Invertibility of Functional Operators with Slowly Oscillating non-Carleman Shifts

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We prove criteria for the invertibility in Lebesgue spaces $L^p, 1 < p < \infty$, of a binomial functional operator of the form

$$A = aI - bW_\alpha$$

where a and b are continuous functions on $(0,1)$, I is the identity operator, W_α is the shift operator, $W_\alpha f = f \circ \alpha$, generated by a non-Carleman shift $\alpha : [0,1] \to [0,1]$, which has only two fixed points $0, 1$. We suppose that the coefficients a, b slowly oscillate at 0 and 1, and α is a slowly oscillating shift, that is, $\log \alpha'$ is bounded and continuous on $(0,1)$ and

$$\lim_{t \to j} (t - j) \frac{d}{dt} \left(\frac{\alpha(t) - \alpha(j)}{t - j} \right) = 0, \quad j \in \{0, 1\}.$$

The main difficulty connected with slow oscillation is overcome using the method of limit operators.

These results are obtained in collaboration with Yuri Karlovich and Amarino Lebre.