Method Of Fundamental Solutions For Modeling Electromagnetic Wave Scattering Problems

Der-Liang Young(1) and Jhih-Wei Ruan(1)

Abstract: *In this paper we attempt to construct the electromagnetic wave scattering field by a given incident wave. For two-dimensional problems the normal incident plane wave scattering by a conducting circular cylinder with infinite dimension in the z-direction are discussed. For three-dimensional problem we focus on the electromagnetic scattering wave by a conducting sphere. The method of fundamental solutions (MFS) for the vector Helmholtz equations in the frequency domain are employed to simulate the electromagnetic wave problems. Both the 2D and 3D homogeneous electromagnetic wave scattering are compared with the analytical as well as other numerical methods, such as the finite element method (FEM) or the boundary element method (BEM). The MFS has demonstrated to render very efficient and accurate results as comparing with the analytical and other numerical solutions.*

1. Introduction

The scattering of electromagnetic waves by objects is an important problem both in academic researches as well as industrial applications. Hsiao et al. [1] applied the boundary element method to simulate three dimensional electromagnetic wave scattering equations. Morgan et al. [2] [3] employed the finite element time domain method to solve the three dimensional scattering problems. Ledger [4] used the edge finite element method to simulate the two-dimensional homogenous and dielectric scattering phenomena. And in the mean time the analytical solutions of scattering problems can be found in other research areas [5].

Different numerical methods have been used to investigate the two and three dimensional scattering problems of electromagnetic waves in our research group. For example, Chen [6] used the conventional BEM to simulate 2D and 3D scattering wave problems. Chiu [7] used the non-singular BEM to study the scattering of electromagnetic waves over a conducting circular cylinder and a conducting sphere. In this study, we will try to use the method of fundamental solution (MFS) as a meshless numerical method to solve the homogenous vector Helmholtz equations in the frequency domain, which represent the governing equations of the electromagnetic wave scattering of a perfect electric conductor. The present method will be used to compare with exact solutions and other numerical results to assess the accuracy and efficiency of the simulations.

(1) Department of Civil Engineering & Hydrotech Research Institute, National Taiwan University, Taipei, TAIWAN 10617

(Corresponding to DL YOUNG, Tel/Fax: +886-2-23626114, Email:dlyoung@hy.ntu.edu.tw)

2. Mathematical formulation

2.1 Governing equations

Maxwell's equations which governing the propagation of electromagnetic waves are reduced to the homogenous vector Helmholtz equations in the frequency domain by assuming that the electric and magnetic fields are both time-harmonic. The homogenous vector Helmholtz equations are considered in the following forms.

$$\nabla^2 \vec{E}_0 + k^2 \vec{E}_0 = 0 \qquad (1)$$

$$\nabla^2 \vec{H}_0 + k^2 \vec{H}_0 = 0 \qquad (2)$$

where $\vec{E}_0$ and $\vec{H}_0$ denote the time-harmonic electric and magnetic field intensity vectors in these equations respectively, and k represents the wave number, is determined by the following formula

$$k = \omega\sqrt{\mu\varepsilon} \qquad (3)$$

where μ and ε denote the permeability and permittivity respectively, whilst ω is the angular frequency of the material.

The total electric and magnetic fields, regarded as being combined with incident and scattered components, were represented in these forms.

$$\vec{E}^t = \vec{E}^i + \vec{E}^s \qquad (4)$$

$$\vec{H}^t = \vec{H}^i + \vec{H}^s \qquad (5)$$

The superscript i and s indicate the incident field and the scattered field respectively. Due to the linearity, scattered fields $\vec{E}^s$ and $\vec{H}^s$ will also satisfy Equation (1) and Equation (2). In other words, the governing equations of scattered fields could be written as

$$\nabla^2 \vec{E}^s_0 + k^2 \vec{E}^s_0 = 0 \qquad (6)$$

$$\nabla^2 \vec{H}^s_0 + k^2 \vec{H}^s_0 = 0 \qquad (7)$$

2.2 Boundary conditions

The region outside the perfect electric conductor is the main concerned domain. Applying the integral form of Maxwell's equations to a small region, we obtain the tangential component of total electric field and the normal component of the total magnetic fields. Both vanish on the surface of a perfect electrical conductor, i.e.

$$\vec{n} \times \vec{E}^s = -\vec{n} \times \vec{E}^i \qquad (8)$$

$$\vec{n} \bullet \vec{H}^s = -\vec{n} \bullet \vec{H}^i \qquad (9)$$

where $\vec{n}$ denotes the unit normal vector to the conductor surface.

3. Numerical algorithm

3.1 Method of fundamental solution(MFS)

Since Equation (6) and (7) are the homogenous vector Helmholtz equations, we may assume that the homogeneous solution is a linear combination of the fundamental solution of the Helmholtz operator [8], i.e.

$$\Phi_h(\mathrm{x}_j) = \sum_{i=1}^{M} \beta_i g(r_{ij}) \tag{10}$$

where

$$g(r) = \begin{cases} -\frac{1}{4}Y_0(kr) - i\frac{1}{4}J_0(kr) & \text{for two dimension} \\ \frac{\cos(kr)}{4\pi r} - i\frac{\sin(kr)}{4\pi r} & \text{for three dimension} \end{cases} \tag{11}$$

are the fundamental solutions of the Helmholtz operator, $r_{ij} = \left|\xi_i - \mathrm{x}_j\right|$ is the distance between a field point x_j and the source point ξ_i, and M is the number of source nodes. Where the derivative of the homogeneous solution is obtained through

$$\frac{\partial \Phi_h(\mathrm{x}_j)}{\partial x_k} = \sum_{i=1}^{M} \beta_i \frac{\partial g(r_{ij})}{\partial x_k} \tag{12}$$

After β_i 's have been solved through the method of the collocation of the boundary and source points, we can find the homogeneous solution through the equation (10).

3.2 Computation of the radar cross section(RCS)

The radar cross section(RCS), obtained by employing the transformation of a near field to the far field, defined as [9]

$$RCS = 10\log_{10}(\sigma)$$

where

$$\sigma_{TM} = \lim_{r\to\infty} 2\pi r \frac{\left|E_z^s\right|^2}{\left|E_z^i\right|^2} \quad \text{in two dimensions} \tag{13}$$

$$\sigma_{3D} = \lim_{r\to\infty} 4\pi r^2 \frac{\left|E^s\right|^2}{\left|E^i\right|^2} = \lim_{r\to\infty} 4\pi r^2 \frac{\left|H^s\right|^2}{\left|H^i\right|^2} \quad \text{in three dimensions} \tag{14}$$

4. Numerical Examples

4.1 Scattering by a conducting circular cylinder

A perfect electric circular cylindrical conductor, one of the most widely used to represent practical scatterers, has been computed to demonstrate the method of fundamental solution of the proposed procedure. The first example we assume that a transverse magnetic (TM^z) plane wave is normally incident upon the perfect conducting circular cylinder with 1 unit radius. The incident wave has a wavelength of 2π unit and propagates in z direction. To achieve this scattering problem, we make the space consist of 40 nodes and take permittivity and permeability equal to 1. Fig.1 shows the contours of the real part of scattered electric field, E_z^s. To compare the exact solutions [5] and the numerical simulations, Fig.2 shows the distribution of exact and computed imaginary part of E_z^s along radius equal to 2 unit. And the radar cross section (RCS) of the scatterer is also shown in Fig.3. Similar to this procedure, we can also get the scattering magnetic field depicted by TE^z waves through equation (12).

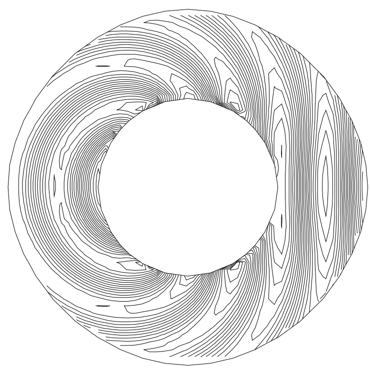

Fig.1 The contour distribution of Re [E_z^s .]

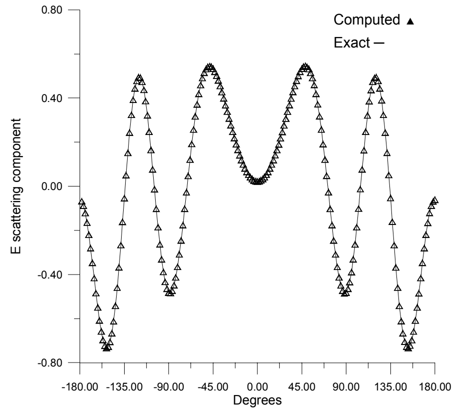

Fig.2 Comparison of Im[E_z^s] between the exact solution (solid line) and the computed solution (triangle) at the radius 2 unit.

4.2 Scattering by a conducting sphere

A perfect electric spherical conductor is a very classic problem for the scattering of electromagnetic waves. We assume that a transverse electric field of a uniform plane wave polarized in the x direction is traveling along the z axis, incident on a conducting sphere with radius 0.5 unit. After the transformation [10] the scattering Intensity of electric field can be expressed as

$$E_x^s = \sin\theta\cos\phi E_r^s + \cos\theta\cos\phi E_\theta^s - \sin\phi E_\phi^s \qquad (15)$$

where E_r^s , E_θ^s and E_ϕ^s can be obtained using the equation (8) and equation (9), therefore we can compute the scattering electric field intensity by setting permittivity and permeability constant equal to 1. Fig.4 shows the contours of the real part of the scattering electric component in z direction on x=0 plane and surface of the sphere. Finally to compare the accuracy and efficiency of the proposed MFS, the radar cross section (RCS) of exact solution [5] and other numerical solution including of the MFS we proposed, the conventional BEM of Chen [6], the non-singular BEM of Chiu [7] and the FEM of Morgan [3] are all illustrated in Fig.5.

It is worthy while to observe that all the solutions give the same accurate results as compared with the exact solutions. However, only 800 nodal points are used in the MFS, 3200 nodal points are employed in both the conventional BEM and non-singular BEM, while 589,505 elements,706,999 edges and 99,991 nodes are used in the FEM formulation.

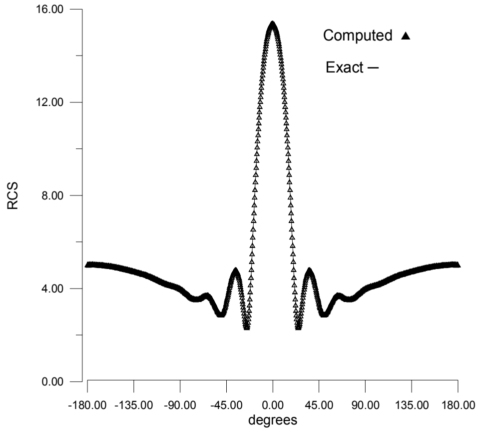

Fig.3. Comparison of the RCS between computed solution (triangle) and exact solution (solid line)

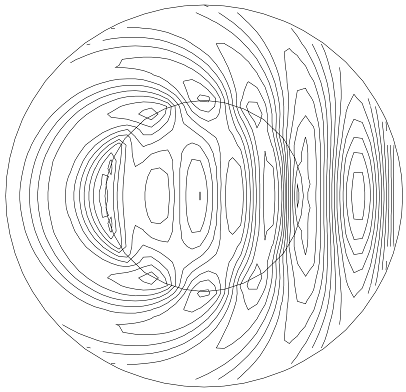

Fig.4. The contour of the Re[E_z^s] on x=0 plane and surface of the sphere.

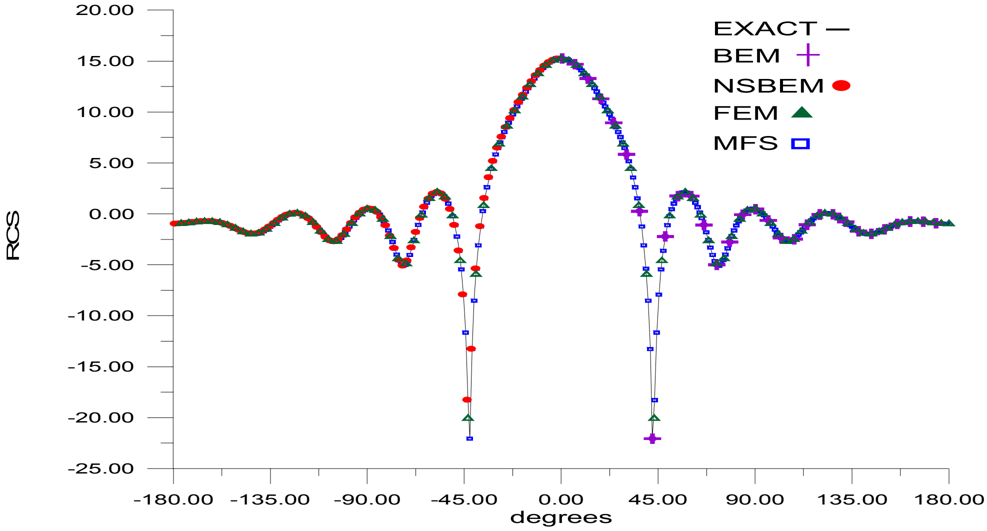

Fig.5. Comparison of the RCS between MFS(square), BEM(cross), NSBEM(circle), FEM(triangle) and exact solution(solid line) at $\phi = 0$

5. Conclusion

The innovative numerical scheme of application of the method of fundamental solution (MFS) to solve the electromagnetic scattering problems has revealed that the MFS is better off than the traditional numerical methodologies Both a perfect electric 2D cylindrical conductor and a 3D perfect electric spherical conductor are performed in this study. The MFS has already satisfied the governing equation and only the boundary nodes need to be collocated so that one dimension can be reduced and also the meshfree merit is achieved. We have demonstrated that for the simulation of an electromagnetic wave scattering of a 3D conducting sphere, the usage of 800 boundary nodes will render the same resolution as those obtained by the conventional BEM, the no-singular BEM as well as the FEM schemes. All the numerical simulations possess the same accuracy as comparing to the exact solutions. However the MFS has shown the simple, efficient, and powerful aspects as far as the numerical algorithm is concerned. The numerical simulation of the scattering of electromagnetic waves by the method of fundamental solution (MFS) has opened a new frontier for the computational electromagnetic wave research.

Acknowledgments

This study is supported by the National Science Council, Taiwan, it is greatly appreciated.

References

[1] G. C. Hsiao, R. E. Kleinman, D.-Q. Wang, Applications of boundary integral equation methods in 3D electromagnetic scattering, Journal of Computational and Applied Mathematics Vol. 104, pp89-110, 1999

[2] K. Morgan, O. Hassan, J. Perarie, A time domain unstructured grid approach to the simulation of electromagnetic scattering in piecewise homogeneous media, Computer Methods in Applied Mechanics and Engineering, Vol. 134, pp17-36, 1996.

[3] K. Morgan, P. J. Brookes, O. Hassan, N. P. Weatherill, Parallel processing for the simulation of problems involving scattering of electromagnetic waves, Computer Methods in Applied Mechanics and Engineering, Vol. 152, pp157-174, 1998.

[4] P. D. Ledger, Edge Elements for Waveguide and Scattering Problem, MS thesis, University of Wales, Swansea, 1999.

[5] C. A. Balanis, Advanced Engineering Electromagnetics, John Wiley & Sons, Inc., 1989.

[6] G. Q. Chen, Application of Helmholtz equation by boundary element method to the waveguide and scattering propagation problems, MS thesis, National Taiwan University, Taiwan, 2002.

[7] C. L. Chiu, Non-singular Boundary Integral Equation for the Analysis of Electromagnetic Problems, MS thesis, National Taiwan University, Taiwan, 2002.

[8] C. C. Tsai, Meshless Numerical Methods and their Engineering Applications, Ph.D. Thesis. National Taiwan University, Taiwan, 2002.

[9] C. A. Balanis, Antenna Theory, John Wiley & Sons, Inc., 1982.

[10] R. F. Harrington, Time-Harmonic electromagnetic Fields, New York, NY: McGraw-Hill Book Company, Inc., 1961.