Lyapunov Exponents and Smooth Ergodic Theory

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Preface

This book provides a systematic introduction to the core of smooth ergodic theory. Despite an impressive amount of literature in the field there is no textbook which contains a sufficiently complete presentation of the theory. This book attempts to fill in this gap. We describe the general (abstract) theory of Lyapunov exponents and its applications to the stability theory of differential equations, the stable manifold theory, absolute continuity of stable manifolds, and the ergodic theory of dynamical systems with nonzero Lyapunov exponents (including geodesic flows).

The book is a revised and considerably expanded version of our *Lectures on Lyapunov Exponents and Smooth Ergodic Theory* [**4**]. We add more examples of dynamical systems with nonzero Lyapunov exponents, including diffeomorphisms on two-dimensional tori and on spheres. Furthermore, we substantially expand the exposition of the crucial absolute continuity property. In particular, we include an example of a foliation that is not absolutely continuous and establish the formula for the Jacobian of the holonomy map. We also add a complete proof of the Multiplicative Ergodic Theorem as well as provide more details in the proofs of several basic results. Finally, a few more figures are added to illustrate the exposition.

We hope that these improvements make the book more accessible to graduate students or anyone who wishes to acquire a working knowledge of smooth ergodic theory and to learn how to use its tools. Indeed, the book can be used as a primary textbook for a special topics course on nonuniform hyperbolic theory or as supplementary reading for a basic course on dynamical systems.

This book is self-contained. We only assume that the reader has a basic knowledge of real analysis, measure theory, differential equations, and topology. We present the basic concepts of smooth ergodic theory and provide complete proof of all main results. We also state some results whose proofs require more advanced techniques which exceed the scope of the book. In our opinion this gives the reader a broader view of smooth ergodic theory and may help stimulate further study. This will also provide nonexperts with a broader perspective of the field.

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