Computable Analysis and Analog Computation

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Numerous models of computability for functions on the real numbers have been developed. One major way of classifying these models is according to how they deal with time, in a "*discrete*" manner or an "*analog*" manner. One of the major discrete models is *Computable Analysis*; from rational approximations to the real number inputs, a rational approximation to the output is computed using standard discrete time Turing Machines (see Grzegorczyk 1955, Ko 1991 and Weihrauch 2000). I will discuss analog models of computation and their relationships to Computable Analysis. In particular I will discuss Shannon's (1941) analog model, the *General Purpose Analog Computer* (GPAC), as well as Moore's (1996) analog model, the *Real Recursive Functions*. The GPAC is a circuit model which can be characterized by systems of polynomial differential equations. The Real Recursive Functions are defined via function algebras in which some "*basic*" real functions are closed under various operations (a fundamental kind of operation is to set up an initial value problem with existing functions in the algebra, and add the resulting solution). Significant work has been done to relate these models to Computable Analysis (see Bournez, Campagnolo, Graça, and Hainry 2005, 2006, 2007). I will discuss our method of "*approximation*," and how it can be used to obtain relationships between these various models of computation. I will present joint work with Manuel L. Campagnolo.