

EXERCISES FOR TRAPA LECTURE #1

1. Find a one dimensional subspace of traceless two-by-two real matrices whose image under the exponential map is compact.

2. Verify that the image of traceless real matrices under the exponential map consists of matrices with determinant one. Is the exponential map surjective onto matrices of determinant one?

3. In the formal expression

$$Z = \log(\exp(X)\exp(Y))$$

we verified in the lecture that

$$Z = X + Y + \frac{1}{2}[X, Y] + \dots$$

Write the cubic terms in  $\dots$  explicitly in terms of iterated brackets of  $X$  and  $Y$ .

4. Classify all finite, connected, undirected graphs with the following properties:

- (a) There are no self-loops in the graph.
- (b) Between any two vertices there is at most one edge.
- (c) Every vertex is labeled with a positive integer.
- (d) The label at any vertex  $v$  is equal to one-half the sum of the labels of the vertices connected to  $v$ .
- (e) There is a distinguished vertex labeled  $1^*$ .