Prolegomena to Any Future Paraconsistency

Some 40 years ago, a remarkable logical approach to the taming of inconsistencies was pioneered by Newton Carneiro Affonso da Costa, in Brazil. The present monograph commemorates this endeavor by updating and extending some chosen aspects of the daCostian approach, centered around the possibility of securing the classical behavior of some assertions made inside a paraconsistent environment.

The birth of paraconsistent logic

> Ai, ai, ai, ai
> Have you ever danced in the tropics?
> With that hazy lazy
> Like, kind of crazy
> Like South American Way
> —Al Dubin e Jimmy McHugh, *South American Way*, 1930s.

Time often helps us separate the wheat from the chaff. With some luck, inconsequent ideas eventually are abandoned and forgotten. It would be a pity, though, that an important approach to a variety of non-classical logics, and one that is so close to our hearts and minds, would end up remembered only for the wrong reasons. An everlasting myth perpetuated by a considerable parcel of the literature on paraconsistency concerns the alleged origin of paraconsistent logic nearby Curitiba, Paraná, to wit, somewhere in between the pinelands and the sea of Southern Brazil. This section aims at debunking that myth, if only for the sake of intellectual honesty in the practice of the 'science of logic'.

Let's initially consider here two expository papers by da Costa and collaborators, namely [43] (1995) and [42] (1999). In [42], for instance, one can find the following assertion [here in my translation]:

> In fact, the first logician to have built paraconsistent systems having a full scope (propositional logic, predicate logic, set theory) is N. C. A. da Costa (cf. [35], [36]).

In a similar vein, [43] mentions, right from the start, "the creation of paraconsistent logic by the first author of the present paper [da Costa], more than thirty years ago", as having shown that it is "possible to develop a logic in which contradictions can be mastered, in which there are inoffensive or, at least, not dangerous contradictions".

In reality, the paper [43] proposes to tell us the history of the 'invention of paraconsistent logic', and to that effect it mentions the 'forerunners' of paraconsistent logic (according to the paper: Łukasiewicz, Vasiliev, Jaśkowski, Nelson, Smiley, but not Orlov), setting them at 'a great distance' from the 'discoverer' of paraconsistent logic (according to the paper: Newton da Costa). In the particular case of Vasiliev, the paper asserts that his work "was not really understood until the seventies, when the first author [da Costa] read an abstract of a paper of his written in English, and perceived that he had the intuition of paraconsistent logic. Then he suggested that one of his students, A. I. Arruda, investigate Vasiliev's works". On what concerns Jaśkowski, the title of 'discoverer' is denied in that same paper because he "has not constructed any discussive logic at the quantificational level. This was done by L. Dubikajtis and the first author [da Costa] in the sixties." Nelson and Smiley are merely mentioned by name, and their works are not commented upon. In addition, a great emphasis seems to be put on the allegation that da Costa developed his paraconsistent calculi "in a completely independent way from the works of Vasiliev, Łukasiewicz and Jaśkowski. At that time, in Brazil, the works of these logicians were inaccessible to him".

I cannot help but find the above statements utterly puzzling. In his initial thesis on paraconsistent logics, da Costa closes the introduction (p.5) by writing that [in my translation]:

> Our research had its origin in studies that we have previously published (see [32, 33] and [30, 31]). But, to the best of our knowledge, very little has been done on the topic, besides certain inquiries by Jaśkowski (see [55, 56] and [65, 66]); some studies by Nelson bear some relation to the object of this thesis, though the orientation of the North-American logician is very much distinct from ours (see [68], where you will find bibliographic references).

This paragraph alone already seems to seriously impair the last contention about da Costa deserving a special merit for having been a lone researcher with no access to the work of other logicians —as he does indeed seem to have had access to all relevant papers, at some point. At any rate, from a historical perspective, why should ignorance or lack of contact with the outside world be attached anything more than a sentimental value at the moment we are assessing one's contribution to science? A more balanced partial account of matters was presented by da Costa himself in his opening address (read by Itala D'Ottaviano) at the Stanisław Jaśkowski's Memorial Symposium, held in Toruń, Poland, in 1998 (cf. [40]):

> I was delighted to notice, in the early 1960's, that the work I had developed in Brazil by that time had close connections with Jaśkowski's. I recall, as if it were today, reading the English abstract of one of his papers, and realising that the two of us were independently producing

> works of a striking similarity. I then sent him a letter, and that is how my long term contact with the Polish community of logicians started.

Many papers produced by the 'Brazilian school', influenced by da Costa, try to maintain somehow that the latter "is actually the founder of paraconsistent logic" (e.g. [3, 51]). However, to rule out any possible doubt about da Costa himself not having recognized Jaśkowski's central role on that foundation (15 years earlier than the former author), it should be noticed that the same address by da Costa, [40], states that "it was not earlier than 1948 that Stanisław Jaśkowski, under Łukasiewicz's influence, would propose the first paraconsistent propositional calculus".

At first, and second, analysis, the distinction proposed in [43] between 'forerunners' and 'discoverers' of paraconsistent logic surely seems discretionary —one could even say inconsistent with all the information brought out by other papers. What are, after all, the criteria used by the authors of [43] in order to determine which researchers belong to each class? Well, on that very respect they defend that the birth of paraconsistent logic "corresponds to its appearance, strictly speaking, as a theory, i.e., as a mathematical theory, studied in itself in a systematic way, and scientifically acknowledged", and they situate this event after da Costa's 1963 thesis. Moreover, they say that "to really constitute a logic", a system of logic "has to be developed at least to encompass quantification and equality, given the role of logic in the articulation of conceptual systems". They conclude by saying that "to this extent, the first author [da Costa] seems to have been the first logician to have done so with paraconsistent logic".

There are several immediate problems posed by the above conceptual schema. Neither 'paraconsistency' nor 'logic' itself, as enterprises that together will allow us to build 'inconsistency-tolerating' 'reasoning mechanisms', seem to depend, for their definition, on anything beyond the propositional object-language level. First-order paraconsistent logics are certainly important for many applications, but that fact alone does not obligate logic or paraconsistency to involve first-order notions from the start. Moreover, as I have argued in [63], if a separation should really be drawn among, on the one hand, those 'forerunners' of paraconsistent logic who have merely advocated for a change of attitude towards the contradictions that would be present in our theories or who have only informally described reasoning mechanisms that would deal with such contradictions, and, on the other hand, those 'founders' of paraconsistent logic who have realized that the most important task to be accomplished was that of avoiding triviality or overcompleteness and, derivatively, the task of controlling the explosion principle of classical logic, then the first class would contain people such as Vasiliev, Łukasiewicz and Wittgenstein, while the second class would contain logicians who have actually built such logic systems, such as Jaśkowski, Nelson and da Costa.

About Jaśkowski (1948 and 1949), as a matter of fact, da Costa says in [40] that "to the best of my knowledge, he was probably the first to formulate, with regard to inconsistent theories, the issues connected with non-triviality". Though da Costa rarely ever mentions nor explains the investigations done by Nelson (1959), there should be no doubt either about Nelson's insight or the importance of his work. Indeed, in [68], a paper based on developments made a decade earlier (cf. [67]), Nelson writes that:

> In both the intuitionistic and classical logic all contradictions are equivalent. This makes it impossible to consider such entities at all in mathematics. It is not clear to me that such a radical position regarding contradiction is necessary. I feel that it may be possible to conceive a logic which does more justice to the uncertainty of the empirical situation insofar as negation is concerned.

Then, after developing in some detail a nice and well-motivated first-order paraconsistent logic with equality, Nelson asserts that "the system has been constructed, of course, to show that the logical operations may be interpreted in such a way that a mathematical system may be inconsistent without being overcomplete". Moreover, not only was this paper by Nelson written in English and was widely circulated, having been related to a number of advances made by then on the study of constructivity in mathematics, but this investigation also gave rise, not so much time later, to further developments by other authors, as a well-known study done by Fitch (cf. [53]) that considers Nelson's logic as a system to overcome paradoxes or a well-known thesis by Prawitz (cf. [70]) that studies a constructive naive set theory based on Nelson's logic. There is no reason thus for the paraconsistent community to continue neglecting, by and large, Nelson's approach. A modern account of Nelson's constructive logic can be found, for instance, in Wansing's [86].

Of course, the fact that Santos-Dumont was *not* the first to invent the airplane does not in any sense diminish the many deeds of Embraer. Analogously, the fact that Newton da Costa was neither the first nor the second author to develop paraconsistent logics should not count against the many interesting intuitions and approaches promoted by the 'Brazilian school' along the years. If, on the one hand, paraconsistent logics still remain nowadays as a rather marginal variety of non-classical logics, on the other hand the papers written by Newton da Costa on the theme (several of the initial studies having been done only in Portuguese and many of them having been published in places that did not render them much visible) have never been the most accessible or the most popular ones on the field, globally speaking. Promoting the approach of the 'Brazilian school' by way of biased historical revisionism would appear to be at most irrelevant: a misleading, pointless and unnecessary strategy. It would only prove that sin *does* exist beneath the Equator, and it would not help in making that approach more accessible or popular. I tend to believe that a better job would be done if we could only stop losing time trying to guarantee a historical or conceptual priority, and concentrate instead on technical and philosophical aspects of relevance. My work in the area aims to make a contribution to this second strategy.

Semantic intuitions

Veritas? Quid est veritas?
—Pontius Pilate (Joannes 18:38).

One of the advantages of saying that you are a 'formalist' is that you do not really have to *understand* the things you are doing. Formalism is often misconstrued in fact as mere 'blind manipulation of symbols'. In principle, a computer could do it better than you. Furthermore, a related confusion often to be found in discussions about formalism is betrayed by assertions to the effect that it makes no sense for a formalist to talk about the 'existence' of mathematical objects. I am certainly no expert in this matter, but *both* positions seem to me to constitute serious misconstructions of David Hilbert's Metamathematics. Anyone who carefully reads Hilbert's lecture 'On the infinite' (cf. [54]) —or, for that matter, several other texts by Hilbert, close collaborators like Bernays, and competent commentators—, will see that Hilbert did *not* defend the idea that mathematical objects 'have no meaning', *neither* did he attack the idea that mathematical objects could have some sort of 'real existence'. In fact, Hilbert fully acknowledged the usual meaning of numbers in number theory and the existence of points and lines in euclidean geometry. Hilbert did worry though about the alleged lack of meaning of some 'ideal elements' such as 'complex numbers' or the 'actual infinite', in spite of how useful these ideal elements have proved to be in mathematics, and stressed the importance of heeding proof theory as a safer way of checking the 'consistency' of our theories, proceeding by finitary steps and abstracting from the meaning of the objects and of the constants of mathematics and logic. I wonder why some people maintain that Hilbert defended a doctrine any stronger than this.

As we are up to this, one should perhaps notice that Van Quine seems to have been quite content with Hilbert's formalist approach to the existence of mathematical objects, as he made just a further small step beyond when formulating his famous ontological motto for mathematics: "To be is to be the value of a variable" (cf. [78]). However, Quine's ontological strictures, as it should be clear, aim not to show that something exists, but rather to tell us about our own ontological commitments when positing our theories. (To help choosing among competing ontologies Quine just suggested that we should accept the "simplest one that fits our experience".) Taking into account the modern proliferation of logical alternatives to classical logic, Newton da Costa suggested to update Quine's slogan by substituting it for: "To be is to be the value of a variable in a particular language with a given underlying logic" (cf. [37, 47]). Moreover, as we will see, if Hilbert took it to his heart that the non-contradictoriness of a mathematical object should count as a necessary and sufficient condition for its very existence, da Costa's approach to paraconsistent logic was soon to update that guideline by substituting Hilbert's 'consistency' by a more generous logical notion, that of 'non-triviality'. But much more will be said below about non-triviality and about (paraconsistent) logic and ontology.

In contrast to the rest of the present monograph, whose main approach to logic is more formal and abstract-oriented, here I want to start by a brief reasonably informal semantic-oriented motivation for paraconsistent logic and for the Logics of Formal Inconsistency. Some definitions of consistency, inconsistency, varieties of explosion and trivialization will be hereby illustrated. Whenever necessary, I will use '$\neg$' as a symbol for unary negation.

Let's consider some convenient set of sentences $\mathcal{S}$, a primitive set of states of affairs G, and two binary predicates defined over subsets of $\mathcal{S}$ representing two notions of consequence: A local notion of consequence $\Vdash^l$ associated to each state $l \in G$, and a global notion of consequence $\Vdash^g$. Given a sentence A and some state l, say that A can be inferred in l in case $\varnothing \Vdash^l \{A\}$. Denote this alternatively by writing $\Vdash^l A$, and in case A cannot be inferred in l denote this by writing $A \Vdash^l$. Given sets of sentences Γ and Δ, say that Δ can be locally inferred from Γ in l in case $\Gamma \Vdash^l \Delta$; in what I call the canonical definition of consequence, this will hold good exactly when there is some $A \in \Gamma$ such that $A \Vdash^l$ or some $B \in \Delta$ such that $\Vdash^l B$. Set-forming braces will often be omitted so as to streamline notation. Similar definitions can be proposed for the global consequence, with the restrictions that (a) given $\Gamma \cup \Delta = \Gamma$, then $\Gamma \Vdash^l \Delta$ iff $\Gamma \Vdash^g \Delta$, and (b) $\Vdash^l \subseteq \Vdash^g$. In the canonical definition of consequence, $\Gamma \Vdash^g \Delta$ holds good exactly when $\Gamma \Vdash^l \Delta$ for every state of affairs $l \in G$. In case $\Gamma \Vdash^x A$ for every $\Gamma \subseteq \mathcal{S}$, we say, if $x = l$, that A is acceptable in l, and we say, if $x = g$, that A is a thesis of G. If a similar thing can be checked about $A \Vdash^x \Delta$, for every $\Delta \subseteq \mathcal{S}$, we say, if $x = l$, that A is refutable in l, and if $x = g$ we say that A is an antithesis of G. Assuming that the above notion of state of affairs is intended to embody intuitive notions of truth and falsehood (you could read $\Vdash^l A$ by 'A is true in l' and read $A \Vdash^l$ by 'A is false in l'), the associated notions of consequence are intended to guarantee that truth is preserved from premises to conclusions and falsehood preserved from conclusions to premises.

Let's concentrate on the canonical notion of consequence. So, when I write hereon something like $\Gamma \Vdash \Delta$, for some $\Gamma \cup \Delta \subseteq \mathcal{S}$, I will mean $\Gamma, \Sigma \Vdash \Pi, \Delta$ for every $\Sigma \cup \Pi \subseteq \mathcal{S}$. Suppose that a particular logic corresponds to each choice of sentences and of a global consequence relation, as defined above. Say that a set of sentences Σ is explosive in case $\Sigma \Vdash^g$. There are many alternate ways in which a set of sentences Σ can explode. This Σ could for instance be a singleton (what I call a bottom particle), or it could be a pair $\{A, \neg A\}$ of contradictory sentences that only explode together, not in separate, defining thus a notion of negation-explosion. Or it could also be some gentle kind of explosion that demands the presence of a larger number of sentences depending exclusively on A. Perhaps we do not have explosion with respect to contradictions made with $\neg$, but we have a supplementing form of explosion with respect to contradictions made with some other primitive or derived negation symbol. Perhaps explosion can be somewhat controlled and $\{A, \neg A\}$ explode only for sentences A of a certain format. Explosion could also be partial, in allowing one to infer not just any sentence, but at least some sentences of a certain format that are not already theses of the underlying global consequence. By definition, paraconsistent logics should fail at least the most basic form of negation-explosion.

Call a state of affairs dadaistic in case it satisfies all sentences of $\mathcal{S}$, and call it negation-inconsistent in case it satisfies some pair of sentences of the form A and $\neg A$. We say that a logic $\mathbf{L}$ is consistent in case two requirements are met: (a) $\mathbf{L}$ admits of no dadaistic state of affairs (that is, $\mathcal{S}$ is explosive), and (b) not all sentences of $\mathcal{S}$ are theses of $\mathbf{L}$. The notion of negation-consistency of a logic $\mathbf{L}$ adds to those requirements the idea that: (c) any contradictory set of sentences is explosive. Obviously, (c) implies (a). In contrast, a paraconsistent logic, though, is (negation-)inconsistent, and in principle it could admit of dadaistic states of affairs. Decent paraconsistent logics will also admit of some non-dadaistic negation-inconsistent states.

There are now many varieties of inconsistency to be considered. A logic $\mathbf{L}$ is called trivial in case any set of sentences can be inferred from any other. With the canonical notion of consequence, that can only be the case if $G = \varnothing$. In case the logic $\mathbf{L}$ is not trivial but every one of its sentences is a thesis (and so, all states of affairs are dadaistic), then $\mathbf{L}$ is called absolutely inconsistent, or dadaistic; in particular, for any given negation symbol, all contradictions are inferable as theses of such a logic. Any logic $\mathbf{L}$ that has some pair A and $\neg A$ of theses is called dialectic. Most paraconsistent logics in the literature are not dialectic. A simple form of negation-inconsistency, for a logic $\mathbf{L}$, is obtained if one admits both non-dadaistic states of affairs (escaping thus absolute inconsistency) and negation-inconsistent states (invalidating explosion). A decent form of negation-inconsistency requires the admission of states of affairs that are at the same time non-dadaistic and negation-inconsistent. An expressively inconsistent logic is decently inconsistent, but it has in common with consistent logics the requirement that no dadaistic models may be admitted. Most important paraconsistent logics are decently inconsistent, but several of them fail to be expressively inconsistent. Finally, a gently inconsistent logic is one for which, for every sentence A, there is a certain number of things that you can say about this sentence so as to make it explosive, that is, there is a minimal set $\underline{\overline{\bigcirc}}(x)$ of sentences depending only on the sentence x such that $\underline{\overline{\bigcirc}}(A)$ (or maybe $\underline{\overline{\bigcirc}}(A) \cup \{A, \neg A\}$) cannot be satisfied by any state of affairs. Logics of Formal Inconsistency constitute a variety of gently inconsistent logics. In such logics the set of sentences $\underline{\overline{\bigcirc}}(A)$ to be added to $\{A, \neg A\}$ in order to make the latter set explosive is said to express the consistency of the sentence A.

On what concerns the above notion of consistency, and to quickly go back to the theme from the beginning of this section, one seems to have now two equally good choices of approach to the Logics of Formal Inconsistency: One can either be a formalist and construe consistency as yet another 'ideal element' to be justified by the role it plays in our structures, or one can adopt instead the semantic intuition that consistency is whatever a theory might be lacking so as to become non-trivially explosive.

The Fundamental Feature of LFIs

> No fairer destiny could be allotted to any physical theory, than that it should of itself point out the way to the introduction of a more comprehensive theory, in which it lives on as a limiting case.
> —Albert Einstein, *Relativity: The Special and General Theory*, chap. 22, 1920.

Agnosticism in logic. As in Poland, the development of mathematical logic in Brazil was strongly influenced by logical positivism, the doctrine that calls *meaningless* any statement that is neither verifiable nor refutable. This intellectual stance has often been taken too far, and it has been used to disqualify a good part of philosophy as 'purely speculative'. All in all, obscure metaphysical jargon was the preferred target of positivists. As an alternative, it became popular to do something that was dubbed 'scientific philosophy', always to begin with an analysis of language, and often to proceed by the use of even more impenetrable jargon. Ray Smullyan gives a humorous definition of a logical positivist as someone who rejects as meaningless anything that *they* cannot understand (cf. [81]). He also tells the story of a lady who, despite not having any formal education as a philosopher, lived in a house full of philosophy books. When asked for the reason of that, she replied that her ex-husband was a logical positivist, and added that it was logical positivism that broke up their marriage. How come? She explained that it was simple: Whatever she said, he told her it was meaningless!

Newton da Costa has since long (cf. [41]) been a faithful espouse of scientific philosophy and a professed supporter of a certain variety of Quinean platonism (check what I wrote about this just above, and check also the note 5 in **Chapter 1.0**). The criticism of the 'standard form of platonism' that da Costa presents in [39] is determinedly directed against the 'speculative character' of a doctrine that "presupposes that mathematical objects are grasped by a material intellectual intuition", a doctrine he rejects for being 'too nonscientific'. Moreover, on the import of logic to philosophy, da Costa insists in [37] that philosophical doctrines cannot be derived directly from logic, or from geometry, or from any other scientific field. On the indirect contribution of logic to philosophy he mentions though the possibility of using logic in "the elaboration of philosophical theories" or in "showing the formal inadequacies of philosophical inquiries". As an example of the former phenomenon, he mentions Tarski's researches on the notion of truth as having shown the tenability of the theory of correspondence, and as an example of the latter phenomenon he mentions Gödel's theorems as having promoted debates on the philosophical status of the formal sciences and a revolution in the domains of proof and of axiomatization.

In spite of his sympathy for realism, however, and the consequent rejection of fictionalism and instrumentalism (cf. [41]), and in spite of the alleged indirect contributions of logic to philosophy, da Costa wants to advocate the 'philosophical neutrality' of paraconsistent logic. In [46], da Costa and

Bueno write that, just as mathematics, logic "cannot justify by itself any metaphysical or, in general, 'speculative' position" [quotation marks by the authors]. Apparently concerned in their paper with 'dialetheist' interpretations of paraconsistency (although no paper on dialetheism is explicitly mentioned), they also write that they want to "stress that one *cannot* prove that 'speculative' philosophical interpretations of paraconsistent logic *cannot be true* (though it might be also difficult to show that they are)".

Dialetheism (cf. [76]) is a doctrine according to which "there are true contradictions" (in mathematics, presumably, or in reality). But, "just as empiricists (such as van Fraassen) are agnostic about (the existence of) unobservable entities in science", da Costa aims to be "agnostic about the existence of true contradictions in nature" (cf. [40]). A reason he presents for that is the so-called 'underdetermination argument': "There are always many paraconsistent logics which can be used to accommodate a given 'phenomenon' —whether it is an 'inconsistent' reasoning or an 'inconsistent' theory" [quotation marks by the author]. For da Costa (check [39], chap. IV.3), "reason, in the sense of a set of principles, does not coincide with any system of logic" [my translation]. In fact, while reason "constitutes itself historically, in harmony with the reality that surround us", each particular logic is supposed to have its own domain of application, as "the logical system underlying each rational context is a consequence of the pragmatic principles of reason, the nature of the context and the historical and cultural factors that shape reason" (id., ibid.). In the particular case of paraconsistent logics and dialetheism, da Costa maintains that "in the present state of science, it is not known whether the universe is consistent or not, in a strict sense, that is, if there are real contradictions" (id. chap. III.5). For "logic does not have a way of deciding, by itself, if there are real contradictions in the world. These can only be verified or refuted by way of experimentation, through the scientific method". Similar arguments are raised by da Costa and Bueno, in [45], against dialetheism and the idea that there would be 'one true logic' —and one of a paraconsistent character. According to the authors: (1) dialetheists have provided no evidence that their logic is the one true logic; (2) each domain has its own appropriate logic, to be chosen with the help of heuristic and pragmatic reasons. These arguments have been criticized, though, by Tanaka (cf. [82]), according to whom the evidence *has* been exhaustively provided, and it's up to da Costa and Bueno to reject it. Moreover, the mentioned heuristic and pragmatic rationale that would allow us to choose *this* logic instead of *that* never seem to be provided by da Costa and Bueno themselves.

From my own perspective, agnosticism might well be the most convenient wager. For the purposes of the present monograph on the foundations of paraconsistent logic, to take a position about dialetheism seems entirely immaterial. In defending agnosticism with respect to the existence of true contradictions, da Costa writes (cf. [37]) that "most systems of paraconsistent logic may also be treated as mere formalisms, by means of which we are able to systematize theories or systems of theories containing contradictions. But, in this case, contradictions are not interpreted as real, but as difficulties caused by the limitations of our knowledge". The point was more fully elaborated by Diderik Batens (cf. [10]), who argues that, even if the world is consistent, we might still need paraconsistent logic to be applied to our *theories*, rather than to the *world*. Moreover, as Batens recalls, "inconsistent theories may very well be the best among the theories available at a particular point in time". Inconsistencies do not have to be 'real', but they may arise, for instance, from conflicting observational criteria, from the language and the relations we choose for describing the world (check also [8]), or from the way we construct our scientific worldview, in which case "it is usually preferable to face an inconsistency rather than to neglect one half of it". Indeed, in the case where one of the conflicting theories is ignored, "if that theory turns out to prevail, one will be forced to reorganize one's worldview in a much more drastic way —the full bet was on the wrong alternative".

For my part, I cannot see what is wrong, in fact, about speculation *per se*, as long as it produces measurable results. Speculation does not presume mysticism, nor does it antagonize science. There is nothing wrong with metaphysics, either. Metaphysical assumptions underlie, in one way or other, any undertake of ours to understand the world. Besides, that technical concepts of formal logic might be invoked in the clarification and the study of such metaphysical assumptions should not be seen as anything like an unwelcome or an unexpected intrusion (check [52]).

A lot of philosophy can be done by 'discursive' means, that is, by sewing arguments through the use of reason rather than intuition or revelation. This is not to deny any rationality for intuition, and it is not to say that one's hunches and epiphanies should be altogether ignored, but only to say that these latter forms of access to knowledge should be used with extreme care, before one can get a better grip on how they are produced, and where they are leading to. Furthermore, discursive philosophy is often done more than well with the use of informal logic, critical thinking, and a more or less informal discourse. As opposed to that, and given the sort of problems I am to tackle in this monograph, what I intend to be doing instead is a kind of 'formal philosophy'.[1] I will not answer any of the great problems of philosophy, or of metaphysics, or even of philosophical logic. Auspiciously, however, I do hope to make some advances on some of the great philosophical problems of logic. Just logic.

[1] "Formal philosophy is called logic", writes Kant in the preface of *Fundamental Principles of the Metaphysics of Morals*, 1785. The use of the term 'formal philosophy' in denoting the employment of logic and formal methods in the study of language and grammar was championed in more recent times by Richard Montague (cf. [84]). The use of this same term, as referring to the use of formal methods in philosophical contexts, seems in fact to be experiencing a revival, nowadays. For the Northern winter of 2005, for instance, an 'International Conference on Formal Philosophy' is being organized by the Danish Research School in Philosophy, History of Ideas and History of Science and the Danish Network for Philosophical Logic and Its Applications.

Recipe for a certain programme in paraconsistency. The first paraconsistent logics ever built by Newton da Costa, the calculi C_n, $1 \leq n \leq \omega$, came to life in 1963.[2] The rationale presented by da Costa for that construction (where C_0 represents classical logic), in 1974, was:

> **The Calculi** C_n. As C_n, $1 \leq n \leq \omega$, are intended to serve as bases for non-trivial inconsistent theories, it seems natural that they satisfy the following conditions: (i) In these calculi the principle of contradiction, $\neg(A\&\neg A)$, must not be a valid schema; (ii) from two contradictory formulas, A and $\neg A$ it will not in general be possible to deduce an arbitrary formula B; (iii) it must be simple to extend C_n, $1 \leq n \leq \omega$, to corresponding predicate calculi (with or without equality) of first order; (iv) C_n, $1 \leq n \leq \omega$, must contain the most part of the schemata and rules of C_0 which do not interfere with the first conditions. (Evidently, the last two conditions are vague.)

In practice, the construction of those calculi was to make use of Kleene's axiomatization of classical logic (cf. [57]), except for the axiom that guarantees *reductio ad absurdum*, a rule to be necessarily failed by paraconsistent logics (check **Chapter 4.1**). Instead of that usual axiom, in general, restricted versions of it, related to requisite (i), were considered by da Costa: Calling A° the formula $\neg(A\&\neg A)$, *reductio* was guaranteed in C_1 as soon as A° could be assumed to hold, in C_2 the same guarantee would be valid as soon as both A° and $(A^\circ)^\circ$ could be assumed to hold, and so on. The result was the definition of an increasingly weaker hierarchy of logics: $C_1 \gnapprox C_2 \gnapprox C_3 \gnapprox \dots$

I will return below to each of the above requisites by da Costa in some detail. For the moment, it suffices to say that, as in any (decent) paraconsistent logic, requisite (ii) is obviously going to be satisfied by the logics C_n, for $n > 0$. Moreover, as I have argued in [27], C_ω, the weakest logic in the hierarchy, is in fact an intruder, as it does not share the main metatheoretical features of the other logics. On what concerns requisite (i) and the choice of the formula $\neg(A\&\neg A)$ to represent the 'principle of contradiction', we will see that it is an ill-advised move, to say the least. Moreover, as I have maintained above, requisite (iii) goes much beyond the mere *proposal* of paraconsistent systems, but it cares instead about their *use*. Finally, requisite (iv) is simply never respected by the logics from the above mentioned hierarchy. A thousand times repeated in the literature, I will argue that the above 'natural conditions' on the construction of paraconsistent logics neither determine in any sense the logics in question (once they are somewhat ill-advised, partially disrespected, and pretty vague), nor do they mention some of the most important features of those logics: Their capacity of expressing consistency and spreading it, and the possibility they open in a paraconsistent environment for classical reasoning to be fully recaptured.

[2]Though da Costa often claims to have started to develop his own brand of paraconsistency 'from 1954 onwards' (cf. [47, 43]) or 'from 1958 on' (cf. [37]), his own criterion for determining the date of birth of paraconsistent logic, as we have seen above, would force us to ignore such dates and stay with 1963, date of his first known publications on paraconsistent logic properly speaking (cf. [35, 34]). Check also the brief historical note at the end of this **Prolegomena**.

The fetish formula, and the Principle of (Non-)Contradiction. A lot of noise is often made in the literature about paraconsistency representing the 'effective derogation of the Principle (or Law) of (Non-)Contradiction'. Da Costa himself claims that "in paraconsistent logic, the principle of contradiction, in one form or another, is qualified or limited" (cf. [40]). We have already seen above that da Costa was worried from the start about the validity of (specific forms of) this principle, and, as a matter of fact, it is easy to see that he shared this preoccupation with many people. For instance, Jean-Yves Béziau writes in [15] that, "roughly speaking, a paraconsistent logic is a logic rejecting the principle of non-contradiction", and similar statements can be found in several other papers by this author (e.g., [17, 18]). Now, that might be true, but, as I show in **Chapter 1.0**, it all depends of course on how you read that principle.[3] Should the Principle of Non-Contradiction say that "no sentence can be true together with its negation", then one would still have to clarify, for instance, what 'true' means: Is it a 'local' or a 'global' notion? Does it mean 'satisfiable' or 'valid'? In the former connotations, the principle reduces to what I here call Principle of Explosion, a direct concern of paraconsistent logics; in the latter connotations the result is a much weaker principle, and one that is respected by the great majority of known paraconsistent systems. That I have decided to use the term in its second connotations does not prove anything definitive about the relation of the 'Principle of Non-Contradiction' to the making of paraconsistent logics. What *can* be proved, though, is that neither formulation of the above principle is related, in general, to the validity of the formula $\neg(A\&\neg A)$, which I shall hereby call the 'fetish formula' of (some) paraconsistentists.

As soon as a fetish formula of the form $\neg(A\&\neg A)$ was proved, da Costa called the formula A *well-behaved* (cf. [35, 36]). As we will see, that meant in practice that the formula A 'behaved classically'. As we have seen in requisite (i) above, such formulas could not all be proved in da Costa's original **C**-systems, under pain of making these logics lose their paraconsistent behavior. Why worry about this particular formula? In [43], da Costa and his collaborators allege that "there are mathematical reasons, related to the construction of the systems, to demarcate between well-behaved and non-well-behaved propositions, and that is why the constraint on rejecting $\neg(A\&\neg A)$ as a logical truth was advanced". They continue rationalizing by pointing to the requisites of da Costa's 1974 paper, mentioned above, and by defending this as a decision with no philosophical motivations: "As opposed to any particular philosophical concerns, the main consideration underlying such a proposal consisted in presenting a logical framework in which the presence of contradictions does not lead to trivialisation, meeting thus, initially at least, a mathematical (not a philosophical) demand". Now, that comment certainly sounds intriguing, as it gives the impression

[3] The theme is still popular. A collection of papers on the 'Law of Non-Contradiction' is indeed about to be published (cf. [71]), and there you can find a few papers that discuss the many possible versions of this law.

that any such a paraconsistent 'logical framework' would have obligated one to follow that proposal of rejecting the fetish formula. But there are many paraconsistent logics for which $\neg(A\&\neg A)$ is a theorem. One of these is a 3-valued maximal paraconsistent logic studied by da Costa himself, together with D'Ottaviano, a **C**-system[4] that they dubbed $\mathbf{J}_3$ (cf. [50]).[5]

The confusion involving the fetish formula and paraconsistent logic is not systematic. Béziau has denounced, for instance, in [16] and in [19] the misidentification of the acceptance of the Principle of Non-Contradiction with the validity of the fetish formula. However, while this same author has identified, as we have seen just above, the failure of the former principle with the very definition of a paraconsistent logic, he has also very often criticized the fact that the fetish formula is a theorem of some paraconsistent logics. In [13], for instance, Béziau criticized Igor Urbas for having proposed in [85] a dual-intuitionistic logic that validated the fetish formula while solving the problem of replacement (see below): "Even if this logic is algebraizable, it has also some 'abnormalities' which are even worse, from the point of view of paraconsistency at least, the fact for example that $\neg(A\&\neg A)$ is a theorem". Calling *full* any paraconsistent logic in which the schema $\neg(A\&\neg A)$ is provable, Béziau asserts in [19] that "it is not clear at all that the idea of a full paraconsistent logic is meaningful", and he adds that the question is still open whether we can find an 'intuitive interpretation' of a paraconsistent negation with respect to which the fetish formula can be proved. All that sounds however more like vague complaints. The only technical reason ever presented by Béziau for mistrusting full paraconsistent logics, in fact, seems to be the one from the paper [14]: That the negations of full paraconsistent logics cannot at the same time validate introduction and elimination of double negation and also satisfy replacement (for a generalization of this result, check Theorem 3.51(vii) from **Chapter 1.0**). The argument only proves, of course, that these classical properties are jointly incompatible inside a paraconsistent logic: Why should one insist on having all of them, though, knowing that paraconsistent logic is bound to throw some classical properties away?

In contrast to the 'Brazilian' inconsistent and inconstant reaction to the fetish formula, one could note for instance that Jaśkowski had already called this formula 'law of contradiction', pointed out that it is a theorem of his discussive logic, and observed that, in spite of the denomination that he adopted for this formula, it "has no close relation to the problem of the logic of inconsistent systems" (cf. [55]). The present monograph will return to this theme every now and then. Check in particular the brief survey of the various reactions to the fetish formula done in subsection 3.8 of the paper TAXONOMY, in **Chapter 1.0**.

[4] At least a **C**-system, *nota bene*, according to my present definition of the term —check sections 2.4 and 2.6 of **Chapter 1.0**.

[5] It should be noticed that those authors have never proved the maximality of this logic, arguably one of its most interesting features. Such a proof can be found in [61, 29].

The replacement property. A logic is said to enjoy the replacement property whenever it allows for equivalent formulas to be freely intersubstituted everywhere. In 1965, da Costa and Guillaume pointed out the failure of that property (and, in particular, the consequent failure of the global form of the rule of contraposition) for the paraconsistent logics from the original C_n hierarchy (cf. [48]). Replacement is an important property, as it constitutes a prerequisite for usual modal interpretations to be available, and it affects the implementation of usual algebraization procedures (in the definition of congruence relations). For quite some time, people were worried about that failure being some sort of structural problem of paraconsistent logics. It is not. Even if many authors have still failed to take notice, examples of paraconsistent logics satisfying the replacement property are known since many years —relevance logics are among the illustrations of importance, as well as dual-intuitionistic logics.

Many of the most well-known **C**-systems fail replacement. But less well known is the fact that Jaśkowski's logic **D2** (another **C**-system, according to my present definitions) also fails replacement, as it is proven, apparently for the first time, in **Chapter 3.2**. Such failure of the replacement property has originated many misdirected criticisms. For instance, according to Priest and Routley, in [77], the failure of contraposition in da Costa's logics C_n, $n > 0$, makes them somewhat problematical, as it "results in the general failure of the principle of the substitutivity of provable equivalents". As an argument, this constitutes already an abuse. As discussed in sections 3.3, 3.5 and 3.7 of **Chapter 1.0**, and illustrated all along **Chapter 3**, contraposition is much more than one needs in order to make a logic respect this substitutivity principle. But Priest and Routley continue, by saying that "this in turn implies that we cannot produce a Lindenbaum algebra for the **C**-systems in the normal way", and they proceed to mention Mortensen's well-known result on the logics C_n (cf. [64]). Here again, as the reader will see in **Chapter 1.0**, the underlying definition of '**C**-systems' that is taken for granted is just too restricted to be of any interest. Besides, it can be shown that there are many other **C**-systems with similar properties that are, nonetheless, perfectly amenable to several varieties of algebraization procedures —some of them quite 'normal'. At last, the authors finish their criticism by asserting that "the fact that there is no Lindenbaum algebra might not seem to be a substantial philosophical (as opposed to technical) problem but in fact it is. For it implies that there are no recursive semantics of a suitable kind. [...] There are well-known arguments for the fact that philosophically adequate semantics must be recursive". There is a lot to be rectified about such final arguments. As the reader will see in the **Chapter 2.1**, all of our **C**-systems possess adequate non-truth-functional bivalent semantics. There is nothing inherently 'non-recursive' about such semantics. Moreover, the competing possible-translations semantics offered in **Chapter 2.2** are entirely recursive, and in fact decidable, and so are the many-valued semantics from **Chapter 1** or the modal semantics from

Chapter 3. At any rate, the insinuated connection between 'Lindenbaum algebras' and 'recursive semantics' remains at best unclear.

On the issue of replacement, da Costa and Bueno write, in [46], that "it is usual to criticise certain paraconsistent propositional logics for not having relations of congruence involving all the connectives. [...] Instead of these logics, some specialists propose distinct ones, which present natural relations of congruence, but which satisfy the law of non-contradiction $\neg(A\&\neg A)$, a law that, *of course*, does not hold in the former ones" [my italics]. The authors seem to suggest a strange dichotomy here: Either the logic satisfies replacement or it validates the fetish formula $\neg(A\&\neg A)$. Now, while **C**-systems like C_1 fail both replacement and the fetish formula, other **C**-systems like $\mathbf{J}_3$ fail the first but validate the latter, and the paraconsistent version of the modal logic K (check **Chapter 3.3**, where the paraconsistent negation is set as a 'non-necessity' operator) satisfies the first while failing the latter. Finally, the paraconsistent versions of modal logics extending KT conform to both replacement and the validity of the fetish formula. So, why 'of course'?

All that said and done, how can the failure of the replacement property be justified, technically or philosophically? Given the multiplicity of paraconsistent logics that are available nowadays, da Costa and Bueno comment, in [46], on what concerns the justification of the success or of the failure of properties such as replacement or theorems such as $\neg(A\&\neg A)$, that 'pragmatic arguments and concrete motives' should be employed, if we are working 'in the domain of applied logic'. That seems a sensible advice. Unfortunately, however, not a single such argument is presented in their paper so as to illustrate how such a choice can be done, in practice.

One last remark. In the case of **dC**-systems (a special kind of **C**-systems, read about this below) satisfying the replacement property, given a classical negation $\sim$ one can often prove that $\sim(A\&\neg A)$ fails to hold (because it will denote the consistency, or 'good behavior', of the formula A). Moreover, the same can be said about $\sim(\neg A\&A)$, or $\sim(A\&(A\&\neg A))$, or any other expression in which $\sim$ applies to a formula equivalent to the inconsistency $(A\&\neg A)$. These formulas will all be equivalent —in fact, congruent. If we are talking about logics that do *not* satisfy the replacement property, however, such variants of the fetish formula can easily fail to be equivalent, to start with. As I point out in **Chapter 1.0**, in a logic like C_1, the formulas $\neg(A\&\neg A)$ and $\sim(A\&\neg A)$ are equivalent, and so are $(A\&\neg A)$, $(\neg A\&A)$ and $(A\&(A\&\neg A))$, but the fetish is monomaniac: While $\neg(A\&\neg A)$ indicates good behavior, seemingly harmless variants such as $\neg(\neg A\&A)$ or $\neg(A\&(A\&\neg A))$ do *not*. In that case, the mentioned logic reveals itself to be, in a certain sense, strongly asymmetric and too dependent on accidental syntactical formulations.

An extended account of the relations of paraconsistency with replacement can be found in **Chapter 1.0** (where replacement is dubbed Intersubstitutivity of Provable Equivalents) and in the introduction to **Chapter 3**.

Good behavior and formal consistency. The path of paraconsistency has never been an easy one to tread. But that was less because of any intrinsic difficulties posed by this variety of non-classical logics than because of, as Florencio Asenjo puts it, "the sterile prejudice of centuries against all contradictions" (cf. [6]). On the one hand, from a purely logical point of view, given the perfect duality between paraconsistent and paracomplete logics, it is hard to imagine an argument for the general rejection of paraconsistency that would not reject, say, intuitionism, for dual reasons; on the other hand, a full acceptance of paraconsistency does seem to require from us the development of a new 'theory of opposition' (or so I argue in **Chapters 3** and **4**). I will not try here to survey the extra-logical arguments that have been presented against paraconsistency. Disgracefully, both among those that work with paraconsistent logic and those that have no idea of what it is about (but still want to write about it) there is still a lot of confusion as to what it accomplishes. For one, Jerzy Perzanowski has adverted to that danger, in writing: "Notice first that the popular name 'paraconsistent logic' is, in a sense, misleading. It suggests that such logics are consistent in a special, weak sense. But, as we know, it is just exactly the reverse. They are simply inconsistent, but unlike the classical logic they are able to work with inconsistencies" (cf. [69]). I will not enter terminological discussions here. I will subscribe though to that same (still shocking?!) intuition about consistency: Consistent logics with a negation symbol are both explosive and non-trivial; paraconsistent logics are inconsistent yet also non-trivial.

According to Batens, "Aristotelian consistency tradition seems to reduce to sheer prejudice" (cf. [10]). Indeed, reasoning mechanisms such as those of classical or of intuitionistic logic unquestionably presuppose consistency as a sort of 'methodological requirement'. I shall here suggest nothing about the 'consistency of the world' or about our possibility of knowing it (or our passionate reaction about its possible failure). As we have seen, da Costa himself believes that it is not up to logic to decide whether the world is consistent or not. But he does say that (check [39], chap.III.3), on what concerns true contradictions, "the central problem consists in knowing [...] if the real world is consistent or not (it seems obvious that it is non-trivial)" [my translation]. (Notice how the 'obvious' part sounds quite 'speculative'.)

Moving to more practical matters, I should say that, after working a few years in the area, I find it ever more difficult to find any interest in da Costa's original hierarchy of logics C_n, $n > 0$, apart from its historical role in the development of paraconsistent logic in Brazil. It seems truly hard to point any technical or philosophical reason that would put these logics in advantage with respect to other competitors, with nicer features. I do believe though that these logical systems were based on remarkable intuitions that can and should be generalized. That belief gave rise to the definition of *Logics of Formal Inconsistency*, explored in the present monograph — logics whose most remarkable feature is the ability to recapture consistent reasoning by the addition of appropriate consistency assumptions.

Suppose you subscribe to the Whiteheadian motto 'one god, one country, one logic'. In that case, if new theories and knowledge happen to supersede old ones by refining rather than by refuting them, you might still want the latter to be preserved in situations in which they were known to work well. Now, suppose instead that you are a pluralist and believe in a 'multiplicity of rationalities', each with its own domain of application. In that case you will not want to commit yourself to the new theory any more than you did commit to the old one. Here again you might be happy that the former theory be capable to reproduce the safe conclusions of the latter, and only contribute to it by suitable localized updates. Non-classical logicians of all breeds will have their reasons either to desire or to abhor the possibility of recapturing classical logic inside their deviant (devious?) logics of choice. Andrew Aberdein classifies in [1] the different attitudes logicians might take with regard to the recapture of classical reasoning as produced by a new non-classical logic **L**: (1) claim that no suitable recapture constraint is expressible in **L**; (2) insist that such recapture of classical logic by **L** is irrelevant; (3) desire to maintain classical logic as a limiting case of its successor **L**; (4) characterize classical logic as a proper fragment of **L**. One would presume, of course, that as soon as a definition of 'recapture' is presented, there should be an objective procedure (if decidable at all) to check whether the logic **L** does or does not recapture classical logic.

Well, how can a theory be recaptured by another, as a 'limiting case'? What would it mean, for instance, to say that hydrogen can be recovered from water, or blue recovered from the sunbeams? That 'hydrogen' and 'blue' are somehow present in 'water' and 'sunbeams', and can be rescued through appropriate mechanisms and devices. Less poetically, what would it mean to say that euclidean geometry is contained in non-euclidean geometry, or that newtonian physics is contained in relativistic physics? That euclidean geometry is obtained from non-euclidean geometry by guaranteeing a certain notion of geometric similarity through variations in scale of the shapes, and newtonian physics is obtained from relativistic physics when the speed of the objects in question is sufficiently low so as to have neglectable effects. Now, how could classical logic be recaptured by a paraconsistent logic?

In [72] and [73], Graham Priest studies a 3-valued paraconsistent logic called LP that is characterizable by adding excluded middle to a De Morgan negation (in a language involving $\wedge$, $\vee$ and $\neg$). As a result, the logic turns out to have no definable implication connective respecting the rule of *modus ponens*. Priest insists though in introducing a sort of *quasi implication* (defined, as in classical logic, by setting $A \supset B \stackrel{\text{def}}{=} \neg A \vee B$) that makes his logic identical to the one that had been proposed by Asenjo, in [5]. Priest argues that LP has, as truth-values, the values 1 and 0 that it shares with classical logic, plus the 'paradoxical' value $\frac{1}{2}$. But, if one restricts the semantics of LP to the classical two-valued (consistent) codomain, he continues, *modus ponens* is respected by the above mentioned quasi implication. Rules and inferences that are validated only in case such a kind of restriction is made are dubbed, by Priest, *quasi valid.* Returning to the issue of his 3-valued quasi implication, Priest asks himself: "How can one reason [in natural language or in mathematics] without *modus ponens*?" (cf. [72], sec. IV.2). The author then exhibits a curious 'methodological maxim' he keeps up his sleeve: "Unless we have specific grounds for believing that paradoxical sentences [i.e. those receiving the intermediary value $\frac{1}{2}$ under some particular valuation] are occurring in our argument, we can allow ourselves to use both valid and quasi-valid inferences" (id., sec. IV.9). How is this maxim actually internalized by the logical machinery of LP? The answer is that it is not. For that the reader would have to wait several years, until the paper [74], where an inconsistency-adaptive (nonmonotonic) version of LP is investigated.[6] Priest wants his logic to somehow recapture classical logic and recover classical reasoning simply by avoiding paradoxical sentences.[7] The logic is, however, just not expressive enough to that end: It is impossible to *say* in LP that a certain sentence is provably 'non-paradoxical'.[8]

Since his first adventures in paraconsistent territory, da Costa has always strived to devise logical mechanisms that would not be contrary to classical logic, but would extend it in some sense. There are various ways of realizing that strategy, and some of them are illustrated in this monograph. Da Costa wants classical logic to be the logic of the 'well-behaved' sentences from the paraconsistent logics that he proposes. I expanded on that idea and made 'good behavior' a primitive 'consistency' connective of what I call **C**-*systems* (a particular case of **LFI**s). The logics in which such new connective can in fact be introduced through a definition in terms of more usual connectives are called **dC**-*systems.* The fundamental feature of all such systems inspired by da Costa's approach consist exactly in the possibility they open for us to recapture consistent reasoning for instance by the addition of appropriate consistency assumptions. So, while classical rules such as disjunctive syllogism (from A and $\neg A \vee B$, infer B) are bound to fail in a paraconsistent logic (because A and $\neg A$ could both be true for some A, independently of B), they can be recovered by an **LFI** if the set of premises is enlarged by the presumption that we are reasoning in a consistent environment (in this case, by the addition of 'consistent-A' as an extra hypothesis of the rule). A more detailed, if relatively informal, explanation of that mechanism can be found in section 1 of **Chapter 3.2** and section 2 of **Chapter 3.3**.

[6] For inconsistency-adaptive logics in general, check [11]; for a sharp criticism of Priest's adaptive strategy, check [9].

[7] Any valid classical inference that is failed by LP is quasi valid. So, if you erase the 'paradoxical' truth-value, you obtain just the classical matrices. This fact is not nearly as informative as it might seem at first look. Call *hyper-classical* any many-valued matrix with that property of defining only classical matrices when we restrict its semantics to the classical codomain. It is easy to see then, using the general abstract results from **Chapter 2.1**, that, given any logic **L** that shares the structural properties of classical logic, **L** is a deductive fragment of classical logic if and only if **L** is characterizable by some set of hyper-classical matrices.

[8] Check the criticism of LP by Batens in section 1.4 of **Chapter 1.0**, and in notes 24 and 25 of the same paper, in section 3.10.

Other aspects of paraconsistency

> Do I contradict myself?
> Very well then I contradict myself,
> (I am large, I contain multitudes.)
> —Walt Whitman, *Leaves of Grass*, 51, 1855.

In 1982, among the 'positive effects' of paraconsistency on the 'philosophical field', da Costa lists (cf. [37]):

1. Better elucidation of some basic concepts of logic, such as, for instance, those of negation, of contradiction, and of the role of the scheme of abstraction in set theory.
2. Deeper understanding of certain theories, specially dialectics and Meinong's theory of objects.
3. Proof of the possibility of strongly inconsistent but non-trivial theories; as a corollary, the common paradoxes are now coming to be seen from very new perspectives.
4. Elaboration of ontological schemes distinct from those of traditional ontology.

Among the 'negative effects', he lists:

1. Proof that some criticisms, formulated against dialectics, are unsound (for example, some critical remarks of Popper).
2. Proof that standard methodological requirements imposed on scientific theories are too stringent and could be liberalized.
3. Evidence that the usual conception of truth, à la Tarski, does not imply that the laws of classical logic (even of the first-order predicate calculus) must be valid.

In view of such a bold list of accomplishments or purposes, it would have been a temerity to think that I would be able to contribute to all fronts. My task in this monograph is much more modest, though. As I explained, the intention is to investigate a certain idea related to the possibility of internalization of the metatheoretical notion of consistency at the object-language level of our logics. It can be seen thus as an exploration of the foundations of paraconsistency made from the point of view of the universal logician, that is, from the point of view of General Abstract Logic.

There are some fundamental intuitions by da Costa and some important aspects of the daCostian systems, however, that were not touched by my present approach, and there are yet some other relevant aspects that I did work on, but that are not reflected in the present selection of papers. I will briefly mention some of these aspects in this section. Each chapter that follows these **Prolegomena** finishes by a **Brief history** where I report on the development of the thereby contained ideas and on some of the venues (congresses, seminars etc) in which I had the opportunity of presenting these ideas and receiving immediate lively feedback on them. This section will also complement that information where it is lacking.

On set theory. I risk being seriously unfair if I talk about da Costa's approach to paraconsistent logic and do not even mention the topic of 'paraconsistent set theory'. Since their very inception, daCostian paraconsistent logics relied heavily on the intellectual provocation brought about by the paradoxes of set theory. As da Costa sees it, this is "one of the most compelling motivations for the construction of paraconsistent logic: the possibilities it opens up in the foundations of set theory" (cf. [40]).

Russell's antinomy is formulated in set-theoretical terms with the help of an explosive negation plus the unrestricted postulate of separation, also known as comprehension scheme. To stay clear from its trivializing conclusions it would seem natural either to make negation non-explosive or to impose restrictions on separation. The classical solutions have all, in one way or another, elected the second alternative —an infinite number of weaker forms of separation is indeed possible and many have been tried. But the first alternative could in principle be naively realized by a paraconsistent logic.

Several paraconsistent set theories have been proposed along the years, many of them based on **C**-systems, notably by da Costa and collaborators (cf. [44]). One of their most reassuring properties is equiconsistency (or, more precisely, 'equi-non-triviality') with classical set theories (cf. [38]). Da Costa attaches a 'paramount importance' to paraconsistent set theories, even for the very definition of logic. In [45] he writes with Bueno that "given that logic is basically concerned with the study and systematization of certain conceptual structures, and that in order to formulate them we need, for instance, set theory, it seems reasonable to demand that a logic, to be taken as such, be developed at least up to this point". Da Costa believes that one of the most important features to be satisfied by a paraconsistent set theory is the provision of a way of recovering classical set theories and reconstructing traditional mathematics, while at the same time settling the foundations for a 'paraconsistent mathematics'.

I should open here a small parenthesis on a puzzling 'motivation' presented by da Costa for the 'devising of paraconsistent logic', namely, "the interplay between semantics and set-theoretic issues". As a matter of fact, the authors of [43] say that the use of classical set theory in the formulation of the semantics for paraconsistent systems is 'philosophically untenable', and they maintain that "there is, in a certain sense, no semantics for a paraconsistent logic without a paraconsistent set theory". Similarly, in [45] the authors write that "one should note that, at least on philosophical grounds, it is needed to have a paraconsistent set theory already articulated if one intends to have a reasonable semantics for paraconsistent logic (given that semantics shall be constructed within set theory)". In both papers they use those remarks in fact to justify why da Costa, "when first presented his paraconsistent systems, not having developed yet a paraconsistent set theory, formulated them in a syntactic, not in a semantic, way" (cf. [43]). The line of argument seems really baffling. So, the authors wish to defend a 'pluralist' outlook on logic, and want at the same time each logic to be dealt with exclusively by a set theory based on this very same logic? Is it really untenable to provide a semantics for a logic such as, say, da Costa's C_3 based on anything else than a set theory built over C_3 itself? What is non-classical about the set theory underlying the theory of valuations that da Costa has proposed for his logics C_n, $1 \leq n \leq \omega$? Or about the many-valued or modal semantics that are adequate for yet some other **C**-systems?

I do not share with da Costa the belief that logic starts at set theory, nor do I see any necessary connection between the foundational problems of paraconsistent logic and the foundational problems of set theory. It is true that paradoxes such as the one raised by Russell's antinomy can serve as a good motivation for the proposal of paraconsistent systems. But, as it is well-known, there are other set-theoretical paradoxes such as Curry–Shaw-Kwei's that can be derived from the unrestricted postulate of separation even when negation is not available, as long as the logic has an implication satisfying both contraction and *modus ponens* (cf. [80]). It is not enough to go paraconsistent so as to avoid such paradoxes. I am unaware of any outstanding progresses recently achieved by the 'Brazilian school' in the investigation of set theories based on the full postulate of separation and on paraconsistent logics weak enough so as to avoid such kinds of trivialization strategies. I myself do not have much to contribute here at present. For all those reasons, the theme 'paraconsistent set theory' is absent from this monograph.

On an infinity of logics. Da Costa started his work on paraconsistent logic already by proposing not one or two such logics, but a denumerable number of them. The present monograph shows only how this number can be multiplied. Is there anything to be learned from such a multiplicity of logical options?

As we have seen above, da Costa defends that each logic has its 'domain of application' and that the existence of various systems of logic only helps in showing that "the rational and logical activities do not coincide, even though any logical activity is, *ipso facto*, rational" (cf. [39]). In [46] he intends to defend a sort of fallibilist interpretation of paraconsistent logic that would seem to lie, as argued in [75], somewhere in between realism and instrumentalism. Da Costa and Bueno say that "given the proliferation of heterodox logical theories, especially the existence of infinite paraconsistent logics containing a considerable part of traditional logic, the defence of an extreme realist view becomes a difficult task". For these authors, "realist conceptions *à la* Frege and Gödel, according to which logic supplies the most general features of the universe, only seem to be defensible on largely speculative grounds". So much here for the philosophy of paraconsistency.

I assume in the present monograph that logic is about 'what-follows-from-what'. It should be noted that da Costa classifies that approach as 'applied logic' —for him, 'pure logic' is about model theory, formal languages, recursion theory, and the like (cf. [45, 46]). But that classification makes him adopt a very specific stance with regards to the choice of logical systems to apply to each specific situation. He asserts that, if one takes into consideration the plethora of non-classical logics that are available, one must conclude that there is 'considerable underdetermination' on what concerns that choice: 'Empirical constraints' and 'pragmatic considerations' should be taken into account "in the determination of the acceptable solutions to the problems under examination". That seems a sensible advice, but it is not clear to me how tractable the choice problem is. Which would be the heuristic and pragmatic commitments to be measured so as to help us in deciding whether a logic like C_3, say, is preferable over C_1, or over any other paraconsistent logic? I can only hope to count on the technical features of each system to help me decide on that.

On maximality. One way of attending requisite (iv) of da Costa's 1974 paper (recall the last section), the one that required that paraconsistent logics should preserve as much of classical logic as possible, is by the systematic search for maximal paraconsistent fragments of classical logic. Already in 1980, Batens denounced the fact that the notion of a maximal paraconsistent logic had not been given enough attention (cf. [8]). None of the logics C_n, $1 \leq n \leq \omega$, is even close to being maximal (though Sette's logic $\mathbf{P}^1$, as studied in [79], is known since long to constitute a maximal extension of the logics from the former hierarchy).

I studied the problem of defining **C**-systems that would constitute maximal paraconsistent fragments of classical logic a few years ago, and presented some preliminary results about that in my contribution to the II World Congress on Paraconsistency, as reported in the paper [62]. These ideas were soon extended into the research note [59], whose main results are reported in section 3.11 of **Chapter 1.0**. A different approach to da Costa's requisite (iv) is taken by inconsistency-adaptive logics (cf. [11]), in which maximality is pursued through nonmonotonic strategies, presupposing consistency by default.

On predicate logic. As we have seen above, and confirmed with requisite (iii) of da Costa's 1974 paper, arguments have been proposed to the effect that (paraconsistent) logic should be (at least) first-order. I have made it clear though that I see no need of requiring that much from the objects I call logics. Now, that surely does not mean that I ignore or refuse to study predicative versions of the Logics of Formal Inconsistency (check what I say about this at section 4 of **Chapter 1.0**). As it happens, given my objectives and the sort of problems I had to attack, not much is said or done about paraconsistent logics in the present monograph that goes beyond the propositional level. In other papers, however, the topic was important: A first-order version of the **C**-system $\mathbf{J}_3$ (under the name of **LFI1**) is used in the papers [29, 49], the latter having been presented by Sandra de Amo at the II International Symposium on Foundations of Information and Knowledge Systems (FoIKS'2002); a process that would allow first-order paraconsistent logics to be obtained by the appropriate combination of propositional paraconsistent logics and classical first-order logic was reported in [24], a paper presented by Carlos Caleiro at the IC-AI'2001, the 2001 International Conference on Artificial Intelligence —in that paper we show in fact how the method could be applied so as to 'first-ordify' the logic C_1.

On databases and mechanized deduction for LFIs. How 'strategic' is (paraconsistent) logic? It is not always easy to do basic science. If you want to get that grant you always dreamt of, and make your research topic count in the class of 'priority topics' of your financing agency, you had better show how useful and applicable it can be. You can also play a bit with that. Physics had its century. Biology is the science of the hour. Aristotle would be happy with that. I have already written a paper on 'taxonomies' of logics. What should the next one be on? Genetic sequencing of logics? Clones of logics? (Note, by the way, that clone theory is already a topic from algebra, and genetic algorithms are old fellows of neural networks.)

Long ago, in [83], the logician Alfred Tarski made a provocative comment on that issue that I cannot resist to quote here:

> I believe, nevertheless, that it is inimical to the progress of science to measure the importance of any research exclusively or chiefly in terms of its usefulness and applicability. We know from the history of science that many important results and discoveries have had to wait centuries before they were applied in any field. And, in my opinion, there are also other important factors which cannot be disregarded in determining the value of a scientific work. It seems to me that there is a special domain of very profound and strong human needs related to scientific research, which are similar in many ways to aesthetic and perhaps religious needs. And it also seems to me that the satisfaction of these needs should be considered an important task of research. Hence, I believe, the question of the value of any research cannot be adequately answered without taking into account the intellectual satisfaction which the results of that research bring to those who understand it and care for it. It may be unpopular and out-of-date to say — but I do not think that a scientific result which gives a better understanding of the world and makes it more harmonious in our eyes should be held in lower esteem than, say, an invention which reduces the cost of paving roads, or improves household plumbing.

On that matter, there is also the story by Michael Faraday, who, after a public demonstration of an electrical experiment, was asked what was the use of electricity. He retorted: "What use, madam, is a new-born baby?"

There might still be some resistance to be found in the philosophical community, but many people in computer science nowadays believe that paraconsistent logic is already running on cables. This monograph probably mentions not a single application of paraconsistent logic to real-life problems. And it will probably not make me rich. I have, nonetheless, worked elsewhere on the application of Logics of Formal Inconsistency, **LFI**s, to problems of computer science. Some initial investigation of ours on the mechanization of deduction for **LFI**s by way of tableaux is reported in [28], a paper presented by Walter Carnielli at the IC-AI'2001. Much more about that, specially on what concerns the many-valued case, can be found in [21], a paper presented by Marcelo Coniglio at the III World Congress on Paraconsistency, and the related papers [23] and [22], inspired on my early draft [60]. In [29] and [49] I have explored the application of certain **LFI**s to the study of 'evolutionary databases', databases that are endowed with inconsistent-tolerant logical mechanisms and that can evolve with time, allowing for some inconsistency to appear also among their integrity constraints. In 2003 I was invited to and participated on the Dagstuhl Seminar 03241 on 'Inconsistency Tolerance', at the Dagstuhl Castle (DE). There I talked about the mechanization and the use of dyadic semantics in providing automated decision procedures for logics that allow for reasoning under uncertainty.[9]

Advertising LFIs. This is not a job just like any other. I had the mission of convincing other people that **LFI**s could do them good —if only to test whether I was really right on that belief. I took the gospel to several places in the last few years, and specially after writing **Chapter 1.0**. Under invitation, I gave some quite general talks on the idea of **LFI**s and the internalization of consistency, at varying degrees of informality and detail, at the Theory of Computation Seminar of the Center for Logic and Computation of the IST, in Lisbon, in January 2002, at the Séminaire Interuniversitaire de Logique Mathématique, at the Université Libre de Bruxelles, in February 2002, at the School of Informatics of the City University of London, in March 2002, at the Institute for Logic, Language and Computation of the University of Amsterdam, in March 2002, and, in Poland, in September 2002, at the Department of Logic and Methodology of Sciences of the University Marie Curie-Skłodowska, in Lublin, and at the Department of Logic of the Institute of Philosophy of the Nicholas Copernicus University, in Toruń. Interestingly, I invariably learned from my audiences much more than what I was able to teach to them.

On the duality between inconsistency and undeterminedness. One of the basic semantic intuitions behind paraconsistency relates to its purported duality with intuitionism —or, more generally, with 'paracompleteness' (cf. [58, 12]). A renitent difficulty concerning the abstract characterization of that duality has always been the persisting use, by a good part of the 'Brazilian school', of old-fashioned syntactical mechanisms that privilege truth over falsehood. I like to compare this asymmetrical situation to all the fiction that surrounded the 'dark side of the Moon', whose first notorious visitor was Jules Verne in his 'Autour de la Lune', from 1869. From where we stand, on Earth, how could we even *know* that the Moon has another side, besides the one that is permanently visible from our planet? I am not

[9] Check `ftp://ftp.dagstuhl.de/pub/Proceedings/03/03241/03241.MarcosJoao.Slides.pdf`.

suggesting that the Moon could have been some sort of Klein Bottle; but, from all we know, the Moon could have turned out to be, for instance, a half-sphere, or have some other strange 'lunoid' shape. The first photographs we ever got from the other side of the Moon were taken by the spacecraft Luna 3, in 1959. Did it have any trouble taking the pictures in the dark? None at all! Most people do not even think about it, but, while it is true that half of the Moon is always in shadow (for the Moon has its phases), the so-called 'dark side of the Moon' gets just as much sunlight on it as the side we do see from the Earth. Even though we are not looking at it, there may well be a lot of interesting things to see at the far side of the Moon. Why should one ignore one half of the Moon? Why should there be in logic a bias towards truth, while falsehood is ignored?

The adoption of a multiple-premise-multiple-conclusion framework, as in **Chapter 2.1** of the present monograph, allows for a very natural fix of the above situation, in restoring symmetry and clearly displaying the duality between paraconsistency and paracompleteness, as it is done in **Chapters 4.1** and **3.3**. Before I could arrive to that conclusion, I had already done some semantic investigations on duality, reported for instance at a talk I presented in October 2000 at the Institut für Logik, Komplexität und Deduktionssysteme of the University of Karlsruhe, in the scope of a ProBrAl project involving Brazil and Germany, at my contribution to the Joint Austro-Italian Workshop on Fuzzy Logics and Applications, held at the Università degli Studi di Milano, in December 2000, at yet another talk I presented in February 2001 at the Theory of Computation Seminar of the CLC / IST, in Lisbon, and at my contribution to the IV Flemish-Polish Workshop on the Ontological Foundations of Paraconsistency, held at the University of Ghent in December 2001. The work documented in the present monograph smoothly sprang from those early investigations, as soon as I managed to fix the right theoretical framework.

Paraconsistent mistakes. That paraconsistent logic admits of some inconsistencies should not mean that you can be as incoherent as you want if you work in this area. That you tolerate inconsistencies should not mean that you eagerly expect for them to be found. One of the most inopportune obstacles that paraconsistency has faced (and still faces) on its way to becoming more popular and well-accepted seems to be the attraction it exerts on practitioners of pseudo-science and other varieties of fashionable nonsense. I will not advertise their work here. In the long run, historians and sociologists of science might help me give support to the impression that this area of logic, more than others, has always been prone to such rubbish. I am more interested here instead in studies that have been produced by reasonably informed researchers but that, nevertheless, got impaired by deadly, and obviously unintentional, mistakes. Several such studies are mentioned in the present monograph, and the reader will see that I have put a lot of effort in fixing the spotted flaws whenever I was able to see the way out.

In August 2003 I organized a round-table called 'Contradictory and not: On the Philosophy of Inconsistency', at the XXI World Congress of Philosophy (XXI WCP), held in Istanbul, and I presented there a talk entitled 'The millionaire contribution of all mistakes' (making use of the nice expression by Oswald de Andrade, one of the heads of the Brazilian Modernist Movement). The talk was based on the exposure of some flaws committed by paraconsistentists-to-be, and on what we can learn from them. I will have certainly committed my own mistakes, here and elsewhere, and I only hope to risk committing even more (for that will indicate that I am, or at least I am trying to be, engaged in a productive scientific life). As a responsible scientist, though, I should try to minimize such errors, and I should be happy to have them pointed out and corrected, when that is the case.

The Future. After half a century, how mature and successful is the paraconsistent enterprise, in our days? On that respect, here is an event that da Costa likes to mention, and with a good reason: "In 1991, fifteen years after its baptism and twenty-eight years after its birth,[10] paraconsistent logic was accepted in the category of theories admitted by the mathematicians: a special section is created for it in *Mathematical Reviews*" (cf. [43]). Indeed, the Mathematical Subject Classification has created for 'Paraconsistent Logic', in 1991, the field 03B53 (where '03' stands for 'Mathematical Logic and Foundations' and 'B' for 'General Logic'). However, in 2000, the above classification changed its description, from 'Paraconsistent Logic' to 'Logics admitting inconsistency (paraconsistent logics, discussive logics, etc.)'. Now, what is there in 'discussive logics' that make them plural and diverse from 'paraconsistent logics'? And what sort of objective description of a class of objects includes an 'etc.' in it? That late change in description only seems to me to attest to the lack of coordination and deep understanding still to be found in the field of paraconsistency. Not maturity.

Another story. In September 1999 I have started a discussion group on paraconsistency,[11] that now counts about 80 members. It does not bear good testimony to this area of research, though, that the members of that group have been unable to or uninterested of pursuing any rich threads of discussion ever since, as one might have expected of researchers in a more consolidated area. Well, let's keep on waiting…

To the moment, our present approach to the Logics of Formal Inconsistency seems to have been reasonably successful in its objectives. It managed to instill a new breath of life into the Brazilian school of paraconsistency, and it has collected a few adherents worldwide (see the next section). No lesser sign of maturity of this particular topic, in my opinion, is given by the publication of a paper called 'Logics of Formal Inconsistency' as a chapter of the second edition of the Handbook of Philosophical Logic (cf. [26]). More is hopefully to come.

[10] The reader will recall from earlier sections that da Costa likes to date the birth of paraconsistent logic from his 1963's habilitation thesis.

[11] Check `http://groups.yahoo.com/group/paraconsistency/`.

Some contributions of the present thesis

> Usando do inglês como língua científica e geral, usaremos do português como língua literária e particular. Teremos, no império como na cultura, uma vida doméstica e uma vida pública. Para o que queremos aprender leremos inglês; para o que queremos sentir, português. Para o que queremos ensinar, falaremos inglês, português para o que queremos dizer.
> —Fernando Pessoa, 'Babel — or the Future of Speech', excerpt from *As Cinco Línguas Imperiais*, 1930s.

If you feel discomfortable with criticism, a life of science is not a life for you. If you are willing to receive it, howsoever, and to learn with it, you should put your work in a visible position and open to debate. Moreover, to minimize problems related to linguistic incompetence (of those who cannot read you), you had better write in the language of the empire. In order to write the present thesis in English at the State University of Campinas, Brazil, I was forced to make it a collection of more or less independent papers. The format has an obvious advantage: No much need to rewrite things after the job is done. But the disadvantages are numerous: A lot of effort should be put in relating those papers to one another, if that be the case; there will certainly be redundancies (and a lot of repetition too); terminology and notation will often vary along the text; there will be a profusion of bibliographical references; the result of late investigations might oppose results previously obtained; one's own linguistic incompetence will be more striking when writing in a foreign language. However, the extra effort might be compensated by a better presentation and understanding of the whole, while the pidgin English, the redundancies and the fluctuation on terminology might end up being no big deal for the perspicacious reader, and other differences in presentation might allow them to better follow the progress of a study and track the evolution of its main ideas. The present text will surely suffer from the former many defects, and enjoy the latter few virtues. The main difficulty one might have with studying such a text, specially if we take into account its number of pages, is in evaluating its original contributions, or even just finding them in the middle of so much else. I will thus briefly offer, in this section, a selection of some of the main contributions of the present study, according to my own preferences.

1. *A formal study of logical principles.* If you have ever heard about paraconsistent logics, you have certainly heard about how these logics allegedly defeat at least one among the so-called 'Principles' or 'Laws' of *ex contradictione*, *ex falso*, *pseudo-scotus* and non-contradiction, and possibly you have also heard about how these logics do respect the principles of non-triviality and non-overcompleteness. The formal framework set in **Chapters 1.0** and **4.1** allows us to distinguish between *all* such principles, and also a number of other principles. *Ex contradictione* and *pseudo-scotus* are related to a certain principle of explosion that needs to be failed by paraconsistent logics. Several distinct varieties of explosion (supplementing explosion, partial explosion, controllable explosion and gentle explosion) are still compatible with paraconsistency. Deviating from the bulk of related literature, my present version of the Principle of Non-Contradiction is also compatible with paraconsistency, yet, as it could be expected, its failure in non-overcomplete logics does require a paraconsistent environment. The fetish formula of paraconsistentists, $\neg(A\&\neg A)$, is shown to have nothing to do with paraconsistency, in general.

2. *Definitions of paraconsistent logic.* Of course, these will depend on how you define 'logic', how you define 'negation', and how you define 'paraconsistent'. Instead of fixing once and for all such definitions, I propose here a novel *negative* approach to them (check **Chapter 4**, but also **Chapters 1** and **3.3**). A logic is just a very general structure having an arbitrary set of 'formulas' as its domain and a convenient 'consequence' relation defined over it, intended to indicate what can be inferred from what. The main property of a *decent* logic consists in failing overcompleteness. A negation is in general a unary symbol aimed at embodying some general notion of 'opposition', and among the main negative properties of a *decent* negation are the rules I call *verificatio* and *falsificatio*, that will guarantee that negation is not a 'positive operator' —intuitively, they will make sure that negation inverts some truth-values. Finally, a *decent* paraconsistent logic should disrespect both *pseudo-scotus* and *ex contradictione*, so that it will not only have an inconsistent model, but a non-dadaistic such model (check **Chapter 4.2**).

3. *Coherence conditions for connectives, and perfect connectives.* Logical constants can often have their meaning set by groups of abstract complementary rules, that show how these constants can be introduced and eliminated, at the right or the left hand side of the consequence symbol. In **Chapter 1.0** I show how the deletion of some of those rules suggests the addition of further negative rules, so as to keep coherence and avoid idle —or 'indecent'— examples of connectives. I show in **Chapter 3.3** how the rules that are lost by paraconsistent or paracomplete logics can often be recovered by the addition of certain subsidiary connectives —such as the connectives of consistency or inconsistency— that complete the partial meaning of negation, restoring its lost perfection.

4. *Definition of* **LFI***s,* **C***-systems, and* **dC***-systems.* The Logics of Formal Inconsistency are introduced in the first chapter and studied throughout the thesis. Their near-ubiquity in the realm of paraconsistency is repeatedly illustrated: Most interesting logics produced by the Brazilian school fit the definition, all non-degenerate normal modal logics can be recast as **dC**-systems, Jaśkowski's discussive logic **D2** and its close relatives also constitute **dC**-systems. A comprehensive survey of the related literature is provided, and the problems related to the algebraization of such logics and the possible validity or invalidity of the replacement rule are equally surveyed. The 'Brazilian plan' is completed in that I hint on how maximal logics obeying all of the initial requisites of da Costa on paraconsistency can be obtained. Examples of **C**-systems that are not **dC**-systems, of **LFI**s that are not **C**-systems, and of paraconsistent logics that are not **LFI**s are also presented. The *Fundamental Feature of* **LFI***s*, as reflected in the so-called 'Derivability Adjustment Theorems' or the translations that allow consistent reasoning to be recaptured inside the inconsistent environments of **LFI**s, is heavily emphasized.

5. *Duality.* A framework of abstract Gentzen-like multiple-premise-multiple-conclusion consequence relations is investigated in **Chapter 2.1**, **3.3** and **4**. That framework allows for a full symmetry to be established between premises and conclusions, and each inference can then be dualized just by reading it from right to left instead of from left to right, or vice-versa. In semantical terms, this corresponds to substituting truth for falsity and vice-versa in each given model. Paracomplete logics are then characterized as duals to paraconsistent logics, and the dual-**LFI**s constitute the so-called Logics of Formal Undeterminedness, **LFU**s.

6. *Definition of structures of possible-translations.* In **Chapter 2.1**, some very generous definitions of Possible-Translations Representation and of Possible-Translations Semantics are offered for the very first time, both in terms of single- and of multiple-conclusion logics. The theory of these structures is shown to extend the general theory of matrices and logical calculi. Several examples of possible-translations semantics as applied to some very weak **LFI**s that are not characterizable through finite matrices nor usual modal semantics are presented in **Chapter 2.2**.

7. *Modal* **LFI***s.* As already mentioned, paraconsistent logics are shown, in **Chapters 3.2** and **3.3** to have a significative intersection with modal logics. Jaśkowski's **D2**, however, is shown in **Chapter 3.2** not to constitute anything like a usual modal logic once it fails the replacement property. Many natural examples of **LFI**s satisfying the full replacement property are still presented, with the help of modal interpretations for paraconsistent negations and consistency connectives. A similar thing is done for **LFU**s.

8. *Logics of Essence and Accident.* Studied apart from the presence of a paraconsistent negation, the modal connectives of consistency and inconsistency are given the reading of connectives that qualify essential and accidental modes of truth. A poor modal language is set by adding such connectives to the classical language, and a minimal logic of essence and accident is adequately axiomatized in **Chapter 3.1**. Some initial results are presented about the definability of the usual modal language from the language of essence and accident as well as the characterizability of classes of frames with the help of the latter language.

9. *Many confusions and mistakes by other authors are pointed out right on the spot*, all along the thesis. Some of the flaws are repaired.

10. *Formal Philosophy.* The whole thesis is an illustration of how philosophical problems can be studied with the help of convenient logical tools, if only we can agree to fix a convenient formalization for the terms under discussion. Philosophy can also claim thus its laboratory and its measuring instruments.

Travelling Salesman

Navigare necesse est, vivere non est necesse.
—thus spake Pompey, according to Plutarch's *Life of Pompey*, 75 AD.

The development of the investigations hereby reported spanned several years of my life. Since I engaged in my PhD research, about 6 years ago, I lived for long periods in 4 different countries, and I received monthly stipends from 7 different academic institutions and foundations, having worked as a Research Fellow, a Research Assistant, and also as a Teaching Assistant. When I finished my Master's Thesis, I had not a single published paper, and I had only participated on a few congresses and workshops here and there. However, as a testimony of astonishing Brazilian scientific autism, I felt no pressure for a more active involvement in scientific life. Notwithstanding, after getting more acquainted with real science as it is done 'out there', I decided to keep away from parochial science the best I could, and to change my behavior and my history. In the course of the last few years, then, I took part directly on 23 different congresses, symposia, workshops and summer schools, in 14 different countries, where I presented 19 contributions. I can in fact add 2 more events and 1 more country to that list if I count the other 6 contributions presented by my co-authors. Besides, in the same period, I presented other 21 invited talks at 8 different countries, addressing audiences from 14 different institutions. Lots of advertise, indeed. And, best of all, I got paid for it.

Not everything you write gets published, and that is just how it should be. I did manage though to have 2 papers already published in international journals and 5 other papers were accepted and await publication in other journals in the near future, I published 8 other papers in books and proceedings of congresses, and 3 further papers of mine are scheduled to appear soon in other volumes. In 9 among the above studies, I had the opportunity to collaborate with 6 different co-authors. I currently have 3 papers submitted for publication in different journals, several preprints available on-line and a few other papers at varying degrees of completion. Not to count many other short abstracts published at the many congresses mentioned above. I was involved in various research projects. The above mentioned papers deal with several different themes. A good sample of those papers organized around a central theme on paraconsistency and Universal Logic builds up the present monograph.

Some further scientific work. I translated a book on modal logic and Gödel's theorems. I co-supervised the work of a student in Scientific Diffusion in mathematics and I co-organized 3 international events of different sizes. I officially refereed 17 papers and books submitted to international journals, publishing houses and journals. Having been no more than a neophite at the II World Congress on Paraconsistency, in 2000, I had the honor of being invited to join the Scientific Committee of the III World

Congress on Paraconsistency, only 3 years later. Best of all, my teachings did not fall on deaf years. They have helped other people make progress. I have collected in recent years a sizable list of papers that cite, refer to, or actively make use of my work, as it can be checked below. That is surely very rewarding for a scientist on the making. Perhaps you will also contribute to my collection?

A list of papers referring to my work

1. João Alcântara, Carlos V. Damásio, and Luís M. Pereira. An encompassing framework for paraconsistent logic programs. *Journal of Applied Logic*, 2005. In print.
2. Martin Allen. Preservationist logics and nonmonotonic logics. Presented at the NASSLLI 2002, June 2002, Stanford / CA, US. `http://www.stanford.edu/group/nasslli/student/allen.ps`.
3. Ofer Arieli. Paraconsistent semantics for extended logic programs. In *Proceedings of the 2002 International Conference on Artificial Intelligence* (IC-AI'02), volume 3, pages 1199–1205. CMSRA Press, 2002.
4. Ofer Arieli. Reasoning with different levels of uncertainty. *Journal of Applied Non-Classical Logics*, 13(3/4):317–343, 2003.
5. Ofer Arieli, Marc Denecker, Bert Van Nuffelen, and Maurice Bruynooghe. Repairing inconsistent databases: A model-theoretic approach and abductive reasoning. In H. Decker, J. Villadsen, and T. Waragai, editors, *Proceedings of the Workshop on Paraconsistent Computational Logic* (PCL 2002), held in Copenhagen, DN, July 2002, as part of the 2002 *Federated Logic Conference* (FLoC'02), volume 95 of *Datalogiske Skrifter*, pages 51–65. Roskilde University, 2002.
6. Ofer Arieli, Marc Denecker, Bert Van Nuffelen, and Maurice Bruynooghe. Coherent integration of databases by abductive logic programming. *Journal of Artificial Intelligence Research*, 21:241–286, 2004.
7. Ofer Arieli, Marc Denecker, Bert Van Nuffelen, and Maurice Bruynooghe. Database repair by signed formulae. In D. Seipel and J. M. Turull-Torres, editors, *Foundations of Information and Knowledge Systems, Proceedings of the* III *International Symposium* (FoIKS 2004), held in Vienna, AU, February 2004, volume 2942 of *Lecture Notes in Computer Science*, pages 14–30. Springer, 2004.
8. Arnon Avron. Tableaux with four signs as a unified framework. In M. C. Mayer and F. Pirri, editors, *Automated Reasoning with Analytic Tableaux and Related Methods, Proceedings of* (TABLEAUX 2003), volume 2796 of *Lecture Notes in Computer Science*, pages 4–16. Springer, 2003. `http://antares.math.tau.ac.il/~aa/articles/tableaux4.pdf`.
9. Arnon Avron. Non-deterministic semantics for paraconsistent **C**-systems. Submitted for publication, 2004. `http://antares.math.tau.ac.il/~aa/articles/c-systems.pdf`.
10. Arnon Avron. A non-deterministic view on non-classical negations. *Studia Logica*, 2005. In print. `http://antares.math.tau.ac.il/~aa/articles/negation-nmatrices.pdf`.

11. Arnon Avron. Combining classical logic, paraconsistency and relevance. *Journal of Applied Logic*, 2005. In print.
`http://antares.math.tau.ac.il/∼aa/articles/combining.pdf`.

12. Arnon Avron. Non-deterministic matrices and modular semantics of rules. In J.-Y. Béziau, editor, *Logica Universalis*, pages 149–167. Birkhäuser, 2005.
`http://antares.math.tau.ac.il/∼aa/articles/modular.pdf`.

13. Arnon Avron. Non-deterministic semantics for families of paraconsistent logics. Presented at the III World Congress on Paraconsistency, held in Toulouse, FR, July 2003. To appear in Proceedings, 2005.
`http://antares.math.tau.ac.il/∼aa/articles/int-c.ps.gz`.

14. Arnon Avron and Beata Konikowska. Proof systems for logics based on non-deterministic multiple-valued structures. Submitted for publication, 2004.
`http://antares.math.tau.ac.il/∼aa/articles/proof-nmatrices.pdf`.

15. Arnon Avron and Iddo Lev. Non-deterministic multiple-valued structures. *Journal of Logic and Computation*, 2005. In print.
`http://antares.math.tau.ac.il/∼aa/articles/nmatrices.ps.gz`.

16. Arnon Avron and Iddo Lev. Non-deterministic matrices. In *Proceedings of the* XXXIV *IEEE International Symposium on Multiple-Valued Logic* (ISMVL 2004), pages 282–287. IEEE Computer Society Press, 2004.

17. Tomaz Barrero. Positive Logic: Plenitude, potentiality and problems (of negationless thinking) (in Portuguese). Master's thesis, State University of Campinas, BR, 2004.

18. Diderik Batens. On some remarkable relations between paraconsistent logics, modal logics, and ambiguity logics. In W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano, editors, *Paraconsistency: The logical way to the inconsistent*, volume 228 of *Lecture Notes in Pure and Applied Mathematics*, pages 275–294. Marcel Dekker, 2002.

19. Diderik Batens and Kristof De Clercq. A rich paraconsistent extension of full positive logic. *Logique et Analyse*. In print.

20. Corrado Benassi and Paolo Gentilini. Paraconsistent provability logic and rational epistemic agents. Submitted for publication, 2004.

21. Leopoldo Bertossi and Jan Chomicki. Query answering in inconsistent databases. In G. Saake J. Chomicki and R. van der Meyden, editors, *Logics for Emerging Applications of Databases*. Springer, 2003.

22. Philippe Besnard and Paul Wong. Modal (logic) paraconsistency. In T. D. Nielsen and N. L. Zhang, editors, *Symbolic and Quantitative Approaches to Reasoning with Uncertainty, Proceedings of the* VII *European Conference* (ECSQARU 2003)*,* held in Aalborg, DK, 2–5 July 2004, volume 2711 of *Lecture Notes in Computer Science*, pages 540–551. Springer, 2003.

23. Jean-Yves Béziau. The future of paraconsistent logic. *Logical Studies*, 2:1–23, 1999.
`http://www.logic.ru/Russian/LogStud/02/LS_2_e_Beziau.pdf`.

24. Jean-Yves Béziau. Non-truth-functional many-valuedness. Draft, 2000. Submitted for publication.

25. Jean-Yves Béziau. The logic of confusion. In H. R. Arabnia, editor, *Proceedings of the International Conference on Artificial Intelligence* (IC-AI'2001), volume II, pages 821–826. CSREA Press, Athens / GA, 2001.

26. Jean-Yves Béziau. Are paraconsistent negations negations? In W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano, editors, *Paraconsistency: The logical way to the inconsistent*, volume 228 of *Lecture Notes in Pure and Applied Mathematics*, pages 465–486. Marcel Dekker, 2002.
27. Jean-Yves Béziau. Paraconsistent logic from a modal viewpoint. In J. Marcos, D. Batens, and W. A. Carnielli, editors, *Proceedings of the Workshop on Paraconsistent Logic* (WoPaLo), held in Trento, IT, August 5–9 2002, pages 121–129. As part of the XIV European Summer School on Logic, Language and Information (ESSLLI 2002), 2002.
28. Manuel Bremer. *An Introduction to Paraconsistent Logics*. Peter Lang, New York / NY, 2005. In pring.
29. Andreas Brunner and Walter A. Carnielli. Anti-intuitionism and paraconsistency. *Journal of Applied Logic*, 2005. In print.
30. Juliana Bueno. Possible-Translations Algebraic Semantics (in Portuguese). Master's thesis, State University of Campinas, BR, 2004.
31. Juliana Bueno, Marcelo E. Coniglio, and Walter A. Carnielli. Finite algebraizability via possible-translations semantics. In W. A. Carnielli, F. M. Dionísio, and P. Mateus, editors, *Proceedings of the Workshop on Combination of Logics: Theory and applications* (CombLog'04), held in Lisbon, PT, 28–30 July 2004, pages 79–85. Departamento de Matemática, Instituto Superior Técnico, 2004. July 28–30, 2004, Lisbon, PT. Preprint available at: `http://wslc.math.ist.utl.pt/comblog04/abstracts/bueno.pdf`.
32. Carlos Caleiro. *Combining Logics*. PhD thesis, IST, Universidade Técnica de Lisboa, PT, 2000. `http://wslc.math.ist.utl.pt/ftp/pub/CaleiroC/00-C-PhDthesis.ps`.
33. Carlos Caleiro, Walter A. Carnielli, João Rasga, and Cristina Sernadas. Fibring of logics as a universal construction. Preprint, CLC, Department of Mathematics, Instituto Superior Técnico, 1049-001 Lisbon, PT, 2004. To appear in *Handbook of Philosophical Logic*, 2nd edition. `http://wslc.math.ist.utl.pt/ftp/pub/SernadasC/04-CCRS-fiblog23.pdf`.
34. Carlos Caleiro, Paulo Mateus, Jaime Ramos, and Amílcar Sernadas. Combining logics: Parchments revisited. In M. Cerioli and G. Reggio, editors, *Recent Trends in Algebraic Development Techniques: Selected Papers*, volume 2267 of Lecture Notes in Computer Science, pages 48–70. Springer-Verlag, Berlin, 2001. `http://wslc.math.ist.utl.pt/ftp/pub/SernadasA/01-CMRS-fiblog9.pdf`.
35. Walter A. Carnielli. Possible-translations semantics for paraconsistent logics. In D. Batens, C. Mortensen, G. Priest, and J. P. Van Bendegem, editors, *Frontiers of Paraconsistent Logic,* Proceedings of the I World Congress on Paraconsistency, held in Ghent, BE, July 29–August 3, 1997, pages 149–163. Research Studies Press, Baldock, 2000.
36. Walter A. Carnielli. How to build your own paraconsistent logic: An introduction to the Logics of Formal (In)Consistency. In J. Marcos, D. Batens, and W. A. Carnielli, editors, *Proceedings of the Workshop on Paraconsistent Logic* (WoPaLo), held in Trento, IT, August 5–9 2002, pages 58–72. As part of the XIV European Summer School on Logic, Language and Information (ESSLLI 2002), 2002.

37. Walter A. Carnielli and Marcelo E. Coniglio. A categorial approach to the combination of logics. *Manuscrito—Revista Internacional de Filosofia*, XXII(2):69–94, 1999.

38. Walter A. Carnielli, Cristina Sernadas, and Alberto Zanardo. Preservation of interpolation by fibring. In W. A. Carnielli, F. M. Dionísio, and P. Mateus, editors, *Proceedings of the Workshop on Combination of Logics: Theory and applications* (CombLog'04), held in Lisbon, PT, 28–30 July 2004, pages 151–157. Departamento de Matemática, Instituto Superior Técnico, 2004. July 28–30, 2004, Lisbon, PT. Preprint available at:
`http://wslc.math.ist.utl.pt/comblog04/abstracts/bueno.pdf`.

39. Henning Christiansen and Davide Martinenghi. Simplification of integrity constraints for data integration. In D. Seipel and J. M. Turull-Torres, editors, *Foundations of Information and Knowledge Systems, Proceedings of the* III *International Symposium* (FoIKS 2004), held in Vienna, AU, February 2004, volume 2942 of *Lecture Notes in Computer Science*, pages 31–48. Springer, 2004.

40. Marcelo E. Coniglio. Categorial combination of logics: Completeness preservation. Preprint, Section of Computer Science, Department of Mathematics, Instituto Superior Técnico, 1049-001 Lisbon, PT, 2001. Submitted for publication.
`http://wslc.math.ist.utl.pt/ftp/pub/ConiglioM/catcomb.ps`.

41. Marcelo E. Coniglio and Walter A. Carnielli. Transfers between logics and their applications. *Studia Logica*, 72(3):367–400, 2002.

42. Marcelo E. Coniglio, Ana T. Martins, Amílcar Sernadas, and Cristina Sernadas. Fibring (para)consistent logics. Research report, Section of Computer Science, Department of Mathematics, Instituto Superior Técnico, 1049-001 Lisbon, PT, 2000. Extended abstract. Presented at the II World Congress on Paraconsistency (WCP'2000), held in Juquehy, BR, May 8–12, 2000.
`http://wslc.math.ist.utl.pt/ftp/pub/SernadasA/00-CMSS-fiblog6.pdf`.

43. Alexandre Costa-Leite. Paraconsistency, Modalities and Knowability (in Portuguese). Master's thesis, State University of Campinas, BR, 2003.

44. Denise Cunha. Unsaid Beliefs, Said About Beliefs (in Portuguese). Master's thesis, State University of Campinas, BR, 2003.
`http://libdigi.unicamp.br/document/?code=vtls000280021`.

45. Newton C. A. da Costa, Décio Krause, and Otávio A. S. Bueno. Paraconsistent logic and paraconsistency: Technical and philosophical developments. *CLE e-Prints*, 4(3), 2004. To appear in D. Jacquette, editor, *Handbook of the Philosophy of Science: Logic*, Elsevier.

46. Sandra de Amo and Mônica S. Pais. A deductive language for databases with inconsistencies. Draft, 2004. Submitted for publication.

47. Milton A. de Castro and Itala M. L. D'Ottaviano. Natural deduction for paraconsistent logic. *Logica Trianguli*, 4:3–24, 2000.

48. Richard L. Epstein. Paraconsistent logics with simple semantics. *Logique et Analyse (N.S.)*. In print.

49. Victor L. Fernández. Society Semantics for n-valued Logics (in Portuguese). Master's thesis, State University of Campinas, BR, 2001.
`http://www.cle.unicamp.br/prof/coniglio/Victesis.ps`.

50. Victor L. Fernández and Marcelo E. Coniglio. Combining valuations with society semantics. *Journal of Applied Non-Classical Logics*, 13(1):21–46, 2003. `http://www.cle.unicamp.br/e-prints/abstract_11.html`.
51. Paola Forcheri and Paolo Gentilini. Paraconsistent informational logic. In J. Marcos, D. Batens, and W. A. Carnielli, editors, *Proceedings of the Workshop on Paraconsistent Logic* (WoPaLo), held in Trento, IT, August 5–9 2002, pages 90–102. As part of the XIV European Summer School on Logic, Language and Information (ESSLLI 2002), 2002.
52. Paola Forcheri and Paolo Gentilini. Paraconsistent informational logic. *Journal of Applied Logic*, 2005. In print.
53. Marcel Guillaume. Da Costa 1964 Logical Seminar: Revisited memories. *CLE e-Prints*, 4(2), 2004. Submitted for publication.
54. Carlos Hifume. A theory of pragmatic truth: The quasi truth of Newton C. A. da Costa (in Portuguese). Master's thesis, State University of Campinas, BR, 2003.
55. Norihiro Kamide. Foundations of paraconsistent resolution. *Fundamenta Informaticae*, 21:1001–1024, 2004.
56. Juliano S. A. Maranhão. Some propositional paraconsistent logics: **C**-extensions, 2001. Tutorial on 'Propositional Paraconsistent Logics', presented at the Institut für Logik und Wissenschaftstheorie, Leipzig, DE. `http://www.uni-leipzig.de/~logik/ppl/ks23-10.pdf`.
57. Paolo Mascellani. *Paraconsistent Logics and Logic Programs Verification.* PhD thesis, Dipartimento di Studi Matematici ed Informatici "R. Magari", Università degli Studi di Siena, IT, 2004.
58. José L. Montes and Camilo E. Restrepo. Paraconsistent Logics: An introduction (in Spanish). Master's thesis, Department of Basic Sciences, Universidad EAFIT, CO, 2000.
59. Francesco Paoli. Tautological entailments and their rivals. Submitted for publication, 2004. `http://www.unica.it/~paoli/Tautent.ps`.
60. Claudio Pizzi and Walter A. Carnielli. *Modalità e Multimodalità.* Franco Angeli, Milan, 2001.
61. Shahid Rahman and Walter A. Carnielli. The dialogical approach to paraconsistency. *Synthese*, 125(1/2):201–232, 2000.
62. João Rasga. *Fibring Labelled First-Order Based Logics.* PhD thesis, IST, Universidade Técnica de Lisboa, PT, 2003.
63. Cristina Sernadas, João Rasga, and Walter A. Carnielli. Modulated fibring and the collapsing problem. *The Journal of Symbolic Logic*, 67(4):1541–1569, 2002.
64. Manuel Sierra A. Sistema de lógica paraconsistente C_1. *Revista Universidad EAFIT* (Colombia), 118:23–34, 2000.
65. Manuel Sierra A. Inferencia visual para los sistemas deductivos **LBPco**, **LBPc** y **LBPo**. Technical report, Logic and Computation Group, Universidad EAFIT, 2002.
66. Manuel Sierra A. Forzamiento semántico de marcas para la lógica básica paraconsistente **LBPc**. *Revista Universidad EAFIT* (Colombia), 130:29–51, 2003.

Short Addendum on Unnecessary Explanations

> If it was so, it might be;
> and if it were so it would be,
> but as it isn't, it ain't. That's logic.
> —says Tweedledee, in Lewis Carroll's *Through the Looking Glass and What Alice Found There*,1872.

Let me tell you one last thing. It might be shocking for you, so please sit down. Here it is. I do not assume logic to be an 'investigation of the laws of thought'. Nor do I buy it, otherwise, as 'a formula language of pure thought'. And do not even think that I understand logic as the 'pursuit of truth'. I certainly do not assume here that logic *by itself* is going to tell you much about the principles of rational thinking, the psychology of reasoning, natural language, epistemology, metaphysics or ontology. (Logic relates to metaphysics, for instance, just as mathematics relates to engineering.) Yet it might *help* you investigating any of those issues. Logic, in my everyday job, is but pure technology —a declaration that would not surprise Aristotle. It helps you in doing philosophy, mathematics, linguistics and computer science in much the same way a laboratory and a computer helps the standard natural scientist in their job. As any other man-made tool, or organon, it extends your power to realize the tasks that your intelligence proposes to you. It would be somewhat surprising that any of the above statements still needed to be made in the days we live. But here they are. For all I said in such a *via negativa*, this might seem all too elusive as a characterization. Oh well, I am not telling you now what I *do* take logic to be. Will you please just have a look at the papers that follow.

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NON SEQUITUR
    QUODLIBET
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. . . .

Late, and hopefully also unnecessary, warning

> I killed my logic teacher when I was 16. Alleging self-defense —and which defense would be more legitimate?— I managed to be absolved by five votes against two, and I went to live under a bridge of the Seine, though I have never been in Paris.
> —Campos de Carvalho, *A Lua vem da Ásia*, 1956.

In case you have not yet noticed, this thesis will be better read by those with a good sense of humor.

Acknowledgements

You already know that I have travelled a lot. That I have read a lot of papers. Met a lot of people. To be fair, the number of people I would have to mention as having helped me in one way or other, by teaching me a few logical tricks, by showing me the way to go, or by just being there when I needed the most, well, that number is certainly LARGE. By way of expression, I would rather call it *innumerable*. But in that case I am forced to abide by the Cantorian dilemma, according to which any list I would try to make of those people would necessarily leave somebody out. So, to be perfectly fair with everybody, I will mention NO ONE here. (Yet many people will be mentioned by name in the next chapters!)

I do have to refer here, though, and in grateful recognition, to the financial support I received in the grants and fellowships that enabled me to survive through the last few years. This thesis would not have been possible had it not been, in this order, for a 24 months CAPES fellowship (at Unicamp / BR), together with a CAPES / DAAD allowance that financed my trip and stay in Karlsruhe (DE) for 4 months, working at the Institut für Logik, Komplexität und Deduktionssysteme, a 12 months Dehousse Doctoral Grant that I received while working as a Research Assistant in Ghent (at UGent / BE), an 11 months CNPq grant (at Unicamp / BR), and finally a 24 months support from the Fundação para a Ciência e a Tecnologia (PT) and FEDER (EU), namely via the grant SFRH / BD / 8825 / 2002 and the Project FibLog POCTI / MAT / 37239 / 2001 of the CLC / IST (at UTL / PT). I should also thank here (contradicting my policy of not mentioning names!) those who were my hosts while I enjoyed those grants, namely, the professors Walter Carnielli (BR), Peter Schmitt and Bernhard Beckert (DE), Diderik Batens and Erik Weber (BE), Carlos Caleiro and Amílcar Sernadas (PT), for providing to me the best working conditions they could throughout this odyssey (and no better place to finish an odyssey than Lisbon, having Ulysses himself granted to it its old name of Olisipo, or Ulyssipo).

Was the whole effort worth? I do hope so (se a alma não é pequena).

> Crime is common. Logic is rare. Therefore it is upon the logic rather than upon the crime that you should dwell.
> —Sir Arthur Conan Doyle, *The Adventures of Sherlock Holmes*, 1892.

Bibliography

[1] Andrew Aberdein. Classical recapture. In V. Fano, M. Stanzione, and G. Tarozzi, editors, *Logica e Filosofia delle Scienze: Atti del sesto convegno triennale*, pages 11–18. Rubettino, Catanzaro, 2001.

[2] H. R. Arabnia, editor. *Proceedings of the International Conference on Artificial Intelligence* (IC-AI'2001), volume II. CSREA Press, Athens / GA, 2001.

[3] Ayda I. Arruda. A survey of paraconsistent logic. In *Mathematical Logic in Latin America: Proceedings of the* IV *Latin American Symposium on Mathematical Logic,* held in Santiago, CL, 1978, pages 1–41, Amsterdam, 1980. North-Holland.

[4] Florencio G. Asenjo. *El Todo y las Partes: Estudios de Ontología Formal.* Madrid, 1962.

[5] Florencio G. Asenjo. A calculus of antinomies. *Notre Dame Journal of Formal Logic*, 7:103–105, 1966.

[6] Florencio G. Asenjo. The logic of opposition. In Carnielli et al. [25], pages 100–140.

[7] D. Batens, C. Mortensen, G. Priest, and J. P. Van Bendegem, editors. *Frontiers of Paraconsistent Logic,* Proceedings of the I World Congress on Paraconsistency, held in Ghent, BE, July 29–August 3, 1997. Research Studies Press, Baldock, 2000.

[8] Diderik Batens. Paraconsistent extensional propositional logics. *Logique et Analyse (N.S.)*, 90/91:195–234, 1980.

[9] Diderik Batens. Linguistic and ontological measures for comparing the inconsistent parts of models. *Logique et Analyse (N.S.)*, 165/166:5–33, 1999.

[10] Diderik Batens. Paraconsistency and its relation to worldviews. *Foundations of Science*, 3:259–283, 1999.

[11] Diderik Batens. A survey of inconsistency-adaptive logics. In Batens et al. [7], pages 49–73.

[12] Jean-Yves Béziau. Logiques construites suivant les méthodes de da Costa. I. Logiques paraconsistantes, paracompletes, non-alèthiques construites suivant la première méthode de da Costa. *Logique et Analyse (N.S.)*, 33(131/132):259–272, 1990.

[13] Jean-Yves Béziau. Logic may be simple. *Logic and Logical Philosophy*, 5:129–147, 1997.

[14] Jean-Yves Béziau. Idempotent full paraconsistent negations are not algebraizable. *Notre Dame Journal of Formal Logic*, 39(1):135–139, 1998.

[15] Jean-Yves Béziau. The future of paraconsistent logic. *Logical Studies*, 2:1–23, 1999.
`http://www.logic.ru/Russian/LogStud/02/LS_2_e_Beziau.pdf`.

[16] Jean-Yves Béziau. What is paraconsistent logic? In Batens et al. [7], pages 95–111.

[17] Jean-Yves Béziau. Y a-t-il des principes logiques? In L. H. de A. Dutra and C. A. Mortari, editors, *Princípios: seu papel na filosofia e nas ciências*, volume 3 of *Rumos da Epistemologia*, pages 47–54. NEL, Florianópolis / SC, 2000.

[18] Jean-Yves Béziau. From paraconsistent to universal logic. *Sorites*, 12:5–32, 2001.

[19] Jean-Yves Béziau. Are paraconsistent negations negations? In Carnielli et al. [25], pages 465–486.

[20] Andrés Bobenrieth-Miserda. *Inconsistencias ¿Por qué no? Un estudio filosófico sobre la lógica paraconsistente.* Tercer Mundo, Santafé de Bogotá, 1996.

[21] Carlos Caleiro, Walter A. Carnielli, Marcelo E. Coniglio, and João Marcos. Dyadic semantics for many-valued logics. Research report, CLC, Department of Mathematics, Instituto Superior Técnico, 1049-001 Lisbon, PT, 2003. Presented at the III World Congress on Paraconsistency, Toulouse, FR, July 28–31, 2003.
`http://wslc.math.ist.utl.pt/ftp/pub/CaleiroC/03-CCCM-dyadic2.pdf`.

[22] Carlos Caleiro, Walter A. Carnielli, Marcelo E. Coniglio, and João Marcos. How many logical values are there? Dyadic semantics for many-valued logics. Draft, 2005. Forthcoming.

[23] Carlos Caleiro, Walter A. Carnielli, Marcelo E. Coniglio, and João Marcos. Two's company: "The humbug of many logical values". In J.-Y. Béziau, editor, *Logica Universalis*, pages 169–189. Birkhäuser, 2005.
`http://wslc.math.ist.utl.pt/ftp/pub/CaleiroC/05-CCCM-dyadic.pdf`.

[24] Carlos Caleiro and João Marcos. Non-truth-functional fibred semantics. In Arabnia [2], pages 841–847.
http://wslc.math.ist.utl.pt/ftp/pub/CaleiroC/01-CM-fiblog10.ps.

[25] W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano, editors. *Paraconsistency: The Logical Way to the Inconsistent,* Proceedings of the II World Congress on Paraconsistency, held in Juquehy, BR, May 8–12, 2000, volume 228 of *Lecture Notes in Pure and Applied Mathematics*. Marcel Dekker, 2002.

[26] Walter A. Carnielli, Marcelo E. Coniglio, and João Marcos. Logics of Formal Inconsistency. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic*, volume 14. Kluwer Academic Publishers, 2nd edition, 2004. In print. Preprint available at:
http://wslc.math.ist.utl.pt/ftp/pub/MarcosJ/03-CCM-lfi.pdf.

[27] Walter A. Carnielli and João Marcos. Limits for paraconsistent calculi. *Notre Dame Journal of Formal Logic*, 40(3):375–390, 1999.
http://projecteuclid.org/Dienst/UI/1.0/Display/euclid.ndjfl/1022615617.

[28] Walter A. Carnielli and João Marcos. Tableaux for logics of formal inconsistency. In Arabnia [2], pages 848–852.
ftp://www.cle.unicamp.br/pub/professors/carnielli/articles/TableauxforLFIs.zip.

[29] Walter A. Carnielli, João Marcos, and Sandra de Amo. Formal inconsistency and evolutionary databases. *Logic and Logical Philosophy*, 8(2):115–152, 2000.
http://www.cle.unicamp.br/e-prints/abstract_6.htm.

[30] E. J. Cogan. Review of N. C. A. da Costa, 'Nota sobre o conceito de contradição', *Anuário da Sociedade Paranaense de Matemática*, 1:6–8, 1958. *Mathematical Reviews*, 21(11):7146, 1960.

[31] E. J. Cogan. Review of N. C. A. da Costa, 'Observações sobre o conceito de existência em matemática', *Anuário da Sociedade Paranaense de Matemática*, 2:16–19, 1959. *Mathematical Reviews*, 24(2A):A654, 1962.

[32] Newton C. A. da Costa. Nota sobre o conceito de contradição. *Anuário da Sociedade Paranaense de Matemática*, 1:6–8, 1958.

[33] Newton C. A. da Costa. Observações sobre o conceito de existência em matemática. *Anuário da Sociedade Paranaense de Matemática*, 2:16–19, 1959.

[34] Newton C. A. da Costa. Calculs propositionnels pour le systèmes formels inconsistants. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences,* Séries A–B, 257:3790–3793, 1963.

[35] Newton C. A. da Costa. *Inconsistent Formal Systems* (Habilitation Thesis, in Portuguese). UFPR, Curitiba, 1963. Editora UFPR, 1993. http://www.cfh.ufsc.br/~nel/historia_logica/sistemas_formais.htm.

[36] Newton C. A. da Costa. On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic*, 11:497–510, 1974.

[37] Newton C. A. da Costa. The philosophical import of paraconsistent logic. *The Journal of Non-Classical Logic*, 1(1):1–19, 1982.

[38] Newton C. A. da Costa. On paraconsistent set theory. *Logique et Analyse (N.S.)*, 115:361–371, 1986.

[39] Newton C. A. da Costa. *Ensaio sobre os Fundamentos da Lógica.* Hucitec, São Paulo / SP, 2nd edition, 1994. First published in 1979. Translated as *Logiques Classiques et Non Classiques*, Masson, Paris, 1997.

[40] Newton C. A. da Costa. Opening address: Paraconsistent logic. *Logic and Logical Philosophy*, 7:25–34, 1999. Proceedings of the Stanisław Jaśkowski's Memorial Symposium, held in Toruń, PL, July 15–18, 1998.

[41] Newton C. A. da Costa. Logic and ontology. *Principia*, 6(2):279–298, 2002. Pré-publicações do CLE, 3, 1982. First printed in Bulgarian in *Critica*, 4:25–34, 1988.

[42] Newton C. A. da Costa and Jean-Yves Béziau. La logique paraconsistante. In J. Sallantin and J.-J. Szczeciniarz, editors, *Le Concept de Preuve à la Lumière de l'Intelligence Artificielle*, pages 107–115. Presses Universitaires de France, 1999.

[43] Newton C. A. da Costa, Jean-Yves Béziau, and Otávio A. S. Bueno. Paraconsistent logic in a historical perspective. *Logique et Analyse (N.S.)*, 150/152:111–125, 1995.

[44] Newton C. A. da Costa, Jean-Yves Béziau, and Otávio A. S. Bueno. *Elementos de Teoria Paraconsistente de Conjuntos*, volume 23 of *Coleção CLE*. CLE / Unicamp, Campinas, 1998.

[45] Newton C. A. da Costa and Otávio Bueno. Consistency, paraconsistency, and truth (logic, the whole logic, and nothing but 'the' logic). *Ideas y Valores*, 100:48–60, 1996.

[46] Newton C. A. da Costa and Otávio Bueno. Paraconsistency: A tentative interpretation. *Theoria (Segunda Época)*, 16(1):119–145, 2001.

[47] Newton C. A. da Costa and Steven French. Paraconsistency. In H. Burkhardt and B. Smith, editors, *Handbook of Metaphysics and Ontology*, volume 2, pages 656–658. Philosophia Verlag, Munich, 1991.

[48] Newton C. A. da Costa and Marcel Guillaume. Négations composées et loi de Peirce dans les systèmes C_n. *Portugaliae Mathematica*, 24:201–210, 1965.

[49] Sandra de Amo, Walter A. Carnielli, and João Marcos. A logical framework for integrating inconsistent information in multiple databases. In *Proceedings of the* II *International Symposium on Foundations of Information and Knowledge Systems* (FoIKS'2002), Schloß Salzau, DE, 19–23 February 2002, Lecture Notes in Computer Science, Berlin, 2002. Springer-Verlag.
`http://www.cle.unicamp.br/e-prints/abstract_9.htm.`

[50] Itala M. L. D'Ottaviano and Newton C. A. da Costa. Sur un problème de Jaśkowski. *Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences,* Séries A–B, 270(21):1349–1353, 1970.

[51] Itala M. L. D'Ottaviano. On the development of paraconsistent logic and da Costa's work. *The Journal of Non-Classical Logic*, 7(1/2):89–152, 1990.

[52] Michael A. E. Dummett. *The Logical Basis of Metaphysics.* Harvard University Press, Cambridge / MA, 1991. The William James lectures: 1976.

[53] Frederic B. Fitch. *Symbolic Logic: An introduction.* Ronald Press, New York / NY, 1952.

[54] David Hilbert. Über das Unendliche. *Mathematische Annalen*, 95:161–190, 1926. Translated into English in P. Benacerraf and H. Putnam (eds.), *Philosophy of Mathematics*, 2nd ed., Harvard University Press, Cambridge / MA, 1983, pp.183–201.

[55] Stanisław Jaśkowski. A propositional calculus for inconsistent deductive systems (in Polish). *Studia Societatis Scientiarum Torunensis*, Sectio A, 5:57–77, 1948. Translated into English in *Studia Logica*, 24:143–157, 1967, and in *Logic and Logical Philosophy*, 7:35–56, 1999.

[56] Stanisław Jaśkowski. On the discussive conjunction in the propositional calculus for inconsistent deductive systems (in Polish). *Studia Societatis Scientiarum Torunensis*, Sectio A, 8:171–172, 1949. Translated into English in *Logic and Logical Philosophy*, 7:57–59, 1999.

[57] Stephen C. Kleene. *Introduction to Metamathematics.* D. van Nostrand, New York, 1952.

[58] Andrea Loparić and Newton C. A. da Costa. Paraconsistency, paracompleteness, and valuations. *Logique et Analyse (N.S.)*, 27(106):119–131, 1984.

[59] João Marcos. 8K solutions and semi-solutions to a problem of da Costa. Draft. Forthcoming.

[60] João Marcos. Many values, many semantics. Draft, 2000.

[61] João Marcos. Possible-Translations Semantics (in Portuguese). Master's thesis, State University of Campinas, BR, 1999.
`http://www.cle.unicamp.br/students/J.Marcos/index.htm.`

[62] João Marcos. On a problem of da Costa. *CLE e-Prints*, 1(8), 2001. To appear in *Logica Trianguli*.
`http://www.cle.unicamp.br/e-prints/abstract_8.htm.`

[63] João Marcos. (Wittgenstein & Paraconsistência), 2001. To appear in *Principia*. Preprint available at:
`http://www.cle.unicamp.br/e-prints/abstract_7.html.`

[64] Chris Mortensen. Every quotient algebra for C_1 is trivial. *Notre Dame Journal of Formal Logic*, 21(4):694–700, 1980.

[65] A. Mostowski. Review of S. Jaśkowski, 'Rachunek zdan dla systemów dedukcyjnych sprzecznych', *Studia Societatis Scientiarum Torunensis*, Sectio A, 5:57–77, 1948. *The Journal of Symbolic Logic*, 14(1):66–67, 1949.

[66] A. Mostowski. Review of S. Jaśkowski, 'O koniunkcji dyskusyjnej w rachunku zdan dla systemów dedukcyjnych sprzecznych', *Studia Societatis Scientiarum Torunensis*, Sectio A, 5:57–77, 1948. *The Journal of Symbolic Logic*, 18(4):345, 1953.

[67] David Nelson. Constructible falsity. *The Journal of Symbolic Logic*, 14:16–26, 1949.

[68] David Nelson. Negation and separation of concepts in constructive systems. In A. Heyting, editor, *Constructivity in Mathematics,* Proceedings of the Colloquium held in Amsterdam, NL, in 1957, Studies in Logic and the Foundations of Mathematics, pages 208–225, Amsterdam, 1959. North-Holland.

[69] Jerzy Perzanowski. Paraconsistency, or inconsistency tamed, investigated and exploited. *Logic and Logical Philosophy*, 9:5–24, 2001.

[70] Dag Prawitz. *Natural Deduction: A proof-theoretical study*, volume 3 of *Stockholm Studies in Philosophy*. Almqvist and Wiksell, Stockholm, 1965.

[71] G. Priest, J. C. Beall, and B. Armour-Garb, editors. *The Law of Non-Contradiction: New philosophical essays*. Oxford University Press, New York, 2005.

[72] Graham Priest. The logic of paradox. *Journal of Philosophical Logic*, 8(2):219–241, 1979.

[73] Graham Priest. *In Contradiction: A study of the transconsistent.* Martinus Nijhoff, The Hague, 1987.

[74] Graham Priest. Minimally inconsistent *LP*. *Studia Logica*, 50:321–331, 1991.

[75] Graham Priest. Review of N. C. A. da Costa, *Logiques Classiques et Non Classiques*, Masson, Paris, 1997. *Studia Logica*, 64:435–443, 2000.

[76] Graham Priest. Dialetheism. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy* (on-line). Summer 2004.
`http://plato.stanford.edu/archives/sum2004/entries/dialetheism.`

[77] Graham Priest and Richard Routley. Introduction: Paraconsistent logics. *Studia Logica*, 43(1/2):3–16, 1984.

[78] Willard V. O. Quine. On what there is. *Review of Metaphysics*, 2:21–38, 1948. Reprinted in the author's *From a Logical Point of View*, Harper and Row, New York / NY, 1953.

[79] Antonio M. Sette. On the propositional calculus $\mathbf{P}^1$. *Mathematica Japonicae*, 18:173–180, 1973.

[80] Moh Shaw-Kwei. Logical paradoxes for many valued systems. *The Journal of Symbolic Logic*, 19(1):37–40, 1954.

[81] Raymond Smullyan. *Some Interesting Memories — A Paradoxical Life.* Thinker's Press, Inc, Davenport / IA, 2002.

[82] Koji Tanaka. Three schools of paraconsistency. *Australasian Journal of Logic*, 1:28–42, 2003.

[83] Alfred Tarski. The semantical conception of truth and the foundations of semantics. *Philosophy and Phenomenological Research*, 4:341–376, 1944.
`http://www.ditext.com/tarski/tarski.html.`

[84] R. Thomason, editor. *Formal Philosophy: Selected papers of Richard Montague.* Yale University Press, New Haven / CT, 1974.

[85] Igor Urbas. Dual-intuitionistic logic. *Notre Dame Journal of Formal Logic*, 37(3):440–451, 1996.

[86] Heinrich Wansing. *The Logic of Information Structures.* Springer-Verlag, Berlin, 1993.

Appendix: Brief historical note

To give a fair account of the early development of paraconsistent logic in Brazil, Newton da Costa, 75 years old, was interviewed in February 2005 about the historical origins of his work on paraconsistency. Prof. da Costa tells us that he developed paraconsistent logic in between 1954 and 1958, presenting his results in seminars and conferences at the Federal University of Paraná (UFPR), at the State University of São Paulo (USP), and at the late University of Brazil. He says that he only exposed his results little by little, because they were revolutionary at the time, in contrast to what happens in our days. Da Costa recalls some early expositions he made on the material for the benefit of Mário Tourasse Teixeira at Rio Claro, São Paulo, and Constantino Menezes de Barros at Rio de Janeiro, in 1958 and 1959. He also mentions a course he gave at the Federal University of Rio de Janeiro (UFRJ) in 1961, followed by, among others, Mário Tourasse, Constantino de Barros and Max Dickmann, and he recalls his surprise for not having been told by anyone that he was crazy —Prof. António Monteiro, for one, is supposed to have told da Costa then that a violation of the principle of non-contradiction was simply inconceivable, as Heyting would have asserted. All that happened allegedly before da Costa got in touch with the abstracts of the studies by Jaśkowski and with the work of Nelson, which da Costa was to mention in his Habilitation Thesis on the theme, in 1963. Da Costa proceeded then to exchange letters, in separate, with both Jaśkowski and Nelson on issues pertaining to their common interests.

The first publications by da Costa containing more precise definitions related to paraconsistency and the axioms of his first paraconsistent logics appeared in 1963 (cf. [35, 34]). However, it should be acknowledged that in 1962 a short and relatively informal notice about these logics had appeared as an abstract of a contribution to the XIV Annual Meeting of the SBPC (the Brazilian Society for Progress in Science), held in Curitiba, Paraná, in July 8–14 of 1962. This abstract was published as:

> Newton Costa. Sôbre um subsistema do cálculo proposicional clássico. *Ciência e Cultura*, 14(3):139, 1962.

Here you can find the content of this abstract, *ipsissima verba*:

> O cálculo proposicional clássico não se presta para servir de base a sistemas dedutivos onde possa haver contradições. Alguns lógicos e matemáticos, como Kolmogoroff e Jaśkowski, procuraram, então, estruturar cálculos proposicionais com tal finalidade. O autor estudou um subsistema do cálculo tradicional, denominado **cálculo C**, que satisfaz, aparentemente, a exigência acima, e que possui as seguintes características: 1) em C no vale o princípio da não contradição; 2) em C, de duas proposições contraditórias, não se pode deduzir, em geral, qualquer proposição;[12] 3) grande parte dos esquemas e regras de dedução

[12] A formulação deste item é algo estranha e pode ser mal entendida se lida por um novato

> mais importantes do cálculo proposicional clássico valem em C; 4) a extensão de C a um cálculo funcional de primeira ordem é imediata; 5) acrescentando-se a C o princípio da não contradição, obtém-se o cálculo tradicional.

I also take the chance here to provide a first translation of this into English:

> Newton Costa. On a subsystem of the classical propositional calculus.
>
> The classical propositional calculus cannot serve as basis for deductive systems where contradictions can be found. Some mathematical logicians, such as Kolmogorov and Jaśkowski, have tried, then, to structure propositional calculi to serve such an end. The author studied a subsystem of the traditional calculus, called **C-calculus**, that would seem to satisfy the above requirement, and that has the following characteristics: 1) in C the principle of noncontradiction does not hold good; 2) in C, from two contradictory propositions, no proposition in general can be deduced;[13] 3) a great deal of the most important schemas and deduction rules from the classical propositional calculus holds good in C; 4) the extension of C to a first-order functional calculus is immediate; 5) if one adds to C the principle of noncontradiction, the traditional calculus is obtained.

. . . .

All that said and done, one should also acknowledge that it would have seemed that paraconsistency was in the air, in the Southern Hemisphere, in the early 1950s. In 1953, a young man called Florencio González Asenjo (nowadays Professor Emeritus of the University of Pittsburgh) delivered at the University of La Plata, in Argentina, a talk entitled 'La idea de un cálculo de antinomias'. Asenjo developed his views on inconsistency in that decade, but when he first published his results in more detail, in between 1965 and 1966 (cf. chap. X.2 of [20]), he was already acquainted both with the review of the first paper by Jaśkowski and with the first French paper by da Costa.[14]

Is Asenjo a 'forerunner' or a 'discoverer' of paraconsistency? Neither of these options? Both? That is yet another combat, not to be fought in the present thesis.[15]

no assunto. No entanto, do que se sabe hoje da lógica paraconsistente, deve-se supor que pela frase "não se pode deduzir, em geral, qualquer proposição" ter-se-á pretendido dizer que "não se pode deduzir, em geral, uma proposição qualquer" (nota minha).

[13] This item is awkwardly formulated already in Portuguese (see the last note). One should suppose, however, that the author wanted to say that one cannot deduce, in C, an arbitrary proposition from two given contradictory propositions.

[14] Prof. Asenjo tells us that he was very pleasantly impressed when he found da Costa's Comptes Rendus paper by accident in a library in Pittsburgh in the early 1964, shortly before he submitted, in the same year, his paper [5] to the NDJFL. Sobociński was the person who called his attention to the work of Jaśkowski, which also ended up cited in the latter paper. Asenjo also calls our attention to an early passage where the work he did in 1953 was mentioned, in a book published in Madrid in 1962 (cf. p.9 of [4]).

[15] I thank Décio Krause, Newton da Costa and Florencio Asenjo for their kind assistance in helping me clear up the above historical imbroglio.