CORRECTIONS AND COMMENTS ON "DIFFERENTIAL TOPOLOGY" BY M. HIRSCH

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The following is a list of corrections to Chapters 1 through 7 of the corrected 5th printing (1994). There are also comments on any statements which were not clear to me. From Chapter 4 on the list is less exhaustive because from that point on I did not follow the text as closely in the course. Several of the typos and imprecisions below were noticed by Manuel Araújo, Eloísa Pires and Aleksandra Perisic (who took the class in 2011). Many thanks to them.

- page 11,line 11: The second coordinates of the vectors should be $D(\varphi^{-1})_a(y)$ and $D(\psi^{-1})_{\psi(z)}D(\psi f\varphi^{-1})_a y$ respectively.
- page 14,Exercise 8: It is implicit in the assumption that the domains of the coordinate changes are open in $\mathbb{R}^n$ (and that the family of subsets of X covers X).
- page 15,line -9: $\varphi(V)$ should be $\varphi(U)$.
- page 15,line -8: There is a φ missing in the "local representation".
- page 27,Exercise 2: I think one must assume in addition that M is paracompact (or equivalently second countable). Otherwise the long line (extended in both directions) is a counterexample: any embedding of $[0,\infty[$ in the long line will be contained in a submanifold homeomorphic to $[0,1]$ and hence not have a closed image.
- page 31,Theorem 4.1: There is an "is" missing after $N-\partial N$. One does not actually need to require that $y \notin \partial N$. However, if $y \in \partial N$ then it can't possibly be a regular value for both f and $f_{|\partial M}$ (unless $f^{-1}(y)=\emptyset$): First note that $f^{-1}(y)$ would have to be contained in ∂M (as submersions are open maps). On the other hand, if $x \in f^{-1}(y)$ and y is a regular value for f and $f_{|\partial M}$ then $\ker Df(x) = T_x f^{-1}(y)$ can't be contained in $T_x\partial M$.
- page 31,Theorem 4.2: Add $r \geq 1$. Can replace "$\partial A = \emptyset$" with "$\partial A = \emptyset$ or A is neat". The conclusion should be that $f^{-1}(A)$ is a neat C^r submanifold.
- page 31,line 13: "The proof of Theorem 3.4, 3.5 goes through with minor changes" This is true provided we are embedding the manifold with boundary in $\mathbb{R}^n$ (with no condition on where the boundary is sent).
- page 31,line 17: I don't know how to prove Theorem 4.3 exclusively with the techniques developed so far (even under the weaker assumption that $r \geq 2$). For the remarks concerning approximation to be applicable, approximation must be understood as approximation within maps which preserve the boundary (if a map is approximated by a neat embedding it must map the boundary to a hyperplane).
- page 32, Exercise 4: One only needs that f and $f_{|\partial M}$ are transverse to A.

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- page 32, Exercise 5: If I is one of the intervals in the statement, then each component of $f^{-1}(I)$ is a submanifold but the components might have different dimensions. A simple counter-example is $M = \{(x,y) \in \mathbb{R}^2\colon x^2+y^2 = 1, y \geq 0\} \cup \{(x,y) \in \mathbb{R}^2\colon (x-\frac{1}{2})^2+y^2 = \frac{1}{4}, y \leq 0\}$, $f\colon M \to \mathbb{R}$ the projection onto the second coordinate and $I = [0, \frac{1}{2}[$.
- page 35,Proposition 1.0: It should be remarked that the proof of Theorem 1.3.5 applies also to manifolds with boundary (these had not been defined at the time Whitney's theorem was proved in section 1.3) and density is valid also in the C^∞ topology (to be defined in a few paragraphs).
- page 36,Lemma 1.3: U should be an open set in a half-space of $\mathbb{R}^m$ so that this also applies for manifolds with boundary. The last line of the Lemma should be "for all $x \in U$, then $g|\overline{W}$ is an embedding." (meaning it is a homeomorphism onto its image and Dg is injective at all points on g). The proof will now work replacing W by $\overline{W}$ on page 37,line 1 and W by U on line 6. The point is that being an embedding on $\overline{W}$ is the same as being an injective immersion, which would not be true for W. Since $\overline{W}$ is not necessarily convex, one needs uniform convergence on a the larger set U for estimating the error in the Taylor formula the way explained in the proof.
- page 37, Proof of Theorem 1.4: In addition to the cover $\{W_i\}$ take also shrinkings of this cover $T_i \subset \overline{T_i} \subset Z_i \subset \overline{Z_i} \subset W_i$. Replace line -10 with "then $g(K_i) \subset V_i$ and $g|_{\overline{Z_i}}$ is a C^r embedding". On line -8, replace K_i with T_i and U_i with Z_i. On the displayed formulas on lines -4,-5 replace K_i with T_i and W_i with Z_i. On line -1, replace K_i with T_i and U_i with Z_i. On page 38, line 1, replace W_i with Z_i and on line 3, K_i with T_i. Finally on page 38, line 5 replace W with Z_i.
- page 37, line -8: Some more explanation could be helpful here: to construct the neighborhood $\mathcal{N}_1$ one needs to use the fact that the set of charts Ψ is locally finite (as in the proof of Theorem 1.5 on the next page).
- page 38, line 11: $f(N)$ should be $f(M)$.
- page 38, line 18: I think it would be helpful to remark here that a map is proper iff it is closed and all the fibers are compact.
- page 38, line -3 and page 39, line 1: Theorem 1.6 should be Theorem 1.7.
- page 40, line 5: "This will be proved by Theorem 3.1.4" should be "This is a simple consequence of Theorem 3.1.4".
- page 40, lines 6,9: Theorem 1.7 should be Theorem 1.8.
- page 40, Ex. 3: I suspect the intended question is whether a specific map whose factors are derivative and evaluation at a point is a homeomorphism.
- page 41, Ex 9: Add the requirement that the covering space is a local diffeomorphism, otherwise it won't be true (as homeomorphisms aren't open).
- page 41, Ex. 15: This is only true for $r < \infty$. If $r = \infty$ and M is not compact the set may restrict the values of all the derivatives, which is not allowed in the C^∞ topology.
- page 41, Ex. 16 b): $\delta\colon C^\infty_S(\mathbb{R},\mathbb{R}) \to C^\infty_S(]0,1[,\mathbb{R})$ is not open as there is no neighborhood of the 0 function in $C^\infty_S(]0,1[,\mathbb{R})$ contained in the image of δ. One can find functions in any neighborhood so that a high derivative will not extend to $\mathbb{R}$ (this is as in the previous exercise). So for this part of the question r should be less than ∞.
- page 43, Theorem 2.1: M can have boundary in this statement.

- page 46, Theorem 2.3: In (a), θ should be C^k (not just on the interior of the support) in order to apply Leibniz's rule in the proof. In the displayed formulas on (a) and (b), k should be replaced by l and "$l \leq k$ finite" added.
- page 46, line -8: "restricted to...". I don't see how this is relevant.
- page 47, line 22: Add "and so that if $K_i \cap K_j = \emptyset$ then $\mathrm{Supp}\,\lambda_i \cap \mathrm{Supp}\,\lambda_j = \emptyset$." (later in the proof one assumes that $\lambda_j = 0$ on K_i if $K_i \cap K_j \neq \emptyset$).
- page 48, Theorem 2.5: Saying that $s > r \geq 0$ would make the statement more clear.
- page 49, displayed formula $(6)_k$: The union should be indexed $1 \leq j \leq k$.
- page 49, 5th paragraph in proof of Theorem 2.6: In the definition of $\kappa(x)$ replace $\overline{U_k}$ with U_k (this still works and removes the need to argue that $\{\overline{U_k}\}$ is locally finite). If W is a neighborhood of x which intersects only finitely many U_k, $g = g_{\kappa(x)}$ on $W \setminus \cup_{\{j>\kappa(x)\colon\, W\cap U_j \neq \emptyset\}} \overline{W_j}$.
- page 52, line 3: Paracompactness and Hausdorfness are not used directly in the previous proof but they are used in the proof that embeddings are open and hence in the proof that diffeomorphisms are open. Thus I am not sure the uniqueness part of Theorem 2.9 holds without the paracompact and Hausdorff assumptions.
- page 53, line -14: "It is convenient to assume $r = 2$" should be "It is convenient to assume $r \geq 2$". Every C^2 immersion can be approximated by C^r immersions but just in the C^2 topology (obviously) so proving the Theorem just for C^2 will not give the result we want. However the proof as given will work for $r \geq 2$ given that Proposition 1.0 (easy Whitney) is also valid in the C^∞ case (by definition of the C^∞ topology).
- page 54, line 4: "is contained in" should be "contains".
- page 54, line 6: I don't know how to prove that $(\mathcal{F}, \mathfrak{N})$ is continuous with the methods developed so far. One needs to prove that given a chain A_α of closed subsets and compatible elements $[g_\alpha] \in F(A_\alpha)$ there exists a $[g] \in F(A)$ where $A = \cup_\alpha A_\alpha$ which restricts to the $[g_\alpha]$. Picking representatives g_α one can use local finiteness to produce an immersion on a neighborhood of A. However we need a map g defined on the whole of M which restricts to an immersion on a neighborhood of A and I don't see how to do this. In view of this I think it is better to adapt the proof of Theorem 2.12 so that it follows the plan of the proof of Theorem 2.6, as we saw in class. Namely, using the notation of the proof, identify the indexing set Λ with an interval in the natural numbers. Define a sequence of functions $g_k \in \mathfrak{N}$ so that $g_0 = f_0$, $g_k = g_{k-1}$ in a neighborhood of $\cup_{i=1}^{k-1} K_i \cup (M \setminus U_k)$ and g_k is an immersion in a neighborhood of $\cup_{i=1}^{k} K_i$. The argument used in the proof of local extendability of the structure functor shows how to obtain g_k given g_{k-1}. The limit $g(x) = \lim_k g_k(x)$ is the required immersion (the sequence is eventually constant in a neighborhood of any given x).
- page 54, line 14: Theorem 1.1 should be Proposition 1.0.
- page 55, line 11: "...the structure functor is continuous". Proving this would give similar problems to those discussed above in the immersion case.
- page 55, Theorem 2.13: This Theorem is proved by different methods on page 63.
- page 56, Exercise 5: delete A_k (unless the question is what are the constants A_k...)

- page 56, fourth line of Lemma 3.1: the subscript U' should be K.
- page 56, line -7: V should be U.
- page 56, Lemma 3.1: I don't know how to prove this lemma with the techniques developed so far (namely the part where you approximate a function by a smooth function keeping the zero set). The proof given for the case when ∂U and ∂F are nonempty is not correct for the following reasons:
  (i) The function $h(x, y)$ is not C^∞ if β is just any convolution kernel. Taking $n = m = 1$ for simplicity, the derivative of $h(y)$ is $-f(y)\beta(0)+$ a C^∞ term and so only has the regularity of f. $h(x, y)$ can be made C^∞ by picking a convolution kernel which has $\beta^{(k)}(0) = 0$ for all k. For plausibility it would also make sense to have the support of β concentrated in $t > 0$.
  (ii) Even if the above changes are made I don't see how to pick β so that the statement "$f_1(x, y) \geq f_1(x, 0)$ which implies $h_1(x, y) \geq h_1(x, 0)$" would be true. I don't see any reason why averaging would preseve the order and given any β I think one could construct an f for which this would not be true.

Here is the proof of Lemma 3.1 in the case when $X = \partial U$ that we saw in class. This is enough to prove Lemma 3.2 and Theorems 3.3 and 3.4 copying the proof for manifolds without boundary (using also $\mathrm{Diff}(M, N)$ open in $C^r(M, \partial M; N, \partial N)$).

Proof. The proof given in the book is correct when at least one of the open sets doesn't intersect the boundary so assume both U and V do. Assume F is defined by the equation $y_1 \geq 0$ and $0 \in V$ and similarly that E is defined by the equation $x_1 \geq 0$. First consider the case $r = 0$. By uniform continuity, given $\epsilon > 0$ there is an open cover of K by open sets $B_i \subset U$ so that $\sup\{\|f(x) - f(x')\| \colon x, x' \in B_i\} < \epsilon$. Let $U' = \cup B_i$. Let λ_i be a smooth partition of unity subordinate to the cover $\{B_i\}$. Pick $x_i \in B_i$ making sure that if $B_i \cap \partial U \neq \emptyset$ then $x_i \in \partial U$. Take

$$g(x) = \sum_i \lambda_i(x) f(x_i).$$

Then $g_1(x) \geq 0$. If $f(\partial U) \subset \partial F$ then $g(\partial U) \subset \partial F$. This is because $x \in \partial U$ and $\lambda_i(x) \neq 0$ implies $f_1(x_i) = 0$ so $g_1(x) = 0$. For $x \in K$, we have $\|g(x) - f(x)\| \leq \sum_i \lambda_i(x)\|f(x) - f(x_i)\| < \epsilon$.

Now assume $r \geq 1$. We start by approximating $f_{|\partial U} \colon \partial U \to F$ on $K \cap \partial U$ by a function k so that $k_1 \geq f_1$ (see the easy cases of the proof to see how this is done). If $f_1 = 0$ on ∂U we can certainly take $k_1 = 0$. Now consider the expression

$$f(x_1, \ldots, x_n) = f(0, x_2, \ldots, x_n) + \int_0^{x_1} \frac{\partial f}{\partial x_1}(t, x_2, \ldots, x_n)dt$$

which is valid in a neighborhood of $K \cap \partial U$.

Let h be a C^{r-1} approximation to $\frac{\partial f}{\partial x_1}$ picked so that $h_1 \geq \frac{\partial f_1}{\partial x_1}$ and define

$$g(x_1, \ldots, x_n) = k(x_2, \ldots, x_n) + \int_0^{x_1} h(t, x_2, \ldots, x_n)dt.$$

Then $g_1(x_1, \dots, x_n) \geq f_1(x_1, \dots, x_n)$ and if $f_1(0, x_2, \dots, x_n) = 0$ then $g_1(0, x_2, \dots, x_n) = 0$.

If h and k are close enough, g will be an ϵ approximation in a neighborhood of $K \cap \partial U$ and one can interpolate with an approximation away from the boundary to get the desired map in a neighborhood of the whole of K. □

- page 57, line 6: There is a missing } in the definition of U'.
- page 57, line 8: $f(x - x, y + t)$ should be $f(x - s, y + t)$.
- page 57, line 10 $f(x, y) \geq f(x, 0)$ should be $f_1(x, y) \geq f_1(x, 0)$.
- page 57, line 11 $h(x, y) \geq h(x, 0)$ should be $h_1(x, y) \geq h_1(x, 0)$.
- page 57, line -6: I don't know how to prove Theorems 3.5, 3.6 for manifold pairs with the techniques developed so far. One difficulty lies in proving a local approximation lemma as the one proved above for open subsets of half spaces. Another difficulty is that $\mathrm{Diff}(M, M_0; N, N_0)$ is not necessarily open in $C^r(M, M_0; N, N_0)$ if M_0, N_0 satisfy the hypotheses given. If these results come up later we will give a proof...
- page 61, line 16: The 2nd and 3rd occurences of θ in this displayed equation should be φ.
- page 62, line -14: $J^\infty(M, N)$ has a complete metric as it is a closed subset of a countable product of complete metric spaces.
- page 62: line -13: The map $j^\infty \colon C^\infty(M, N) \to C^0(M, J^\infty(M, N))$ is not continuous when we give the range and the target the strong topologies (because, on the target of j^∞, the strong topology may restrict the value of all the derivatives of a function from M to N when M is not compact). It is true that a weakly closed subset of $C^\infty_S(M, N)$ is a Baire space. One can adapt the proof of Theorem 4.2. For that it is convenient to pick complete metrics d_r on the jet spaces $J^r(M, N)$ so that, for $\pi \colon J^r(M, N) \to J^{r-1}(M, N)$ the projection, $d_r \geq d_{r-1} \circ (\pi \times \pi)$ (this can always be done by adding $d_{r-1} \circ (\pi \times \pi)$ to a complete metric on $J^r(M, N)$). Then inductively construct functions f_n and neighborhoods $N_n = \{g \in Q \colon d_{k_n}(j^{k_n} g, j^{k_n} f_n) < \epsilon_n\}$ with k_n an increasing sequence of natural numbers as in the proof of Theorem 4.2. This will ensure uniform convergence of f_n and all its derivatives to a smooth function in the given open set in Q.
- page 63, line 16: "The definition of r-jet is unchanged." This is not true (it is not true for tangent vectors which are particular kinds of 1-jets). In order for the rest of this section to make sense one needs to define jets as equivalence classes of jets on coordinate charts as was done in class, the latter being defined as jets from $\mathbb{R}^m$ to $\mathbb{R}^n$ even if the source and target points are on the boundary.
- page 64, line 1: Theorem 3.4 should be Theorem 4.4.
- page 64, Exercise 3, line 3: brackets missing around x's.
- page 65, Exercise 12: This doesn't make much sense as paracompact and Hausdorff implies normal...
- page 65, Exercise 14: On the first line, x_n should be $f(x_n)$.
- page 65, Exercise 16: It should be $2 \dim M \leq \dim N$.
- page 66, Statement of Theorem 5.2: $\delta(\kappa, x)$ should be $\delta(\kappa x)$.
- page 71, line 14: $\lambda | s$ should be λ / s.

- page 71, line 17: X should be Σ^1.
- page 74, Exercise 3: $\lambda^{-1}([0,y])$ should be $\lambda^{-1}([y,1])$.
- page 75, line 10: It is worth pointing out that the localization axiom implies, in particular that $\mathcal{X}$ is functorial in L, U, V meaning that, if $L \subset L', U \subset U'$ and $V' \subset V$ there is a restriction map $\mathcal{X}_{L'}(U', V') \to \mathcal{X}_L(U, V')$ and an inclusion map $\mathcal{X}_L(U,V) \to \mathcal{X}_L(U, V')$.
- page 75, line 14: The definition of *rich* shoul be changed in the $r = \infty$ case to the following: There exists $k < \infty$ so that
  (i) $\mathcal{X}_L(U,V)$ is open in the topology induced by $C^k_W(U,V)$,
  (ii) $\mathcal{X}_L(U,V)$ is dense in $C^\infty_W(U,V)$.
  The statement that $\mathcal{M}$ is strongly open on the same page, line -11 will then be true in the C^∞ case. Moreover, this stronger definition of rich C^∞ mapping class applies in all applications of the globalization theorem later.
- page 75, statement of Theorem 2.2: "mapping functor" should be "mapping class".
- page 75, line -12: "$\mathcal{M}$ is weakly open..." This is not true (take for instance $\mathcal{X}_L(U,V) = C^r(U,V)$). In order to fix the proof, pick U_i from the beginning so that $\overline{U_i}$ is compact and $f(\overline{U_i}) \subset V_i$. Then $\mathcal{N} = \{g \in C^r(M,N) \colon g(\overline{U_i}) \subset V_i \text{ if } K_i \neq \emptyset\}$ is a neighborhood of f in the weak topology if L is compact and in the strong topology in general. Now the proof works if we define $\mathcal{M}$ to be the set of $g \in \mathcal{N}$ which restrict to $\mathcal{X}_{K_i}(U_i, V_i)$ because the restriction maps $C^r_S(M,N) \supset \mathcal{N} \to C^r_W(U_i, V_i)$ are continuous.
- page 75, line -7: "For each i, let $\epsilon_i > 0$..." This whole paragraph is irrelevant and can be deleted.
- page 76, Lemma 2.3: In order for this Lemma to apply in the proof of Theorem 2.1 to submanifolds A with boundary it is necessary to replace $\mathbb{R}^a$ by a half-space $H \subset \mathbb{R}^a$.
- page 76, proof of Lemma 2.3: There are two problems with this proof. Firstly, $C^r(U,V)$ is not open in $C^r_W(U,\mathbb{R}^n)$ (it is in the strong topology) and secondly, it is not true that condition (ii) on line 3 of the proof is open (there is also a typo in this condition; it should be $f(x) \in \mathbb{R}^a$). In order to fix this let $W_1 = f^{-1}(V \setminus H)$ and $W_2 = \{x \in U \colon \pi \circ f \colon U \to V \to \mathbb{R}^n \to \mathbb{R}^n/\mathbb{R}^a \text{ is submersive at } x\}$. This is an open cover of K. Pick a shrinking $T_i \subset \overline{T_i} \subset W_i$ of the cover. Then there exist a neighborhood $\mathcal{N}$ of f in $C^r_W(U,V)$ so that $g \in \mathcal{N} \Rightarrow g(\overline{T_1}) \subset V \setminus H$ and g is submersive on $\overline{T}_2$. Since $K \subset (\overline{T_1} \cup \overline{T_2})$ it follows that $\mathcal{N} \subset \pitchfork_K (U, V; V \cap H)$. To prove density, given $g \in C^\infty(U,V)$ and $\mathcal{N} = \mathcal{N}(g, \Phi, \Psi, \{K_i\}_{i\in\Lambda}, \{\epsilon_i\}_{i\in\Lambda})$ a neighborhood of g in $C^r_W(U,V)$ (so Λ is finite) let $L = K \cup \cup_{i\in\Lambda} K_i$. Let $\lambda \colon U \to [0,1]$ be a smooth map so that $\lambda(x) = 1$ in a compact neighborhood of L and 0 outside an even bigger compact neighborhood. Take a sequence y_k as in the proof and consider the map $g_k = g - y_k\lambda$. Then for k big enough $g_k \in \mathcal{N} \cap \pitchfork^\infty_K (U, V; V \cap H)$.
- page 76, line -7: $\pitchfork^r_K (U, \mathbb{R}^n; E)$ should be $\pitchfork^r_K (U, \mathbb{R}^n; \mathbb{R}^a)$.
- page 77, proof of Theorem 2.1: In this proof it is assumed that the submanifold A is without boundary. It applies also to A with boundary once Lemma 2.3 is modified as explained in the previous remark.
- page 77, line 17: The displayed equation is not true. A true statement would be that $\pitchfork^r (M, N; A) = C^r(M,N) \cap \pitchfork^r (M, \mathbb{R}^q; \nu_A)$ where ν_A denotes a

suitable closed neighborhood of the 0 section in the normal bundle to N restricted to A (embedded in $\mathbb{R}^q$). The proof would then go through as stated. Alternatively, the proof of openness in Lemma 2.3 goes through without any changes if V is not an open set of $\mathbb{R}^n$ but instead the image of an open set W in a manifold with boundary by a diffeomorphism that sends $W \cap A$ to a half space H (this will not necessarily send ∂W to a hyperplane (cf. Section 1.4)). The openness part of the proof of Theorem 2.2 then applies to show that $\pitchfork_L^r(M, N; A)$ is weakly/strongly open when L is compact/closed.

- page 77, line -4: M_j should be subscript.
- page 78, Theorem 2.6: It is perhaps worth pointing out that the most important case is $M = D^q$ and $K = S^{q-1}$ so that $V_{k,n}$ is $(n-k-1)$-connected.
- page 79, line -1: $\dim N + \dim A - \dim M$ should be $\dim M + \dim A - \dim N$.
- page 80, line 6: "By (a)..." should be "By (b)...".
- page 80, line 8: "is and only if" should be "if and only if".
- page 80, line -7: "It now suffices..." It doesn't suffice to take $V = \mathbb{R}^n$ because $C_W^s(U, V)$ is not open in $C_W^s(U, \mathbb{R}^n)$. This is the same problem as in the proof of Lemma 2.3 and can be fixed in the same way: a neighborhood $\mathcal{N}$ of f in $C_W^r(U, V)$ restricts the values of f only on a compact set $L \subset U$. Let W be open with $L \subset W \subset \overline{W} \subset U$ and $\overline{W}$ compact, and $\lambda\colon M \to [0,1]$ be smooth with $\lambda(x) = 1$ in a neighborhood of L and $\lambda(x) = 0$ for $x \notin W$. If $Z \subset J_0^r(\mathbb{R}^m, \mathbb{R}^n)$ is a small enough neighborhood of 0 the map $F\colon Z \times U \to \mathbb{R}^n$ defined by $F(j_0^r g, x) = f(x) + \lambda(x) g(x)$ will have image contained in V and we can then apply the argument explained below.
- page 80, line -6: "$\pitchfork^s(U, \mathbb{R}^n; j^r, A)$ is open and dense in $C_W^r(U, \mathbb{R}^n)$" should be "$\pitchfork_K^s(U, \mathbb{R}^n; j^r, A)$ is open and dense in $C_W^r(U, \mathbb{R}^n)$ for each $K \subset U$ compact."
- page 80, line -5: "It is enough to prove this for s finite, $s > r$." Although this is true, I think it is simpler to omit this phrase and replace s with r on lines 5,7,13,15,17,18,21,22 of page 81 (10 occurrences).
- page 81, lines 16-24: The formulas for F^{ev} and β are not correct. $j_0^r(g + f)$ should be $j_x^r(g+f)$ translated to the origin of $\mathbb{R}^m$ via the canonical isomorphism $J_x^r(\mathbb{R}^m, \mathbb{R}^n) \to J_0^r(\mathbb{R}^m, \mathbb{R}^n)$. This affects the next 8 lines of the argument. Instead, use that the map sending a symmetric polynomial $g \in J_0^r(\mathbb{R}^m, \mathbb{R}^n)$ to its jet at $x \in \mathbb{R}^m$ is an analytic diffeomorphism (in fact it is affine) $J_0^r(\mathbb{R}^m, \mathbb{R}^n) \to J_x^r(\mathbb{R}^m, \mathbb{R}^n)$ (because the r-th order Taylor expansion at any point $x \in \mathbb{R}^m$ of a polynomial of degree r determines the polynomial). Because of this, the map β which sends $j_0^r(g)$ to $j_x^r(g+f)$ is a diffeomorphism and in particular has surjective derivative. If one does not want to replace s with r as suggested in the previous item then β will still be a submersion because its restriction to J_0^r is a diffeomorphism.
- page 82, line -11: In exercise 2, just before the displayed formula, $Tf(M_{x,k})$ should be $Tf(M_{x_k})$.
- page 82, Exercise 3: $f^{-1}(A_0 \cup \ldots \cup A_q)$ should be $f^{-1}(A_0), \ldots, f^{-1}(A_q))$.
- page 83, Exercise 5: Probably want to assume f is C^2 and M and N without boundary in order to apply the jet transversality theorem.

- page 83, Exercise 10: "if V is compact" should be "if M is compact". Also the end of the statement should be replaced by "...if and only if, for each $v \in V$ there exists a neighborwood W of v so that $F_{|W}$ is constant outside a compact set $K_v \subset M$."
- page 83, Exercise 11: It seems to me that the answer is $r > \max\{0, \dim M + \dim A - \dim J^r(M,N)\}$ which does not depend on s but does depend on $\dim A$.
- page 83, Exercise 13: In order to be able to apply Ex. 3 of Section 2.2, it seems to me that p should be a C^∞ map instead of a C^1 submersion.
- page 88, line -10: $\phi\colon U \to \mathbb{R}^n$ should be $\varphi\colon U \to \mathbb{R}^n$.
- page 92, Exercise 4.1.1: B should also be Hausdorff.
- page 103, Exercise 8: π_{k-1} should be π_{n-1} (two occurrences).
- page 108, line -2: Grassman is missing an n at the end.
- page 110, line 12: To apply exercise 2.1.7 directly would need $f_{|M}$ to be proper, i.e. would need M to be closed. However, M is locally closed so one can find an open set $V \subset \mathbb{R}^n$ such that M is closed in V and can then apply the exercise to the inclusion of M in V.
- page 111, line 9: One may have to replace U by a smaller neighborhood of the zero section in order to ensure that the map f defined by this expression is a diffeomorphism onto its image.
- page 111, line 15: It is perhaps better to define isotopy as a smooth map $F\colon P \times [0,1] \to Q$ such that F_t is an embedding for each $t \in [0,1]$. This is equivalent to the stated definition if and only if P is compact. In the rest of the book, the stronger condition that $F \times \pi_2$ be an embedding is never checked and, at the beginning of Chapter 8, the definition of isotopy is restated as the (inequivalent) weaker condition.
- page 112, line -13: In the displayed formula $H(x,y)$ should be $H(x,t)$.
- page 113, lines 1-4: The map s should take value in linear maps, i.e. $s\colon U \times \mathbb{R}^k \to L(\mathbb{R}^k, \mathbb{R}^k)$. This paragraph needs to be changed accordingly.
- page 113, line 13: $f_1^{-1}H(x, 1-t)$ should be $f_1 H(x, 1-t)$.
- page 113, line 21: The denominator should be $1 + |y|^2$ instead of $1 + y^2$.
- page 115, line 2: "We can extend the embedding of N to an embedding of V..." I think this deserves more explanation.
- page 116, Theorem 6.5: Add assumption $\partial M = \partial V = \emptyset$.
- page 121, line -13: This convention for the induced orientation on the boundary is not the standard convention coming from Stokes' Theorem.
- page 123, Lemma 1.2: The last line of the Lemma is not italicized.
- page 129, line -6: "the double of M". The differential structure on the double has not been defined! This should at least be added as an exercise in Section 4.6.
- page 130, Exercise 5(a),(b): I am not sure this is right. The inverse image of a loop might not be connected so can only show that the map is surjective on H_1, not π_1...
- page 130, Exercise 5(c): $f\sharp$ should be $f_\sharp$.
- page 130, Exercises 7 (c) and (d): A cylinder embedded with k is a Möbius band type surface where a line segment rotates k full turns around the central circle.

- page 130,131: Exercises 7 (b) and 8: In exercise 8 one should give the points x and y opposite orientations. Similarly, in exercise 7 (b) it's best to assume that M and N have positive dimension. Otherwise, if M is a disjoint pair of points in the interior of N and we give both points the positive orientation, the linking number of M with N will be ± 2.
- page 137, line 13: $\partial W \cup_{i=1}^{n} \partial D_i$ should be $\partial W \cup \cup_{i=1}^{n} \partial D_i$.
- page 139, Exercise 4: It seems to me that, with the given definitions, $\#(f,g) = (-1)^{m+mn}\#(f,g(N))$ and $\#(g,f) = (-1)^{nm+n+m}\#(f,g)$.
- page 139, Exercise 9: I think the definition of $L(G)$ does not make sense as there are no "corresponding orientations". It makes sense when G is of the form $x \mapsto (x, g(x))$.
- page 147, line -5: $J^1(M,\mathbb{R})$ should be $J^1(M,\mathbb{R})_0$.
- page 150, line 8: In the displayed equation $\psi - 1$ should be ψ^{-1}.
- page 158, line 3: In the displayed equation $D^{n-k}(\sqrt{2\epsilon})$ should be $D^{n-k}(\sqrt{3\epsilon})$ (this is what is in accordance with Figure 6-4).
- page 158, line -8: Γ_2 should be $D^k(\sqrt{3\epsilon}) \times D^{n-k}(2\sqrt{\epsilon})$.
- page 160, Theorem 3.3: There is no reason to assume that the critical points have the same index.
- page 166, line -11: $f^{-1}[a_1 a_1]$ should be $f^{-1}[a, a_1]$.
- page 172, Exercise 1: $(-1)^{i+j}\beta\alpha$ should be $(-1)^{ij}\beta\alpha$.
- page 175, Exercise 2: The Thom space was only defined for bundles over manifolds without boundary so probably want to assume the base is without boundary.
- page 213, line 10: The definition of "locally finite" is mistaken. What is defined here is called "point finite". Locally finite means that every point x has a neighborhood W_x which intersects only finitely many of the open sets in the cover $\mathcal{V}$.