PROBLEM SET FOR ENUMERATIVE GEOMETRY

Exercise 1. Compute the degree of the locus of completely reducible cubic curves in $\mathbb{P}^2$. Compute the degree of the locus of cubic curves in $\mathbb{P}^2$ that are three concurrent lines.

Exercise 2. Compute the degree of the locus of plane curves of degree d that contain a line as a component.

Exercise 3. Let V be a 4 dimensional vector space with basis e_1, e_2, e_3, e_4. Show that $\omega = e_1 \wedge e_2 + e_3 \wedge e_4 \in \bigwedge^2 V$ is not completely decomposable. Do this in two different ways. First, show that the corresponding point does not satisfy the Plücker relation for $G(2,4)$. Second, show that the dimension of the kernel of the map $V \to \bigwedge^3 V$ given by wedging by ω is not two.

Exercise 4. Write down the cell decomposition of $G(2,5)$ and $G(3,6)$ explicitly. Write down all the Plücker relations for $G(2,5)$.

Exercise 5. Show that the Euler characteristic of $G(k,n)$ is $\binom{n}{k}$. Compute the Betti numbers of $G(k,n)$.

Exercise 6. Work out the explicit geometric description of all the Schubert varieties in $G(2,5)$ and $G(3,6)$. Compute the intersection products of the Schubert classes in $G(2,5)$ and $G(3,6)$.

Exercise 7. The intersection of two general quadric three-folds in $\mathbb{P}^4$ is a del Pezzo surface S of degree 4. Compute the number of lines on S.

Exercise 8. Let C_1 and C_2 be two general twisted cubic curves. Compute the number of lines that are simultaneously secant to both C_1 and C_2.

Exercise 9. Compute the degree of the Grassmannian $G(2,n)$ under the Plücker embedding (hint: Catalan numbers may become useful).

Exercise 10. Find the number of conics that contain r general points and intersect $8-2r$ general lines in $\mathbb{P}^3$. As a challenge, find the number of conics that contain r general points, intersect s general lines and $11-2s-3r$ general planes in $\mathbb{P}^4$.