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Invertibility of functional operators with non-Carleman shift in rearrangement-invariant spaces

Let ω be a diffeomorphism (shift) of $[0,1]$ onto itself such that $\omega(0)=0$ and $\omega(1)=1$, but $\omega(t)\neq t$ for $t\in(0,1)$. This talk is devoted to criteria for the two- and one-sided invertibility of functional operators

$$A = aI - bW_\omega,$$

where a and b are continuous functions on $[0,1]$, I is the identity operator, W_ω is the shift operator: $W_\omega f = f\circ\omega$, in a reflexive rearrangement-invariant space $X(0,1)$ which Boyd indices α_X,β_X and Zippin (fundamantal) indices p_X,q_X satisfy

$$0<\alpha_X=p_X\leq q_X=\beta_X<1.$$

These spaces are a wide generalization of classic Lebesgue, Orlicz, and Lorentz spaces.

Moreover, we prove criteria for the two-sided invertibility of A in Lebesgue spaces $L^p(0,1), 1<p<\infty$, under weaker assumptions on a,b and ω'. We assume that a,b and ω' are continuous on $(0,1)$ and slowly oscillate at 0 and 1, but they may not have one-sided limits at 0 and 1.

A part of these results are obtained jointly with Amarino Lebre and Yuri Karlovich.