Combinatory vs. axiomatic completeness

(Abstract)

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The Curry-Howard isomorphism establishes an amazing connection between systems of typed λ-calculus and logical systems. In particular, the types of typable λK-terms form the set of provable formulas in intuitionistic logic[1]. The same correspondence exists between the λI-calculus and the system $R_{\rightarrow}$ of relevance logic, as well as between the `BCK`-λ- and `BCI`-λ-calculus and `BCK`- and `BCI`-logic respectively. Furthermore, it is well known that the principal types of some standard combinator bases for the four systems of λ-calculus form, together with the inference rules modus ponens and substitution, complete axiom sets for the respective logical systems. The aim of this contribution is to present a study of this pattern in the four cases, i.e. the relationship between the combinatory completeness of a set of typable combinators, with simple types, and the axiomatic completeness of their principal types. We show that the two problems are equivalent in the `BCI`-λ- as well as in the `BCK`-λ-calculus, and that for the two remaining cases axiomatic completeness is a sufficient, but not necessary, condition for combinatory completeness.

[1] Strictly speaking, we refer to Hilbert's positive implicational logic.