausdorff metric. In a joint paper with Franz Hofbauer conditions implying the continuity of $S \mapsto \omega(S)$ at T are investigated.

Dynamics of Timoshenko beams by Difference Equations using NURBS

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A new approach to a dynamic analysis of one dimensional systems using NURBS (Non-Uniform Rational B-Splines) is presented. The basic concept of the Isogeometric Analysis has been given in [1]. The Isogeometric Analysis may be viewed as a logical extension of the classical FEM and geometrically it is based on the Computer Aided Design approach (CAD).
In this work the efficiency of this approach using the difference formulation to the analysis of Timoshenko beam is shown. For a typical separated interior element the stiffness and mass matrix for different order of B-spline basis functions are obtained.
If we take into account the third order of B-spline basis functions (p=3) and the regular division of one dimensional system, we get the difference equilibrium equation in free vibrations motion of the form:

$$(\Delta^6 + 6\Delta^4)w_r - \frac{\Omega^2}{840}(\Delta^6 + 126\Delta^4 + 1680\Delta^2 + 5040)w_r = 0$$

where $\Omega^2 = \rho A\omega^2 a^4/EI$ is the frequency parameter, w_r - the transverse displacement amplitude, Δ^n - the n-th order central difference operator.
The above equations are equivalent to the infinite set of equations and they can be solved analytically. Having found the solution in a closed form it is very easy to carry out a parameter study and to examine parameter effects on wave propagation phenomenon. The results are compared with exact and FEM calculations.

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Bifurcation Theory for Nonautonomous Difference Equations

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Although, bifurcation theory of equations with autonomous and periodic time dependence is a major object of research in the study of dynamical systems since decades, the notion of a nonautonomous bifurcation is not yet established. In this talk, an approach to overcome this deficit is presented in the context of nonautonomous difference equations. Based on special notions of attractivity and repulsivity, nonautonomous bifurcation phenomena are studied. We obtain generalizations of the well-known one-dimensional transcritical and pitchfork bifurcation.

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Regularly Varying Sequences and Second Order Difference Equations

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(Joint work with S. Matucci, *University of Florence, Italy*)

We will present some of the basic properties of regularly and rapidly varying sequences. Then we will apply them to study the asymptotic behavior of second order difference equations. In particular, we will give necessary and sufficient conditions guaranteeing that the solutions of these equations are slowly varying or regularly varying (with a nonzero index) or rapidly varying. This characterization is exhaustive in a certain sense. We will also discuss relations with known classification of nonoscillatory solutions and with the notion of recessive solutions. Finally we will mention some possible directions for a future research.

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Conjugacy of Difference Equations in the Neighbourhood of Invariant Manifold

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In Banach space $\mathbf{X} \times \mathbf{E}$ the system of difference equations

$$\begin{aligned} x(t+1) &= g(x(t)) + G(x(t), p(t)), \\ p(t+1) &= A(x(t))p(t) + \Phi(x(t), p(t)) \end{aligned} \tag{1}$$

is considered. Sufficient conditions under which there is an Lipschitzian invariant manifold $u \colon \mathbf{X} \to \mathbf{E}$ are obtained. Using this result we find sufficient conditions of conjugacy (1) and

$$\begin{aligned} x(t+1) &= g(x(t)) + G(x(t), u(x(t)), \\ p(t+1) &= A(x(t))p(t). \end{aligned} \tag{2}$$

This result allows one to replace the given system by a much simpler one. Relevant results concerning partial decoupling and simplifying of the noninvertible difference equations are given also.

Positive Periodic Solutions for Neutral Difference Equations with Variable Delay

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In this talk, we investigate a type of neutral difference equation with variable delay. By applications of Krasnosell'skii's fixed point theorem and some new techniques, various sufficient conditions for the existence of one or twin positive periodic solutions are established.

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Exact Analysis of the Adiabatic Invariants in Time-dependent Harmonic Oscillator

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The theory of adiabatic invariants has a long history, and very important implications and applications in many different branches of physics, classically and quantally, but is rarely founded on rigorous results. It began with the classical paper by Einstein in 1911, following a suggestion by Lorentz in the same year. We treat the general one-dimensional harmonic (linear) oscillator with time-dependent frequency whose energy is generally not conserved, and analyse the evolution of the energy and its statistical properties, like the distribution function of the final energies evolved from an initial microcanonical ensemble. This distribution function turns out to be universal, i.e. independent of the nature of the frequency as a function of time. The theory is interesting from the mathematical point of view as it comprises elements of the theory of dynamical systems, the probability theory and discrete mathematics, and sheds new light on the understanding of the adiabatic invariants in dynamical systems.

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Non-exponential Stability and Decay Rates in Nonlinear Stochastic Difference Equation with Unbounded Noises

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We consider stochastic difference equation

$$x_{n+1} = x_n\biggl(1 - hf(x_n) + \sqrt{h}g(x_n)\xi_{n+1}\biggr), \quad n = 0, 1, \dots,$$

where functions f and g are nonlinear and bounded, random variables ξ_i are independent and $h > 0$ is a nonrandom parameter.
We establish results on asymptotic stability and instability of the trivial solution $x_n \equiv 0$. We also show, that for some natural choices of the nonlinearities f and g, the rate of decay of x_n is approximately polynomial: we find $\alpha > 0$ such that x_n decay faster than $n^{-\alpha+\varepsilon}$ but slower than $n^{-\alpha-\varepsilon}$ for any $\varepsilon > 0$.
It also turns out that if $g(x)$ decays faster than $f(x)$ as $x \to 0$, the polynomial rate of decay can be established exactly, $x_n n^\alpha \to const$. On the other hand, if the coefficient by the noise does not decay fast enough, the approximate decay rate is the best possible result.

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Hopf Bifurcation with $\mathbf{S}_N$-Symmetry

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In this talk we will discuss Hopf bifurcation with $\mathbf{S}_N$-Symmetry. Stewart [2] proved the existence of two branches of periodic solutions ($\mathbf{C}$-axial), up to conjugacy, in systems of ordinary differential equations with $\mathbf{S}_N$-symmetry. He considers the standard action of $\mathbf{S}_N$ on $\mathbf{C}^{N,0}$ which is an $\mathbf{S}_N$-simple space. We derive, for $N \geq 4$, the $\mathbf{S}_N$-equivariant map up to degree five. We use the Equivariant Hopf Theorem to prove the existence of branches of periodic solutions and we determine conditions on the parameters that describe the stability of the different types of bifurcating periodic solutions.

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Trees in Arithmetic Dynamics

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Several combinatorial features of one-dimensional discrete dynamical systems are better understood by means of trees of symbolic orbits. Some algebraic properties of these are considered in the case of β-transformations and in other related dynamical systems.

Some Considerations on Fuzzy Dynamical Systems

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We consider fuzzy differential equations, that is, differential equations in the space of fuzzy real numbers E^1, which contains the functions $u : \mathbb{R} \longrightarrow [0,1]$ satisfying that:

i) u is normal: there exists $x_0 \in \mathbb{R}$ with $u(x_0) = 1$.

ii) u is fuzzy convex: for all $x,\ y \in \mathbb{R}$ and $\lambda \in [0,1]$,

$$u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}.$$

iii) u is upper-semicontinuous.

iv) The support of u, $[u]^0 = \overline{\{x \in \mathbb{R} \,:\, u(x) > 0\}}$ is a compact set.

E^1 is a complete metric space considering the distance d_∞ defined, for $u,\ v \in E^1$, by

$$d_\infty(u,v) = \sup_{\alpha \in [0,1]} d_H\left([u]^\alpha, [v]^\alpha\right),$$

where d_H denotes the Hausdorff distance between nonempty compact convex subsets of $\mathbb{R}$ and

$$[u]^\alpha = \{x \in \mathbb{R} \,:\, u(x) \geq \alpha\},\ \ \alpha \in (0,1],$$
$$[u]^0 = \overline{\{x \in \mathbb{R} \,:\, u(x) > 0\}}$$

are the level sets of the fuzzy number u.
We analyze the behavior of the solutions for initial value problems associated to certain fuzzy differential equations, paying special attention to its vantages and disadvantages in relation with the modelization of real phenomena subject to uncertainty and comparing the results obtained with the corresponding results in the ordinary case. We also analyze the dynamics of a class of fuzzy mappings, showing the analogies and differences between the fuzzy and the classical case.

Difference Operators of Generalized Askey Wilson Type

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We present facts about difference operators of generalized Askey Wilson type and apply them to the theory of discrete Fourier transforms and discrete Schrödinger operators.

Population Models with Allee Effect

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We discuss several Mathematical models that exhibit the Allee effect. This implies there are two attracting (stable) stationary population levels, the zero population and a positive one S, separated by an unstable or repelling stationary population level A which we call the Allee point. Populations starting out below the Allee point are driven to extinction while those starting out above are attracted to S.
In order to model an environment that varies with time we consider the special case in which the parameters in the model vary periodically. The question then arises as to whether there is an unstable or "Allee" periodic state P_A and an attracting or stable periodic state P_S.
For certain population models we explore conditions guaranteeing the existence of P_S.

On a k order System of Lyness Type Difference Equations

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We consider the following system of Lyness type difference equations

$$x_1(n+1) = \frac{a_k x_k(n) + b_k}{x_{k-1}(n-1)},\ x_2(n+1) = \frac{a_1 x_1(n) + b_1}{x_k(n-1)},\ x_i(n+1) = \frac{a_{i-1} x_{i-1}(n) + b_{i-1}}{x_{i-2}(n-1)},$$

where $i = 3, 4, ..., k$, a_i, b_i, $i = 1, 2, ..., k$ are positive constants, $k \geq 3$ is an integer and the initial values are positive. We study the existence of invariant, the boundedness, the persistence and the periodicity of the positive solutions of this system.

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Maximal and Minimal Solutions of Neutral Equations

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In this talk we set together some results obtained for higher order nonlinear neutral difference equations. Classification of nonoscillatory solutions of considered equations and its companion sequence is presented. Sufficient conditions for existence of the minimal and maximal solutions of nonlinear neutral difference equations, are given. In each case Schauder Fix Point Theorem is used but the different operators are employed depending on asymptotic property which is treated. Finally for a class of even order nonlinear neutral difference equations the conditions under which the eventually positive solutions of considered equation can be classified into three nonempty distinct categories are given, eg. the sufficient condition under which considered equation has solution which converges to zero as well necessary and sufficient conditions under which considered equation has solution which tends to nonzero constant and which diverges to infinity.

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Riccati Difference Equations with Real Period-2 Coefficients

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We give a detailed analysis of the "forbidden set"of the Riccati difference equation

$$x_{n+1} = \frac{\alpha_n + \beta_n x_n}{A_n + B_n x_n},\ n = 0, 1, \ldots$$

where the coefficient sequences

$$\{\alpha_n\}_{n=0}^{\infty},\ \{\beta_n\}_{n=0}^{\infty},\ \{A_n\}_{n=0}^{\infty},\ \{B_n\}_{n=0}^{\infty}$$

are periodic sequences of real numbers with (not necessarily prime) period-2, and where the initial condition $x_0 \in \Re$. The character of the solutions through the "good points"of the equation is also presented.

Longtime Dynamics of Eulerian Time Discretizations for Differential Delay Equations with Feedback Properties

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One of the central issues arising in the study of time-discretizations of differential equations is to show that the longtime dynamics of the discrete approximation is - in some sense - "close to" the longtime dynamics of the related differential equation. For example, if the discrete problem has a periodic cycle does it follow that the continuous time problem has a periodic solution, and vice versa? Under a suitable hyperbolicity assumption, the answer is yes! However, what happens when the hyperbolicty is absent? In this lecture, we will describe a class of cyclic systems of differential delay equations with a feedback property, and show that the longtime dynamics of the discrete problem forms an excellent model for the longtime dynamics of the continuous time problem. It is not known whether this class of cyclic systems has the hyperbolicity properties needed to reduce the issue to the classical methods.

Difference Equations and Nonlinear Boundary Problems for Hyperbolic Systems

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According to Sharkovsky et al, [3], the solution of a class of linear hyperbolic systems with nonlinear boundary conditions and consistent initial conditions can be written via the iteration of a map of the interval. In this work we characterize the solution of such problems, with a vortex as initial condition and the iteration of a bimodal map of the interval, using bimodal topological invariants.

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Difference Equations with Continuous Time: Theory and Applications

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We build basics of the qualitative theory of continuous time difference equations $x(t+1) = f(x(t))$ with the method of going to the infinite-dimensional dynamical system induced by the equation. For the study this system we suggest a general approach to analyzing the asymptotic dynamics of nondissipative systems on continuous functions spaces. Using this approach allows us to derive properties of the solutions from that of the ω-limit sets of trajectories of the corresponding dynamical system. In particularly, typical continuous solutions are showed to be those tending (in the Hausdorff metric for graphs) to upper semicontinuous functions whose graphs are self-similar and, in wide conditions, fractal; there may exist especially nonregular solutions described asymptotically exactly by random processes. We introduce the notion of self-stochasticity in deterministic systems — a situation when the global attractor contains random functions. Substantiated is a scenario for a spatial-temporal chaos in distributed parameters systems with regular dynamics on attractor: the attractor consists of cycles only and the onset of chaos results from the very complicated structure of attractor "points" which are elements of some function space (different from the space of smooth functions). We develop a method to research into boundary value problems for partial differential equations, that bases on the reduction to difference equations.

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Genus and Braid Index Associated to Sequences of Renormalizable Lorenz Maps

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(Joint work with Nuno Franco, CIMA-UE and Department of Mathematics, University of Evora)

We describe the structure of renormalizable Lorenz knots and links and obtain explicit formulas for the braid index and genus of, respectively, Lorenz knots and links associated with n- renormalizable Lorenz maps.

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Why it is important to understand the dynamics of triangular maps?

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The dynamics of triangular (or skew-product) maps of the square, given by F : $(x, y) \mapsto (f(x), g(x, y))$ became interesting since 1979 when Kloeden proved that the Sharkovsky's theorem concerning coexistence of cycles of a continuous map of an interval is valid also for them. We briefly recall the history of research, pointing out the main results. Then we provide a survey of recent results, related the problem of classification of triangular maps formulated by A. N. Sharkovsky more than twenty years ago. We give information concerning special techniques that were developped in order to solve this problem. Finally, we show that these techniques proved to be very useful in the theory of (abstract) topological dynamical systems. Some open problems and conjectures will be presented.

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Embedding Systems in Larger Monotone Systems versus Comparison Methods

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The method of embedding certain non-monotone dynamical systems into larger monotone systems and comparing solutions of the original system with upper and lower solutions of the larger system has been continuously rediscovered. It can lead to global stability results for the original system. Is this method better than an alternative comparison method which constructs monotone upper and lower systems of the same dimension as the original system? We explore these different but related approaches to give a tentative answer to the question.

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Minimal Sets in Discrete Dynamics

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Minimal sets in discrete dynamical systems given by a compact metric space and its continuous (in general non-invertible) selfmap will be discussed. We start with a short survey of some of the known results about the topological structure of minimal sets in various spaces; in particular we will pay an attention to the case when the whole space is a minimal set. Then minimal sets on 2-manifolds will be discussed in more details. A recent result that proper minimal sets on compact connected 2-manifolds are nowhere dense will be presented together with main tools from the proof.

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Oscillations of Linear Difference Equations with variable delay

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Consider the first order linear difference equation with delay argument

$$\Delta u(k) + p(k)\, u(\tau(k)) = 0, \quad k \in N, \tag{1}$$

where $\Delta u(k) = u(k+1) - u(k)$, $p : N \to R_+$, $\tau : N \to N$, $\tau(k) \leq k-1$ and $\lim\limits_{k \to +\infty} \tau(k) = +\infty$. It is proved that all solutions of (1.1) are oscillatory if

$$\limsup_{k \to +\infty} \sum_{i=\tau(k)}^{k} p(i) > 1. \tag{2}$$

or

$$\liminf_{k \to +\infty} \sum_{i=\tau(k)}^{k-1} p(i) > \frac{1}{e}. \tag{3}$$

New oscillation criteria are also presented for Eq. (1) when the conditions (2) and (3) are not satisfied.

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Distributional (and other) Chaos Almost Everywhere

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In this talk I will give a survey of results concerning the measure of scrambled sets for continuous maps on compact metric spaces. The talk will concern chaos in the sense of Li and Yorke, ω-chaos and distributional chaos.

Basic Properties of Partial Dynamic Operators

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(Joint work with H.B. Thompson, *University of Queensland*)

Motivated by the importance of maximum principles in the theory of partial differential equations and the numerical analysis, we establish simple maximum principles for basic partial dynamic operators on general time scales. As in the case of ordinary dynamic operators we reveal a set of results and counterexamples which illustrate the distinct behaviour in continuous and discrete case. Finally, we provide some immediate consequences and prove uniqueness results to problems involving partial dynamic operators.

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On Lengths of Semicycles of Solutions of Difference Equations

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Studying semicycles of nonlinear difference equations is a tool usually used in the investigation of the boundedness and attractivity of solutions of difference equations. To the best of our knowledge there are only results concerning semicycle analysis on some particular equations. We present some general results regarding lengths of semicycles of solutions of some difference equations about their equilibria. Several applications of these results are given.

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Numerical Results for Some Schrödinger Difference Equations

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We perform numerical calculations for some Schrödinger difference equations. The main interest lies in the numerical visualization of the oscillations and the eigenvalues. For the treatment of more sophisticated potentials, we introduce an adaptive basic linear grid and determine and illustrate the eigenvalues of the Schrödinger operators by considering this adaptive grid.

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Asymptotic and Oscillatory Properties of Solutions of Neutral Difference Systems

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In this talk, we discuss the asymptotic and oscillatory behavior of two -dimensional forced neutral delay difference system of the form.

$$\begin{aligned} \Delta(x_n - a_n x_{n-k}) &= -p_n y_n + e_n \\ \Delta y_n &= q_n f(y_{n-l}) \end{aligned}$$

with appropriate conditions on the known quantities.

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On the Analytic Structure of the Complex Blasius Problem

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The complex solution of the Blasius problem is studied using a functional-analytic technique. By use of this method, an equivalent discrete version of the Blasius problem is found and numerical results, concerning its real as well as its complex valued solution, are given. The obtained results indicate that the complex Blasius function exhibits an oscillatory behavior. Moreover, a conjecture regarding the singularities of its solution in the complex plane is strengthened.

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Lyapunov-type Inequalities for Nonlinear Systems on Time Scales

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We state and prove some Lyapunov-type inequalities for nonlinear systems on an arbitrary time scale $\mathbb{T}$ by using elementary time scale calculus, so that the well-known case of systems of nonlinear differential equations (when $\mathbb{T} = \mathbb{R}$ in [7]) and the recently developed systems of nonlinear difference equations (when $\mathbb{T} = \mathbb{Z}$ in [8]) are unified. These inequalities enable us to obtain a criterion of disconjugacy for this type of nonlinear systems. Special cases of our results contain the classical Lyapunov inequality for both differential and difference equations.

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Discrete Non-linear Model of Information Evolution

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Consideration of discrete dynamic systems describing non-linear processes, features and structure has a special actuality in the modern world because of non-linear scientific paradigm importance. In the present work we propose and investigate non-linear dynamic model of information evolution and propagation. In contrast to known continuous dynamic model [1] we consider the discrete model, in which is possible an interaction of elements - information types - from different time layers. In general case, the information type is described by the vector, which components are ranked. We have conducted this model analysis and computation experiment. In particular they have shown that one of the possible evolution scenarios can be a stabilization of one of information types at a value corresponding to one of the preceding time layers. Such a value is established after passing of the corresponding numerical information characteristics through its maximum value. This work was supported by the Russian Fund for Basic Research (Grant No. 06-01-00548-a) and the Russian Humanitarian Scientific Fund (Grant No. 06-03-00-59-a).

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Lyapunov-Schmidt Revisited

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In this review talk we describe some older and some newer results on the use of the Lyapunov-Schmidt reduction for the bifurcation analysis of periodic orbits, both in discrete and in continuous systems. We give a formulation of the reduction result which emphasizes the structure preserving properties and the relation with center manifold reduction and normal form theory.

On Some Nonlinear Difference Equations with Periodic Coefficients

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We study the second order nonlinear difference equation

$$x_n = \max\left\{\frac{1}{x_{n-1}}, \frac{E_n}{x_{n-2}}\right\}, \quad n = 0, 1, \ldots$$

where the coefficients E_n are positive and periodic with arbitrary period k. The special case when $k \leq 3$ has been completely investigated by G. Ladas et al. It was conjectured that if $E_n < 1$ for all n, then every positive solution of this equation is periodic with period 2. Here we prove that this conjecture is true.

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Palais-Smale Approaches to Semilinear Elliptic Equations in Unbounded Domains

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In this talk, we are going to introduce a new method to construct a Palais-Smale Sequence. Moreover, we are going to find a condition for a domain such that in which a Palais-Smale Sequence admits a nonzero limit. The nonzero limit turns out to be a nonzero solution of the associate semilinear elliptic equation.

Perturbation Method for Linear Difference Equations with Small Parameters

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We consider a boundary value problem for a linear difference equation with several widely different coefficients. We study the existence and uniqueness of its solution and we give successive asymptotic approximations for this solution, obtained by a simple iterative method. This method improves the *singular perturbation method*, it offers considerable reduction and simplicity in computation since it does not require to compute *boundary layer correction solutions*.

(2+1)-Dimensional Nonisospectral Lattice Hierarchy and Generalized Discrete Painlevé Hierarchy

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This is a joint work with Dr. P R Gordoa and Dr. A Pickering. By considering proper (2+1)- dimensional nonisospectral discrete linear problems, new (2+1)- dimensional nonisospectral integrable lattice hierarchies—2+1 nonisospectral relativistic Toda lattice hierarchy, 2+1 nonisospectral negative relativistic Toda lattice hierarchy, and new 2+1 nonisospectral Toda lattice hierarchy — are presented. We will show that the reductions of new 2+1 nonisospectral lattice hierarchies lead to (2+1)- dimensional nonisospectral Volterra lattice hierarchy and (2+1)- dimensional nonisospectral negative Volterra lattice hierarchy. We also present new (1+1)- dimensional nonisospectral integrable lattice hierarchies and new ordinary difference hierarchies. Those difference hierarchies yield the generalized discrete Painlevé hierarchies including a dP_I hierarchy, an asymmetric dP_I hierarchy, an alternative dP_I hierarchy and an alternative dP_{II} hierarchy.

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