MAXIMAL KNOWN RANK OVER Q
(B. CONRAD, LISBON SUMMER SCHOOL, 2011)

The largest known rank for an elliptic curve over $\mathbf{Q}$ was discovered by Elkies in 2006, and it has rank at least 28 (but the exact rank is not known). The curve E is:

$$y^2 + xy + y = x^3 - x^2 - ax + b$$

where

$$a = 20067762415575526585033208209338542750930230312178956502$$

and

$$b = 44816117950305564670329856903907203748559443593191803612660082962919394487322434 29.$$

This elliptic curve has $E(\mathbf{Q})_{\mathrm{tors}} = 1$, and here are 28 explicit points which are proved to be independent by computing a certain real determinant and verifying that it is non-zero (via accurate approximation):

$$\begin{aligned}
P_1 &= (-21241500912543810732921374 63, 25985449205189959903051551107078062891153 1)\\
P_2 &= (23345098660347017568847545 37, 18872004195494469180868316552803627931531)\\
P_3 &= (-16717360540623690638790386 63, 25170937726114428780850694724131912604913 1)\\
P_4 &= (21391302601391566664929821 37, 36639509171439729202421459692941297527531)\\
P_5 &= (15347067644671207238854773 37, 85429585346017694289021032862781072799531)\\
P_6 &= (-27310794878756770333415750 63, 26252181548433219164128407262390214338 7531)\\
P_7 &= (27757262668445716497054585 37, 12845755474014060248869487699082640369931)\\
P_8 &= (14943857293271889575418338 17, 88486605527733405986116494514049233411451)\\
P_9 &= (18684382286208873585090652 57, 59237403214437708712725140393059358589131)\\
P_{10} &= (20089451088257437748665425 37, 47690677880125552882151750781541424711531)\\
P_{11} &= (23483605409180251696516329 37, 17492930006200557857340332476448804363531)\\
P_{12} &= (-14720840070904811744700086 63, 24664345065350371419994744154975979846913 1)\\
P_{13} &= (29241286077080612133632889 37, 28350264431488878501488356474767375899531)\\
P_{14} &= (53749938910660618932939345 37, 28618890842726338645117503191647989373153 1)\\
P_{15} &= (17096907682333545233340085 57, 71898834974686089466159700529215980921631)\\
P_{16} &= (24509540113535931440725951 87, 44452281735326343570492625506107147365 31)\\
P_{17} &= (29692547092735591674646749 37, 32766893075366270801333682543160469687531)\\
P_{18} &= (27119149349416926013328829 37, 20684366127783816986504139815065906135 31)
\end{aligned}$$

$$
\begin{aligned}
P_{19} &= (2007858607799685452877832893 7, 27796085411378066046560517256246240300915 31)\\
P_{20} &= (2158082450240734774317810697, 3499437340196402680996966224180090125473 1)\\
P_{21} &= (2004645458247059022403224937, 4804932978070464552243986699988847546 7531)\\
P_{22} &= (2975749450947996264947091337, 3339898982607532232020893441010485786 9131)\\
P_{23} &= (-2102490467686285150147347863, 2595763914598757895716773931716872032 27531)\\
P_{24} &= (3115831799150630349021945 37, 1681043852299806035401094729156601534 73931)\\
P_{25} &= (2773931008341865231443771817, 1263216283464992100241411627376927581 3451)\\
P_{26} &= (2156581188143768409363461387, 3512509296402290889700415051637517808 7331)\\
P_{27} &= (3866330499872412508815659137, 1211977556559442262930369267150258473 22531)\\
P_{28} &= (2230868289773576023778678737, 2855876003059748566338702060076864002 8531)
\end{aligned}
$$