Differential Geometry - 1st Semester 2011/12

Ficha 1 (due Wednesday September 21)

1) Let $M \subset \mathbb{R}^n$ be a subset that satisfies the following properties:

for each $p \in M$ there exists an open subset $U \subset \mathbb{R}^n$ containing p and a homeomorphism $\psi : V \to M \cap U$ (with $V \subset \mathbb{R}^k$ an open subset) such that ψ is a smooth map, and such that $\psi'(q) : \mathbb{R}^k \to \mathbb{R}^n$ is injective for all $q \in V$.

Show that M is a smooth manifold of dimension k.

2) Let M and N be two smooth manifolds, and let $\psi : M \to N$ be a smooth map.

Show that $d\psi : TM \to TN$ is a smooth map.

3) Consider the map $\Phi : \mathbb{P}^2 \to \mathbb{R}^3$ defined by

$$\Phi([x, y, z]) = \frac{1}{x^2 + y^2 + z^2} \, (yz, xz, xy) \, .$$

Show that Φ is a smooth map, and that it describes an immersion with the exception of 6 points.