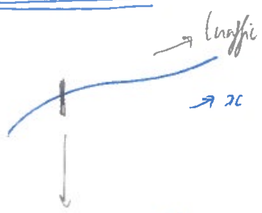
Conservation Laws 4/7



→ $\rho(x,t)$ → # cars for unity of lenght

→ $v(x,t)$ → average velocity of the cars

$\mu(x,t,v)$

→ $Q(x,t)$ → flux of cars (taxa q atravessa)

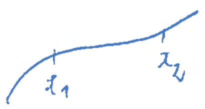
↳ how many cars will cross the section on one second

$Q = \rho \cdot v$

se v for $\neq$, então $\bar{v} = \int v \, \mu(x,t,v)\, dv$

$$\rho = \int \mu(x,t,v)\, dv$$

Conservation of mass (cars don't disappear)



M → # cars between x_1 and x_2

$$M(t) = \int_{x_1}^{x_2} \rho(x,t)\,dx$$

conservation of cars

$$M(t_2) - M(t_1) = \int_{t_1}^{t_2} \{Q(x_1,t) - Q(x_2,t)\}\,dt \quad , \quad -\int_{x_1}^{x_2} Q_x\, dx = Q(x_1,t) - Q(x_2,t)$$

Definition of $M(t)$ ⇒

$$\int_{x_1}^{x_2} \{\rho(x,t_2) - \rho(x,t_1)\}\,dx \Rightarrow \qquad \rho(x,t_2) - \rho(x,t_1) = \int_{t_1}^{t_2} \rho_t\, dt$$

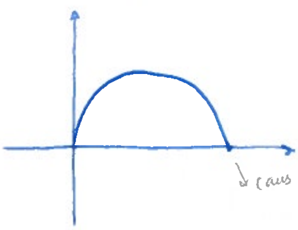
3D

? $\boxed{\rho_t + Q_x = 0} \Leftrightarrow \boxed{\rho_t + (\rho v)_x = 0} \Leftrightarrow \rho_t + \nabla \cdot (\rho v) = 0$

$$\int_{x_1}^{x_2}\int_{t_1}^{t_2} \underbrace{(\rho_t + Q_x)}_{\text{assuming it is continuous}}\, dx\, dt = 0$$

empirical

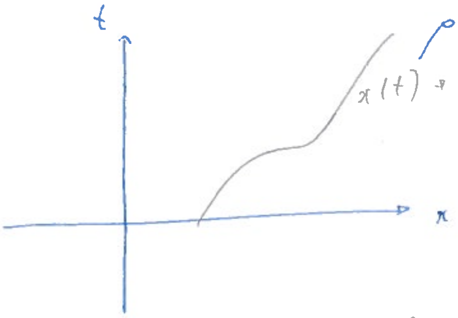
$Q = Q(\rho)$



↳ cars can't move because there are many

$\boxed{\rho_t + Q'(\rho)\,\rho_x = 0}$ (*)

Method of characteristics



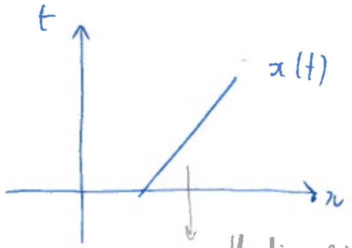
we will find an easy way of solving → particular line

$$\frac{d\rho}{dt}(x(t),t) = \rho_t + x'(t)\rho_x$$

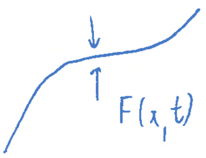
comparing with ⊛, they are = if $\boxed{x'(t) = Q'(\rho(x(t),t))}$

then

$\rho_t + \underbrace{x'(t)}_{Q'}\rho_x = 0$, $x'(t) = \frac{dx}{dt}$, then $\begin{cases} \frac{dx}{dt} = Q'(\rho) \\ \frac{d\rho}{dt} = 0 \end{cases}$ if ρ is constant, then Q' is constant ⇒ straight line



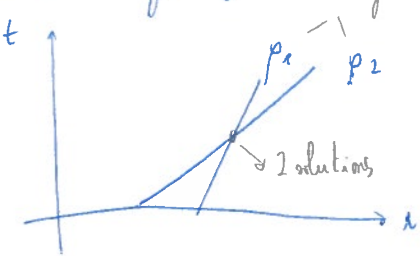
line: characteristic, we can solve the problem

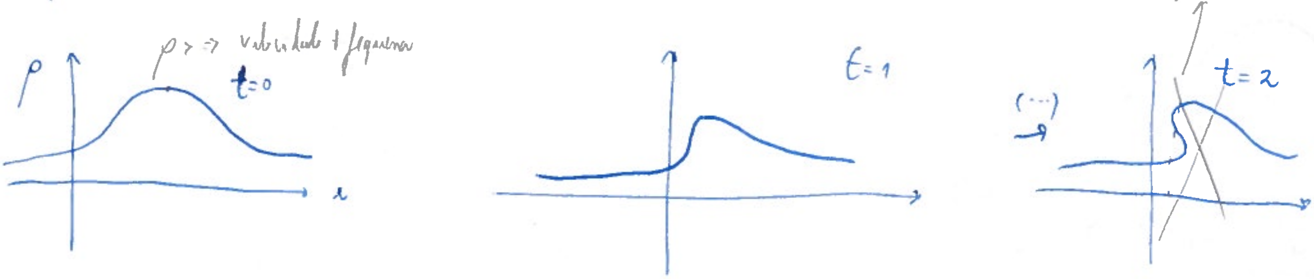


$F(x,t)$ → how many cars are coming or leaving / unit time ⇒ $\rho_t + Q_x \neq 0$

What changes when applying MC? $\frac{d\rho}{dt} \neq 0$, but we can still use it

What could go wrong?





$\frac{dx}{dt} = Q'$ → line along which information travels

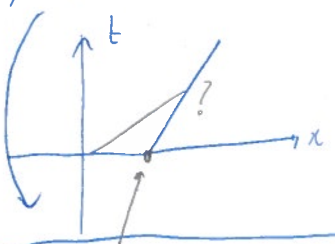
speed of propagation of information

Is information faster or slower than we are?

$Q(\rho) = \rho\, U(\rho)$

$Q' = U + \underbrace{\rho U'}_{\text{negative} < 0} \Rightarrow Q' < U$ $\boxed{Q' < U}$ information is slower

$\rho(x,t)$



ρ is constant in this line

① $\boxed{\rho_0(x - Q'(\rho)t, 0) = \rho(x,t)}$, $\rho_0 = \rho(x,0)$ vertical derivative

When is the first time then 2 characteristics will meet?

$\rho(x)' \to \infty$?

derivative of ① $\rho_x = \rho_0'\,[1 - tQ''(\rho)\,\rho_x] \Rightarrow \rho_x = \frac{\rho_0'}{1 + tQ''(\rho)}$

$\rho_x\,[1 + \rho_0' t Q''] = \rho_0' \Rightarrow \boxed{\rho_x = \frac{\rho_0'}{1 + \rho_0' t Q''}}$

$\Rightarrow Q''$ is $< 0 \Rightarrow \rho_x$ eventually will blow up.

What is the biggest value that $\rho_0' Q''$! this will give the time when this will break