Luís Barreira

# Ergodic Theory, Hyperbolic Dynamics and Dimension Theory

December 12, 2011

Springer

# Preface

This book is an introduction to the interplay of three main areas of research: *ergodic theory*, *hyperbolic dynamics*, and *dimension theory* of dynamical systems. This includes an introduction to the thermodynamic formalism, which is an important tool in dimension theory.

My main aim was to provide in a single volume a rigorous self-contained introduction to dimension theory of hyperbolic dynamics, including sufficiently high-level introductions to ergodic theory and the thermodynamic formalism. This caused that several topics of ergodic theory and hyperbolic dynamics had to be excluded, essentially to keep the size of the book under control. However, it should be emphasized that the same happened to several topics of dimension theory, while making an effort to reach a good compromise between the various areas. On the other hand, it was necessary to include topics of ergodic theory and hyperbolic dynamics that are often absent in introductory texts, such as the construction of Markov partitions for repellers and an introduction to the thermodynamic formalism.

The book is directed primarily to graduate students interested in dynamical systems, as well as researchers in other areas who wish to learn ergodic theory or dimension theory of hyperbolic dynamics, at an intermediate level and in a sufficiently detailed manner. In particular, the text can be used as a basis for a graduate course on ergodic theory and the thermodynamic formalism (using Chapters 2–5), and for a graduate course on dimension theory of hyperbolic dynamics (using Chapters 6–9, eventually referring to Chapters 2–5 for any prerequisites).

The book can also be used for independent study: it is self-contained, and with the exception of some basic well-known statements and results from other areas, all statements are included with detailed proofs. Moreover, each chapter can essentially be read independently. It is only assumed some familiarity with basic material from measure theory and integration theory, which anyway is recalled in Appendix A.

The material is divided into four parts:

* Part I is dedicated to the foundations of ergodic theory, and its interplay with symbolic dynamics and topological dynamics. In Chapter 2 we introduce the basic notions and results of ergodic theory, including Poincaré's recurrence theo-

rem and Birkhoff's ergodic theorem. We also give a large number of examples. In Chapter 3 we discuss additional topics of ergodic theory, including the existence of invariant measures for a continuous transformation of a compact metric space.

* Part II is an introduction to entropy theory and the thermodynamic formalism. In Chapter 4 we introduce the notions of metric entropy and topological entropy. We also establish the Shannon–McMillan–Breiman theorem and the Variational principle for the topological entropy. Chapter 5 is an introduction to the thermodynamic formalism. In particular, we establish the Variational principle for the topological pressure.
* Part III is dedicated to hyperbolic dynamics. In Chapter 6 we start by discussing some basic properties of hyperbolic sets. In addition, we discuss in detail some properties of the Smale horseshoe and of the hyperbolic automorphisms of $\mathbb{T}^2$. We also construct Markov partitions for any repeller. In Chapter 7, after establishing the existence of stable and unstable manifolds, we use the shadowing property to construct Markov partitions for any locally maximal hyperbolic set.
* Finally, Part IV is an introduction to dimension theory of hyperbolic dynamics. In Chapter 8 we give an introduction to the basic notions of dimension theory. In particular, we introduce the notions of Hausdorff dimension and box dimension. We also show how pointwise dimension can be used to estimate the dimension of a measure. In Chapter 9 we study the dimension of repellers and hyperbolic sets for conformal transformations, using Markov partitions.

In addition, the book contains more than 150 exercises, of variable level of difficulty.

There are no words that can adequately express my gratitude to Claudia Valls for her help, patience, encouragement, and inspiration without which it would be impossible for this book to exist.

Lisbon, December 2011 *Luís Barreira*

# Contents

## Part III Hyperbolic Dynamics



## Part IV Dimension Theory