ALGEBRAS OF SINGULAR INTEGRAL OPERATORS IN REARRANGEMENT-INVARIANT SPACES WITH MUCKENHOUPT WEIGHTS

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In this talk we study Fredholmness of singular integral operators with piecewise continuous coefficients in reflexive rearrangement-invariant spaces with weights $X(\Gamma, w)$ over arbitrary Carleson curves Γ. Suppose a weight w belongs to the Muckenhoupt classes $A_{\frac{1}{\alpha_X}}(\Gamma)$ and $A_{\frac{1}{\beta_X}}(\Gamma)$, where α_X and β_X are Boyd indices of a rearrangement-invariant space $X(\Gamma)$. We prove that these conditions guarantee the boundedness of the Cauchy singular integral operator S in the weighted rearrangement-invariant space $X(\Gamma, w)$. Under some "disintegration condition" we construct a symbol calculus for the Banach algebra generated by singular integral operators with matrix-valued piecewise continuous coefficients. We prove criteria for Fredholmness and get a formula for the index of any operator from this algebra in terms of its symbol. We give nontrivial examples of spaces, for which this "disintegration condition" is satisfied. One of such spaces is a Lebesgue space with a general Muckenhoupt weight over an arbitrary Carleson curve. Another nontrivial example of space satisfying this "disintegration condition" is a reflexive Orlicz space with distinct Boyd indices over a smooth curve. Note that for weighted Lebesgue spaces corresponding results were obtained by I. Gohberg and N. Krupnik for power weights (the end of sixties); and I. Spitkovsky, A. Böttcher and Yu. I. Karlovich for arbitrary Muckenhoupt weights (the middle of nineties).