# On the Interpolation Constant for Sublinear Operators in Orlicz Spaces*

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In this talk we deal with the interpolation of bounded sublinear operators from Lebesgue spaces $L^p$ and $L^q$ into an Orlicz space $L^\varphi$, where

$$1 \le p < q \le \infty \quad \text{and} \quad \varphi^{-1}(t) = t^{1/p}\rho(t^{1/q-1/p})$$

for some concave function $\rho : \mathbf{R}_+ \to \mathbf{R}_+$. If $q = \infty$ assume in addition that $\rho_*(\mathbf{R}_+) = \mathbf{R}_+$, where $\rho_*(t) := t\rho(1/t)$. Our main aim is to get estimates for the interpolation constant $C$.

**Theorem.** If a sublinear operator $T$ is bounded in Lebesgue spaces $L^p$ and $L^q$, then it is bounded in the Orlicz space $L^\varphi$ (with both, the Luxemburg and the Orlicz norm) and

$$\|T\|_{L^\varphi \to L^\varphi} \le C \max\left\{\|T\|_{L^p \to L^p}, \|T\|_{L^q \to L^q}\right\},$$

with the interpolation constant $C$ satisfying
(a) $C \le 2^{1-1/p} < 2$ for $1 \le p < q = \infty$;
(b) $C \le (2\gamma_{p,q})^{1/p} \le 2^{(2-1/q)/p} < 4$ for $1 \le p < q < \infty$, where

$$\gamma_{p,q} = \inf\left\{x + \left(\frac{p}{q}x^{p-1}\right)^{1/(q-1)} : \ x^p + \left(\frac{p}{q}x^{p-1}\right)^{q/(q-1)} = 1\right\} \in [2^{1-1/p}, 2^{1-1/q}]$$

is the constant introduced by G. Sparr [1, Lemma 5.1].

## References

[1] G. Sparr, *Interpolation of weighted $L_p$-spaces*, Studia Math. **62** (1978), 229 – 271.

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