An introduction to ordinally informative proof theory
Abstract

Ordinally informative proof theory goes back to the work of G. Gentzen who first proved in [1] that ε_0 is the supremum of the order–types of primitive recursively definable well–orderings on the the natural number whose well–foundedness is provable from the axioms of Peano arithmetic. In this course we give an introduction to ordinally informative proof theory. We start by repeating Gentzen's result in a more modern setting using infinitary logic calculi. We explain the connection between the Schütte–Feferman ordinal Γ_0 and the limits of predicativity. We use the example of an axiom system for inductively defined sets of natural number to point out the difficulties which arise in the ordinal analysis of impredicative systems. In the end we use the example of Peano arithmetic to indicate how an ordinal analysis can be used to characterize the provably computable functions of an axiom system.

References

[1] G. GENTZEN, *Beweisbarkeit und Unbeweisbarkeit von Anfangsfällen der transfiniten Induktion in der reinen Zahlentheorie*, **Mathematische Annalen**, vol. 119 (1943), pp. 140–161.

[2] W. POHLERS, **Proof theory. An introduction**, Lecture Notes in Mathematics, vol. 1407, Springer-Verlag, Berlin/Heidelberg/New York, 1989.

[3] ——, **Proof theory. A first step into impredicativity**, Universitext, Springer-Verlag, Berlin/Heidelberg/New York, to appear.