

# Differential Geometry - 1st Semester 2011/12

## Ficha 5 (due Monday November 21)

- 1)** Let  $M$  be a smooth manifold, and  $\omega \in \Omega^{k-1}(M)$  a  $(k-1)$ -form.

Prove Stokes theorem for a smooth singular  $k$ -chain  $c$ :

$$\int_c d\omega = \int_{\partial c} \omega .$$

- 2)** Compute  $H^k(M)$  for

- a)  $M = \mathbb{T}^d$ ,
- b)  $M = \mathbb{P}^d$ .

- 3)** Consider the long sequence of the *Zigzag Lemma*,

$$\dots \longrightarrow H^k(A) \xrightarrow{f} H^k(B) \xrightarrow{g} H^k(C) \xrightarrow{d^*} H^{k+1}(A) \longrightarrow \dots$$

Show that  $\text{Im } d^* = \ker f$  and  $\text{Im } g = \ker d^*$ .

- 4)** Given an exact sequence of vector spaces

$$0 \longrightarrow C^0 \longrightarrow \dots \longrightarrow C^k \longrightarrow \dots \longrightarrow C^d \longrightarrow 0 ,$$

show that

$$\sum_{i=0}^d (-1)^i \dim C^i = 0 .$$

- 5)** Prove the *The Five Lemma*.