# LisMath seminar 2014/2015, 2nd semester

Venue: Complexo Interdisciplinar ULisboa, room B3-01, Friday 16-17h

**Schedule:**

- March 13: Pedro Oliveira

  `Hawking's singularity theorem in general relativity`

- March 20: Pedro Pinto

  `Old and New Results in the Foundations of Elementary Plane Euclidean and Non-Euclidean Geometries`

- March 27: Sílvia Reis

  `Vanik-Chervonenkis theory and the independence property in model theory`

- April 10: Hillal Elshehabey

  `Numerical Simulation of Heat and Mass Transfer in Fluid Flow`

- April 17: João Enes

  `Indefiniteness in semi-intuitionistic set theories: On a conjecture of Feferman`

- April 24: Alexandra Symeonides

  `Time reversible stochastic processes and the relevant Feynman-Kac formula`

- May 08: Fábio Silva

  `"Problems" in inverse semigroups`

- May 15: Filipe Gomes

  `Symmetric functions in noncommuting variables and supercharacters of unitriangular groups`

- May 22: Juan Quijano

  `A brief introduction to groupoids`

- May 29: João Dias

  `Supercharacters of algebraic groups: the geometric approach`

**Topics/Speakers:**

- `Hawking's singularity theorem in general relativity.`

  `Speaker:` Pedro Oliveira

  `Bibliography:`

  [1] S. Hawking, The occurrence of singularities in cosmology. iii. causality and singularities, Proc. Roy. Soc. Lon. A 300 (1967), 187201

  [2] G. Naber, Spacetime and singularities  an introduction, Cambridge University Press, 1988.

  [3] L. Godinho and J. Natário, An Introduction to Riemannian Geometry: With Applications to Mechanics and Relativity (Universitext), Springer, 2014

- `Time reversible stochastic processes and the relevant Feynman-Kac formula.`

  `Speaker:` Alexandra Symeonides

  `Abstract:`

  We will present a family of time reversible stochastic processes known (among other names) as Bernstein processes [1,2]. These processes are much closer, in their properties, to the solutions of deterministic (Lagrangian or Hamiltonian) dynamical equations, they are in fact absolutely continuous with respect to the Wiener process, i.e. they are processes with a drift term. The relevant Feynman-Kac formula for this class of processes will be proved. Some particular cases of the formula will be given as examples: the well-known Feynman-Kac formula and Doob's relation between Wiener and Ornstein-Uhlenbeck processes [3,4].

  The consequences of this probabilistic perturbation theory and the underlying time reversible processes should go beyond stochastic analysis. We will give a hint of two applications, one motivated by mathematical quantum physics and the other by a stochastic version of geometric mechanics.

  The first concerns a rigorous probabilistic interpretation of Feynman informal perturbation theory in his Path Integral approach [7], which needs the development of a rigorous integration by parts formula, also inspired by Feynman, done in terms of special reversible probability measures on path space [6]. Furthermore, to investigate the consequences of this probabilistic perturbation theory in the more general context of stochastic analysis.

The second application concerns Geometric Mechanics, that in the recent years started to investigate perturbations under random noise, preserving as much as possible the geometric content. Roughly, what has been done by this community up to now is to add noise to the Hamiltonian equation for the momentum (i.e random force) [8], or add noise to the configuration (i.e the trajectories are random processes) [5]. This goal is close to the one of stochastic perturbation, whose foundations lie in the dynamics of the above mentioned Bernstein processes, e.g. [10]. In this context, a stochastic Euler-Poincaré reduction has been recently proved [7]: the corresponding equations of motion are dissipative perturbations of Hamiltonian systems, and should be relevant, in particular, in Hydrodynamics. So far the main application regards the Navier-Stokes equations (as perturbations of Euler equations) but many other systems can be considered and studied in this perspective. This study is still in its beginnings and many mathematical questions remain to be solved.

`Bibliography:`

[1] J.C. Zambrini,Variational processes and stochastic versions of mechanics, Journal of Math. Physics 27,p2307-2330 (1986)

[2] Albeverio, Yasue and Zambrini, Euclidean quantum mechanics: analytical approach Annales de l'I.H.P., section A, tome 50, no3 (1989), p. 259-308.

[3] P. Lescot, J.C. Zambrini. Probabilistic deformation of contact geometry, difusion processes and their quadratures. Progress in Probability 59, Birkhauser Verlag Basel (2007), 203-226.

http://gfm.cii.fc.ul.pt/people/jczambrini/lescot-zambrini-ascona.pdf

[4] A.B. Cruzeiro and J.C. Zambrini. Ornstein-Uhlenbeck processes as Bernstein difusions, Proceedings of the Conference on Stochastic Analysis (Barcelona), Birkhauser, Boston, Inc. (1993) (P.P. no 32).

[5] J.C. Zambrini,The researchprogram of Stochastic Deformation (with a view toward Geometric Mechanics), http://arxiv.org/abs/1212.4186

[6] A.B. Cruzeiro and J.C. Zambrini, Malliavin Calculus and Euclidean Quantum Mechanics 1. Functional calculus, Journal of Functional Analysis,91,1,p.62 (1991)

[7] Richard P. Feynman, Albert R. Hibbs. Quantum Mechanics and Path Integrals: Emended Edition.Dover Publications, Incorporated, 2012

[8] A.B. Cruzeiro, M. Arnaudon and X. Chen, Stochastic Euler-Poincaré reduction, http://arxiv.org/abs/1204.3922

[9] Joan-Andreu, Lazaro-Cami, Juan-Pablo Ortega. Stochastic Hamiltonian dynamical systems. http://arxiv.org/abs/math/0702787

[10] Fernanda Cipriano, A Stochastic Variational Principle for Burgers Equation and its Symmetries. Stochastic Analysis and Mathematical Physics II, 4th International ANESTOC Workshop in Santiago, Chile. Birkhauser, R. Rebolledo (2003), p.29.

- `A New Look At The Path Integral Of Quantum Mechanics.`

  `Bibliography:`

  [1] Edward Witten, arXiv:1009.6032

- `Analytic Continuation Of Chern-Simons Theory.`

  `Bibliography:`

  [1] Edward Witten, arXiv:1001.2933

- `Indefiniteness in semi-intuitionistic set theories: On a conjecture of Feferman.`

  `Speaker:` João Enes

  `Bibliography:`

  [1] Michael Rathjen, arXiv:1405.4481v1

- `Old and New Results in the Foundations of Elementary Plane Euclidean and Non-Euclidean Geometries.`

  `Speaker:` Pedro Pinto

  `Bibliography:`

  [1] Marvin Greenberg, The American Mathematical Monthly, vol. 117, no. 3, March 2010, pp. 198-219

  [2] section 3 of arXiv:1405.4481v1

- `Vanik-Chervonenkis theory and the independence property in model theory.`

  `Speaker:` Sílvia Reis

  `Abstract:`

  Vapnik-Chervonenkis dimension is a measure of complexity of a family of sets. Informally, it measures how many subsets of an arbitrary finite set

the family can recognise. VC-theory was developed as a foundation for statistical learning theory in the early 1970's, and still provides the theoretical basis for the classical approach to classification problems in machine learning. At heart of VC-theory lies a simple but surprising combinatorial lemma, that was proved independently and simultaneously in three different papers [1],[2],[3].

Model theory provides a different perspective on VC-theory. The compactness theorem allows to investigate asymptotic behaviour of finite sets via infinite sets with a particularly nice homogeneous structure (indiscernible sequences). This allowed Shelah to prove in his original paper [3] results that were thought by combinatorists to be open for over ten years afterwards (the connection between Shelah's paper [3] and the work [1],[2] was only made in mid-80's).

The goal of the seminar would be to introduce the basics of VC-theory, and to explain the connection between this subject and model theory. The seminar will follow mostly a recent account [4], but the student will be encouraged to look at the original papers for more background and details.

`Bibliography:`

[1] V. Vapnik and A. Chervonenkis. "On the uniform convergence of relative frequencies of events to their probabilities." Theory of Probability and its Applications, 16(2):264280, 1971.

[2] Shelah, Saharon, "A combinatorial problem; stability and order for models and theories in infinitary languages", Pacific Journal of Mathematics 41: 24726, 1972.

[3] Sauer, N., "On the density of families of sets", Journal of Combinatorial Theory, Series A 13: 145147, 1972.

[4] Hans Adler, "An introduction to theories without the independence property", Archive for Math Logic, to appear.

- `The Approximate Subgroup Theorem.`

  `Abstract:`

  This is an exciting recent result, stating (very informally) that a subset of an arbitrary large enough finite group that "resembles" a subgroup is indeed essentially a coset of a subgroup. The proof has two components, model theoretic in nature. First, one uses "stable group theory" in order to show that the theorem is true for a "large" subset of an infinite group G, where the concept of size is replaced with a well-behaved G-invariant measure. Second, using an ultra-product construction and a basic transfer principle, one deduces the desired result for finite groups.

The seminar will follow the very well written self-contained original article [1], and my notes [2] that contain some additional details.


`Bibliography:`

[1] E. Hrushovski, Stable group theory and approximate subgroups, J. Amer. Math. Soc. 25 (2012), 189243.

[2] A. Usvyatsov, "Notes for SLM on Hrushovski's paper "Stable group theory and approximate subgroups", http://ptmat.fc.ul.pt/ alexus/papers.html.


- `Continuous model theory.`


`Abstract:`

This topic covers a model theoretic approach to classes of metric structures, particularly Banach spaces, Banach algebras, probability algebras with extra-structure. One motivation for this theory comes from important results of Krivine et al on applications of the ultra-product construction to Banach space geometry. One particular example is the paper [3] where it is shown that a "stable" Banach space contains an almost isometric copy of an $l_p$ space.


The seminar will follow [1] and [2], and certain parts of [3] in order to exemplify connections between model theory and Banach space geometry.


`Bibliography:`

[1] I. Ben-Yaacov, A.J.Berenstein, C.W.Henson, A. Usvyatsov,"Model theory for metric structures", Model theory with Applications to Algebra and Analysis Vol. 2, London Math Soc. Lecture Note Series Nr 350, Cambridge Univ Press 2008.

[2] I. Ben-Yaacov, A. Usvyatsov, "Continuous first order logic and local stability", Trans. Amer. Math. Soc. 362 (2010), no. 10, 5213-5259.

[3] Jean-Louis Krivine and Bernard Maurey, Espaces de Banach stables, Israel Journal of Mathematics 39 (1981), no. 4, 273295.


- `Symmetric functions in noncommuting variables and supercharacters of unitriangular groups.`

`Speaker: Filipe Gomes`

`Bibliography:`

[1] M. Aguiar et al. "Supercharacters, symmetric functions in noncommuting variables, and related Hopf algebras". Advances in Mathematics 229(4), (2012), 2310-2337

[2] C. André, "Supercharacters of unitriangular groups and set partitions combinatorics" ECOS2013, 2da Escuela Puntana de Combinatoria, Universidad Nacional de San Luis, Argentina, July 22-August 2, 2013.

- `Supercharacters of algebraic groups: the geometric approach.`

  `Speaker: João Dias`

  `Bibliography:`

[1] M. Boyarchenko, "Character sheaves and characters of unipotent groups over finite fields", American Journal of Mathematics (to appear)

[2] M. Boyarchenko & V. Drinfeld, "Character sheaves on unipotent groups in positive characteristic: foundations" arXiv:0810.0794v2

- `A brief introduction to groupoids.`

  `Speaker: Juan Pablo Quijano`

  `Abstract:`

Groupoids were introduced by Brandt in his 1926 paper [1] and since then they have been used in a wide variety of areas of mathematics, from ergodic theory and functional analysis to homotopy theory, algebraic geometry, differential geometry, differential topology and group theory. Specially in category theory and homotopy theory, a groupoid generalises the notion of group in several equivalent ways: A groupoid can be seen as a group with a partial function replacing the binary operation or a category in which every morphism is invertible. In this talk, going through the notion of symmetry, I would like to justify the argument that the theory of groupoids does not differ widely in spirit and aims from the theory of groups and how groupoids describe symmetry. Of course, I will give the basic definitions and important examples in a wide range.

`Bibliography:`

[1] H. Brandt, Über eine Verallgemeinerung des Gruppenbegriffes, Math. Ann. 96, 360- 366 (1926).

[2] R. Brown, From Groups to Groupoids: A Brief Survey, Bull. London Math. Soc. 19, 113- 134 (1987).

[3] A. Weinstein, Groupoids: Unifying Internal and External Symmetry, Notices Amer. Math. Soc. 43 (1996).

- `"Problems" in inverse semigroups.`

  `Speaker: Fábio Silva`

  `Abstract:`

  The word problem, the rational word problem and the regular idempotent problem for inverse semigroups will be discussed.

  `Bibliography:`

  [1] Brough, T. Inverse semigroups with rational word problem are finite, arXiv:1311.3955

  [2] Kambits, M. Anisimov's theorem for inverse semigroups, IJAC, to appear.

  [3] Lawson, M. V. Inverse semigroups, the theory of partial symmetries, World Scientific, 1998

  [4] Munn, W.D. Free inverse semigroups, Proc. London Math. Soc. 29 (3), 385-404, 1974

- `The maximal number of nodes on algebraic surfaces`

  `Bibliography:`

  [1] Beauville, Arnaud, Sur le nombre maximum de points doubles d'une surface dans P3 ($\mu(5) = 31$). (French) Journes de Gometrie Algbrique d'Angers, Juillet 1979/Algebraic Geometry, Angers, 1979, pp. 207215, Sijthoff & Noordhoff, Alphen aan den RijnGermantown, Md., 1980.

  [2] Pignatelli, Roberto; Tonoli, Fabio, On Wahl's proof of $\mu(6) = 65$.

  Asian J. Math. 13 (2009), no. 3, 307310.

- `Numerical Simulation of Heat and Mass Transfer in Fluid Flow`

  `Speaker: Hillal M. Elshehabey`

  `Key words:` Mathematical Modeling, Heat and Mass Transfer, Numerical Solutions, Nanofluids.

  `Abstract:`

  Computational Fluid Dynamics has become an indispensable tool in the scientific research as well as in the modern engineering design and development. The increasing computer power during the last decades allowed for more and more detailed numerical simulations, which deepened the insight and understanding of physically highly complex flow problems involving complicated geometries, chemical reactions, phase change, etc.. Nanofluid,

which is a mixture of nano-sized particles (nanoparticles) suspended in a base fluid, is used to enhance the rate of heat transfer via its higher thermal conductivity compared to the base fluid. Due to this enhancement of the material properties, nanofluids are widely used in our life such as in transportation (engine cooling and vehicle thermal management), electronics cooling, nuclear systems cooling, heat exchanger, biomedicine, heat pipes, fuel cell, Solar water heating, etc... In this seminar, we shall present some fluid models, following [1] and [2], and then we will discuss applications to nanofluids, based on references [3], [4], and [5].

Bibliography:

[1] D.J Acheson, Elementary fluid dynamics, Oxford University Press (1990).

[2] W. Layton, Introduction to the numerical analysis of incompressible viscous flows, SIAM, Philadelphia (2008).

[3] H.M. Elshehabey, F.M. Hady, S.E. Ahmed, R.A. Mohamed, Numerical investigation for natural convection of a nanofluid in an inclined L-shaped cavity in the presence of an inclined magnetic field. International Communications in Heat and Mass Transfer: (2014) 57, 228-238.

[4] F.M. Hady, S.E. Ahmed, H.M. Elshehabey, R.A.Mohamed, Natural Convection of a Nanofluid in Inclined, Partially Open Cavities: Thermal Effects. Journal of Thermophysics and Heat Transfer, (2015) (in press).

[5] H.M. Elshehabey, S.E. Ahmed, Buongiorno's mathematical model for MHD mixed convection of a fluid driven cavity with sinusoidal temperature distributions on both sides filled with nanofluid, (2014) (submitted).