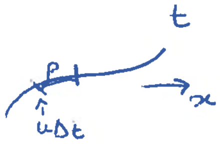
Esteban Tabak
04/07/2016



$\rho U \Delta t = Q \Delta t$

$\rho(x,t)$
U (x,t)
$(r(x,t,u))$
$Q(x,t)$
$Q = \rho U$

$U = \int u \cdot r(x,t,u)\, du$

$\rho = \int r(x,t,u)\, du$

$M(t) = \int_{x_1}^{x_2} \rho(x,t)\, dx$

$M(t_2) - M(t_1) = \int_{t_1}^{t_2} [Q(x_1,t) - Q(x_2,t)]\, dt$

$= \int_{x_1}^{x_2} \underbrace{[\rho(x,t_2) - \rho(x,t_1)]}_{\int_{t_1}^{t_2} \rho_t\, dt}\, dx = -\int_{x_1}^{x_2} Q_x\, dx$

$\rho_t + Q_x = 0$

$\rho_t + (\rho U)_x = 0$

$\rho_t + \nabla \cdot (\rho U) = 0$

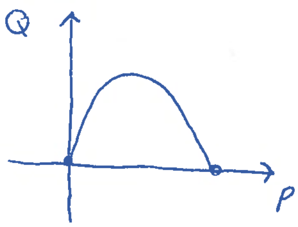
$\int_{x_1}^{x_2} \int_{t_1}^{t_2} (\rho_t + Q_x)\, dx\, dt = 0$

$\downarrow$

$\rho_t + Q_x = 0$

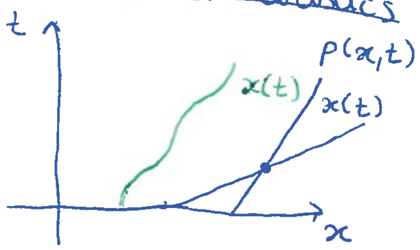
$Q = \rho U$

$Q = Q(\rho)$



$\rho_t + Q'(\rho)\, \rho_x = 0$

Method of Characteristics

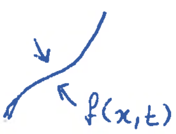


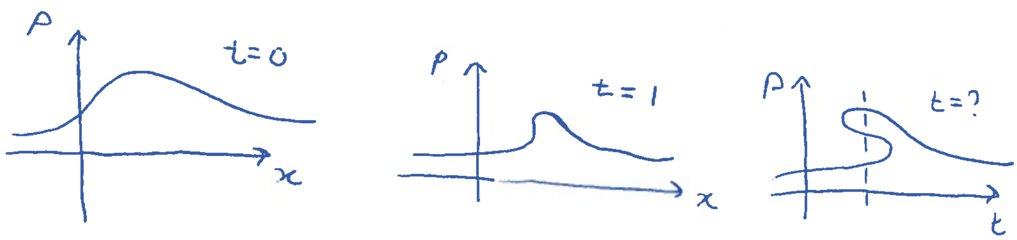
$\frac{d\rho}{dt}(x(t),t) = \rho_t + \underbrace{x'(t)}_{\text{choose as } Q'(t)} \rho_x = 0$

$x'(t) = Q'(\rho(x(t),t))$

$\frac{dx}{dt} = Q'(\rho)$

$\frac{d\rho}{dt} = 0$





$$\frac{dx}{dt} = Q'$$

$$Q(q) = \rho\, U(\rho)$$

$$Q' = U + \underbrace{\rho U'}_{<0} \leq U$$

$$\rho(x,t) = \rho(x - Q'(q)t, 0) = \rho_0(x - Q'(\rho)t) \qquad \rho_0(x) = \rho(x,0)$$

infinite derivative

$$\rho_x = \rho_0'\left[1 - tQ''(\rho)\rho_x\right]$$

$$\rho_x = \frac{\rho_0'}{1 + t\underbrace{\underbrace{Q''(\rho)}_{<0}\rho_0'}_{<0}}$$

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T2
06/07/2016