SCHUBERT POLYNOMIALS AND SYMMETRIC FUNCTIONS
NOTES FOR THE LISBON COMBINATORICS SUMMER SCHOOL 2012

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1. Schubert polynomials exercises

Exercise 1.1.
1. *∂_i satisfies the "twisted Leibniz rule":*
$$\partial_i(pq) = (\partial_i p)q + (r_i p)(\partial_i q).$$
In these ways, ∂_i behaves somewhat like a derivative.
2. *Let p be a polynomial such that $\partial_i p = 0$ for all $i \neq n$. Show that p is a symmetric polynomial in $x_1, \ldots, x_n$.*

Exercise 1.2.
1. *Show $\ell(\pi \circ (i \leftrightarrow i+1)) = \ell(\pi) \pm 1$, with the sign depending on whether i is an ascent or descent of π.*
2. *Show $\ell(\pi) = \ell(\pi^{-1})$.*
3. *What is the maximum value of $\ell(\pi)$, $\pi \in S_n$?*

Exercise 1.3.
1. *Determine $\mathcal{S}_{(i \leftrightarrow i+1)}$.*
2. *Determine $\mathcal{S}_\pi$ for $\pi \in S_3$, thought of as the evident subgroup of the group of finite permutations of $\mathbb{N}$.*

Exercise 1.4. *Show that 12321, 13231, and 31231 are reduced words for the same permutation in S_4, and find all the other reduced words for that permutation.*