SCHUBERT POLYNOMIALS AND SYMMETRIC FUNCTIONS
NOTES FOR THE LISBON COMBINATORICS SUMMER SCHOOL 2012

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1. REDUCED WORDS AND SCHUBERT POLYNOMIALS EXERCISES

Exercise 1.1. *Show the following conditions on a permutation π are equivalent:*

(1) $\pi(i) = i$ *for* $i \leq m$.
(2) *Some reduced word for* π *does not use the letters* $1, \ldots, m-1$.
(3) *No reduced word for* π *uses the letters* $1, \ldots, m-1$.

Exercise 1.2. *Let* $\pi \circ \rho = \sigma$ *be a product of two permutations. Show that* $\ell(\pi) + \ell(\rho) \geq \ell(\sigma)$*, and*

$$\partial_\pi \partial_\rho = \begin{cases} \partial_\sigma & \text{if } \ell(\pi) + \ell(\rho) = \ell(\sigma) \\ 0 & \text{if } \ell(\pi) + \ell(\rho) > \ell(\sigma). \end{cases}$$

Exercise 1.3. *Show that* any *two words (not necessarily reduced) for* π *can be related by the commuting move, the braid move, and insertion/deletion of pairs* $i\,i$.

Exercise 1.4. *If* $\pi \in S_n$*, show* $\ell\left(\pi^{-1} w_0^n\right) = \binom{n}{2} - \ell(\pi)$.

Exercise 1.5. *Compute all the Schubert polynomials for* S_4*, starting with* $\mathcal{S}_{\underline{4321}}$ *and going down using divided difference operators.*

Exercise 1.6. *Compute all the Schubert polynomials for* S_4*, starting with* $\mathcal{S}_{\mathrm{Id}} = 1$ *and going* up *using the transition formula.*

Exercise 1.7. *Check the pipe dream formula for each* $\pi \in S_3$.