Chapter Two

Possible-Translations Semantics for Logics of Formal Inconsistency

This chapter is composed of two contributions: **2.1** brings the more general paper 'Possible-translations semantics', henceforth PTSurvey; **2.2** brings the more specific paper 'Possible-translations semantics for some weak classically-based paraconsistent logics', henceforth WeakPTS. The next pages are written by way of an introduction. To fully understand and follow them, it might help that you have read the subsequent papers first. Or that you keep an eye here and another one there, like a dragon.

Resumo de PTSurvey

Este texto almeja dar uma visão panorâmica das semânticas de traduções possíveis, definidas, desenvolvidas e ilustradas como um formalismo muito abrangente para se obter ou representar semânticas para todo tipo de lógicas. Com tal ferramenta, uma ampla classe de lógicas complexas se revela muito naturalmente (de)componível em termos de alguma combinação adequada de lógicas mais simples. Vários exemplos serão mencionados, e alguns casos particulares de semânticas de traduções possíveis, dentre os quais se encontram as semânticas de sociedade e as semânticas não-determinísticas, serão referidos.

Resumo de WeakPTS

Esta nota fornece interpretações por meio de semânticas de traduções possíveis para um grupo de lógicas paraconsistentes fundamentais estendendo o fragmento positivo da lógica proposicional clássica. As lógicas PI, C_{min}, **mbC**, **bC**, **mCi** e **Ci**, entre outras, são todas inicialmente apresentadas por meio de semânticas bivalentes e sequentes, e são a seguir destrançadas por meio de semânticas de traduções possíveis —o conjunto de matrizes 3-valoradas das lógicas ingredientes é exibido, em cada caso, juntamente com o conjunto de funções de tradução admissíveis. Enunciados precisos e todos os detalhes não-óbvios das demonstrações são apresentados. Outros detalhes são deixados para o leitor.

Contents

> I hope that posterity will judge me kindly, not only as to the things which I have explained, but also as to those which I have intentionally omitted so as to leave to others the pleasure of discovery.
> —René Descartes, *La Géométrie*, 1637.

Theoretical and practical aspects of the Logics of Formal Inconsistency (**LFI**s) were both studied in detail in **Chapter 1.0**: A huge number of **LFI**s were presented there, most of them, though, in purely syntactical terms. Can one always provide adequate and informative semantics for those very same logics? The weakest samples among our previous **LFI**s are logics that are neither finite-valued nor do they have canonical modal semantics (once they fail replacement). It is not difficult to provide, however, bivalent semantics for those logics, mocking somehow their syntactical formulations. Such 2-valued non-truth-functional semantics are often not that much illuminating. The present chapter will show how those same logics can be alternatively interpreted in terms of another paradigm of formal semantics: the *possible-translations semantics* (PTS). The papers contained in the present chapter are helpful but somewhat sketchy: One is an extended abstract and another evolved from a research report aimed at helping interested readers find their way. This choice of presentation is hopefully condoned by the fact that PTS is only a subsidiary topic in the present thesis.

One size fits all

The paper PTSurvey (cf. [24]) starts by proposing a structure called 'possible-translations representation' (PTR) as an extremely general framework for specifying the notion of a consequence relation.[1] In principle, whatever non-degenerate definition of logic one might propose, it is always possible to come up with another thing one might want to call a 'logic' and that eludes that definition. Nonetheless, to a first approximation, any logic based on sets of formulas and on (single- or multiple-conclusion) consequence relations will have an adequate PTR, given the present comprehensive design of the latter concept. The basic idea is that of splitting a logic with the help of a collection of 'factors' (other logics) into which it can be 'translated', producing, by suitable combination of these translations, a 'conservative translation' that should provide an adequate (sound and complete) representation for the initial logic. In case the definition of the involved factors can somehow be alleged to involve semantic notions, then the corresponding PTR is said to constitute a 'possible-translations semantics' (PTS).

[1] For the case of single-premise 'simple' PTRs based on grammatical translations (that is, translations homomorphic over the algebra of formulas), my definition coincides with the definition of a *syntactical semantics* presented in [20]. My deepest thanks to Dov Gabbay for calling my attention to that.

The above described splitting process is by no means unusual. For an example from the history of linguistics, the paper mentions the Rosetta Stone and the quest for providing an adequate interpretation for the ancient Hieroglyphic writing. It was only by collecting bits and pieces of meaning from several translations of Hieroglyphic texts into other languages, with varying degrees of fidelity and authenticity, that a conclusive meaning could finally be /extracted from/attached to/ the ancient Egyptian religious texts.

Traduttore, traditore! The well-known Italian proverb that haunts skilled translators from all over punctuates the horny dilemma they are confronted with on an everyday basis: Should translation be as mechanized as possible, or should one let creativity come in? Notice that this question allegedly applies to translations both of technical and of literary texts. To invert the situation and put it in more charitable terms for the translators, can translation actually help *explicate* the meaning of the 'original' text? One could defend, for instance, that the true meaning of a poem is fully conveyed only by the set of all of its translations —and such a theory is beautifully confirmed by [21] or [27]. In a sense, nobody is a native speaker of the language of ideas: We are translating all the time to make ourselves understood, and to try understand the Other. Be that as it may, on what concerns machine translation the current situation remains at best dismaying. One might always recall for instance the story according to which a machine was being built in Shinar to translate English into Chinese, and vice-versa. To test its first prototype, a mighty hunter suggested the phrase "Out of sight, out of mind" to be fed into it. After translating it into Chinese, and then back into English, the final output produced was: "Blind idiot"... An alternative version of the story brings another Babylonian machine built to translate English into Russian, and vice-versa. To test it, the phrase "The flesh is weak, but the spirit is willing" was suggested and fed into it. It was then translated into Russian and back into English, using the latest technological advances on universal grammar, HPSG, GB, and context-sensitivity. The output produced was: "The meat is rotten, but the vodka is good". Yes, Babel is a reality. So far so good for automated translation.

Now, for some examples from the field of logic, the paper PTSurvey considers next some usual abstract definitions of the very notion a logical system. SCT (single-conclusion tarskian) and MCT (multiple-conclusion tarskian) consequence relations are characterized there both abstractly and semantically. In fact, there are traditional adequacy results that prove the equivalence of those two characterizations, in general: Every /SCT/MCT/ consequence relation characterized semantically also respects the abstract clauses defining an /SCT/MCT/ consequence relation; conversely, every logic respecting the appropriate abstract clauses can also be characterized semantically (check [33], but also [30] and [32]). The first result is easy and I leave it as an exercise. The canonical construction employed in the proof of the second result is shown in the paper to make use of a very specific PTS for each /SCT/MCT/ abstract logic. This PTS is simple and is based on a collection of many-valued factors; I also show (applying an idea from Suszko, in [31]) how they can all be reduced to factors that are at most 2-valued. So, to be sure, every /SCT/MCT/ logic is shown after all to have adequate PTS based on many-valued or on 2-valued factors.

Several degenerate examples of logics and of translations are also provided. Some more specific classes of semantics are mentioned as particular cases of PTS, but the demonstration of that claim is left for a future version of the paper. Keep your eyes open, if you're not a fool. And don't drink too much, or else you might miss the churrasco.

How much is that in 'real money'?

While the preceding paper was quite general and abstract, the next paper, WeakPTS (cf. [25]) gets much more down to earth, and provides several examples of PTS as applied to some of the weakest among the **LFI**s from **Chapter 1.0** as well as to some other very weak paraconsistent logics deprived of a consistency connective. Nine paraconsistent logics, six of them **LFI**s, are here split with the help of PTS based on a common set of 3-valued matrices, varying only the set of admissible translations so as to suit the case of each logic. These logics are this time introduced directly in terms of the axioms governing their sets of admissible non-truth-functional bivaluations. Special attention should be paid to the logic **mCi**, suggested at the final section of [18] but here axiomatized for the first time. Its basic intuition is that formulas preceded with a consistency connective should 'behave classically'.

While the move from a logic to another, in this paper, can usually be made by adding or erasing a few axioms, the difference between my presentation of **mCi** and of its extension **Ci** is more remarkable: While the former uses an infinite number of axioms of a certain format, the latter uses only a finite number of them, for the other ones turn then to be derivable. The underlying idea is the following. All of our current Logics of Formal Inconsistency are based on classical logic and extend the weak non-gently explosive logic PI. Moreover, all of them, as we have seen in the previous chapter, can define a classical negation —this was shown there for the case of **bC**, but the same definition given in Theorem 3.48 of the Taxonomy works equally well for **mbC**, as I point out in WeakPTS. Now, if $\div$ denotes this classical negation and $\sim$ denotes here the primitive paraconsistent negation, an inconsistency connective $\bullet$ that behaves as dual to the primitive consistency connective $\circ$ can always be defined simply by setting $\bullet\alpha \stackrel{\text{def}}{=} \div\circ\alpha$. What the logic **mCi** does in extending the logic **mbC**, and what the logic **Ci** does in extending the logic **bC**, is exactly guaranteeing that this definition can alternatively be written as $\bullet\alpha \stackrel{\text{def}}{=} \sim\circ\alpha$. Once the logic **mCi** has a weaker control over the paraconsistent negation than the logic **Ci**, given that only the latter allows for $\sim\sim$-elimination, an infinite number of axioms came on handy in this paper in order to guarantee the fine interaction of $\sim$ with $\circ$.

In the WeakPTS, sequent-style formulations of all the above mentioned logics are offered from the start. Sequent systems for paraconsistent logics originated from the 'Brazilian school' approach are known at least since [28], and they received a new impulse as some of the most traditional **C**-systems and some variations on them were endowed with adequate sequent-style formulations in [3, 4, 5]. The connections between sequent systems and bivaluations are well-known (cf. [6]), and I do not go here into the trouble of proving the equivalence between these two forms of presentation for the above logics. My paper *does* show in some detail, however, how to prove the equivalence between the presentations of those logics in terms of bivaluations and in terms of the proposed PTS. To that effect, the use of a non-canonical measure of complexity of the formulas is helpful, as it was done in [9, 10].

A traditional key to proving that two semantics 'do the same job' consists in building a sort of bisimulation between them, showing that a model from one semantics can be simulated by a model from the other semantics, and vice-versa. On the one hand, bivaluational models are defined by attributing the value 1 ('true') or the value 0 ('false') to each formula of the language. On the other hand, we can understand a PTS-model as a pair consisting of a translation into a factor logic together with a model from that factor. As usual, the more models you have, the less inferences and theses your logic is likely to validate. Intuitively, to prove soundness you have thus to make sure that you do not have 'too many models', not to fail validating something that should be validated. To prove completeness you ought, conversely, to have a sufficient number of models, so as not to 'validate too many things'. Our 'convenience' result, following an idea from [13], shows that the set of bivaluational models, in each case, can simulate the corresponding set of PTS-models. Soundness is a corollary to that. The 'representability' result does the converse and completeness follows as a corollary. Now, this is the only really delicate point: Given a bivaluation, the choice of the simulating translation from among the admissible alternatives is not always obvious. I show in the paper how it can be done in each case, and I leave the rest of the easy but long inductive proofs on the reader's charge. There is no real novelty: In [23] I have illustrated such sort of proofs and their heuristics in painstaking detail.

Given that each particular formula of our paraconsistent logics will originate a finite number of translations into the corresponding factors, and given that those factors are 3-valued, it should be clear to everybody how the above mentioned PTS provide decision procedures for those 9 logics. Once the present paper did not illustrate the procedures in any detail, however, I will briefly do that in what follows, before closing this subsection, so as not to leave any doubt as to how they work. (You can safely jump the forthcoming ramblings if you have already fully understood the methods involved.)

As in our [9], let's consider here a metalinguistic equational logic in which the symbol ',' represents an '... and...', '|' represents an '... or...', '$\to$' rep-

line	p	$\sim p$	$\sim\sim p$	$\sim\sim p \supset p$	$p \supset \sim\sim p$
1	0	~~0~~	...	...	...
2		1	0	1	1
3			~~1~~	...	...
4	1	0	~~0~~	...	...
5			1	1	1
6		1	0	1	0
7			1	1	1

Figure 1: Illustration of quasi matrices.

resents an 'if... then...', and '$\leftrightarrow$' represents an '... if and only if...'. If one now takes the set of all bivaluation mappings $b : \mathcal{S}_{\mathbf{CPL}} \longrightarrow \{0, 1\}$ such that:

(b1.1) $\quad b(\alpha) = 1, b(\beta) = 1 \quad \leftrightarrow \quad b(\alpha \wedge \beta) = 1$

(b1.2) $\quad b(\alpha) = 1 \mid b(\beta) = 1 \quad \leftrightarrow \quad b(\alpha \vee \beta) = 1$

(b1.3) $\quad b(\alpha) = 0 \mid b(\beta) = 1 \quad \leftrightarrow \quad b(\alpha \supset \beta) = 1$

(b2**c**) $\quad b(\alpha) = 0 \quad \leftrightarrow \quad b(\sim\alpha) = 1$

then one obtains an adequate semantic characterization for classical propositional logic (**CPL**). It is easy to tinker with the above axioms on bivaluations, thus defining new logics instead of **CPL**. For instance, as we can see in the present paper, the logic C_{min} (a.k.a. PIf) is obtained if one just drops (b2) and puts the following bivaluational axioms in its place:

(b2) $\quad b(\sim\alpha) = 0 \quad \rightarrow \quad b(\alpha) = 1$

(b6) $\quad b(\sim\sim\alpha) = 1 \quad \rightarrow \quad b(\alpha) = 1$

While the bivaluations of **CPL** determine a well-known decision procedure by way of 2-valued matrices, this time another decision procedure can still be obtained for PIf by way of 2-valued 'quasi matrices'. In a sense, it all works pretty much as if we started writing every possible attribution of the truth-values 0 and 1 to the subformulas of a given formula, following its canonical complexity measure, but then we erased each attribution that disrespected the above bivaluational axioms.

In practice, suppose we would like to test the validity in PIf of the formulas $\sim\sim p \supset p$ and $p \supset \sim\sim p$. Then we would get something like in Figure 1. Lines 1 and 4 of Figure 1 are erased in consideration of the bivaluational axiom (b2), and line 3 is erased in consideration of (b6). We see that $\sim\sim p \supset p$ is a tautology of PIf given that all remaining lines of the quasi matrix satisfy this formula. On the other hand, $p \supset \sim\sim p$ is not satisfied by line 6.

Now, in case a further bivaluational axiom is added such as:

(b6$^{\mathbf{r}}$) $\quad b(\sim\sim\alpha) = 0 \quad \rightarrow \quad b(\alpha) = 0$

then the resulting semantics characterizes the logic $PIfe$, according to the present paper. Notice that now the line 6 of the quasi matrix from Figure 1 will be erased in consideration of (b6$^{\mathbf{r}}$), so that $p \supset \sim\sim p$ turns to be a tautology of $PIfe$.

Of course, to show that the above sketched decision procedures really work in the general case, one has to show that every possible bivaluation of PIf is thereby represented, and only those bivaluations are so represented, that is: (i) each bivaluation is simulated by a line of a quasi matrix (one does not erase more lines than needed); (ii) each line of a quasi matrix can be extended into a bivaluation that simulates it. I will here leave that proof as an exercise and move on instead to show how the decision procedure for the corresponding PTS works. This time the formulas of the logics in focus, in a sense, 'lose their individuality' and start to mean the same as the 'sum of all their translations.'

1	2	3	4	5	6	7
p	$\sim_1 p$	$\sim_2 p$	$\sim_1\sim_1 p$	$\sim_1\sim_2 p$	$\sim_2\sim_1 p$	$\sim_2\sim_2 p$
F	T	T	F	F	F	F
t	F	t	T	F	T	t
T	F	F	T	T	T	T

1	8	9	10	11
p	$\sim_1\sim_1 p \supset p$	$\sim_1\sim_2 p \supset p$	$\sim_2\sim_1 p \supset p$	$\sim_2\sim_2 p \supset p$
F	t	t	t	t
t	t	t	t	t
T	t	t	t	t

1	12	13	14	15
p	$p \supset \sim_1\sim_1 p$	$p \supset \sim_1\sim_2 p$	$p \supset \sim_2\sim_1 p$	$p \supset \sim_2\sim_2 p$
F	t	t	t	t
t	t	F	t	t
T	t	t	t	t

Figure 2: Illustration of a PTS-decision procedure.

Have a look at Figure 2. Notice that the validity of $\sim\sim p \supset p$ already in PIf is corroborated if you look at all lines of translations **8**–**11**. However, the second line of the translation **13** in PIf shows a counter-model for the formula $p \supset \sim\sim p$. This counter-model is no longer allowed in case you turn your eyes to $PIfe$, as the set of possible translations for this logic does not include those translations that produce columns **5**, **6**, **9**, **10**, **13** and **14**. As a byproduct of the 'bisimulative' proofs of the above mentioned convenience and representability results in the paper, the reader can see that there is a transformation taking each line and each translation (that is, a partial PTS-model) of a PTS-decision procedure into a corresponding line of a quasi matrix (that is, a partial bivaluational model), and, conversely, a (usually non-surjective) transformation taking each line of a quasi matrix into a line and a translation of a PTS-decision procedure. These transformations were also discussed in detail in section 2.3.3.7 of [23].

Não tem tradução

Say that a PTR/PTS has a *fixed vocabulary* in case all of its factors are identical —you might well consider a set of different translations from a source logic having the same logic as target. Each translation still provides you, in principle, with a different *scenario* for the evaluation of your original logic. Recall from PTSurvey (**Chapter 2.1**) that a semantics is called *unitary* in case its set of bivaluations or its set of translations is a singleton, and a semantics is called *large* in case it contains at least as many valuations or factors as formulas of the underlying language of the logic being interpreted. Moreover, following [19], call a translation *literal* in case it leaves atomic sentences unaltered, and call it *grammatical* in case it takes each connective of the source logic into a 'homonymous' connective of the target logic, that is, in case it is based on convenient homomorphisms between the underlying algebras of formulas.

From PTSurvey, we know that each valuation by itself determines a logic (based on a unitary semantics). So, classical logic, for instance, can alternatively be characterized by a simple PTS $\langle \mathsf{Log}, \mathsf{Tr}\rangle$ with a fixed vocabulary $\langle \mathcal{S}_k, \vDash_k\rangle$ such that: (a) every $\mathcal{S}_k = \mathsf{Alg}(\{\top,\bot\}, \sim, \wedge, \vee, \supset)$, where Alg is the algebra freely generated by the binary symbols $\sim, \wedge, \vee, \supset$ over the carrier $\{\top,\bot\}$, with all symbols interpreted as in classical logic, and $\vDash_k$ is defined accordingly; (b) $\mathsf{Tr} = \{t_j : \mathcal{S} \to \mathcal{S}_k\}_{j\in J}$ is the set of all mappings t_j such that:

$t_j(p) \in \{\top,\bot\}$, for p atomic,

$t_j({\sim}\alpha) = {\sim}t_j(\alpha)$,

$t_j(\alpha \bowtie \beta) = t_j(\alpha) \bowtie t_j(\beta)$, for $\bowtie \in \{\wedge, \vee, \supset\}$.

Notice that atomic sentences are thereby translated into constants, or 0-ary connectives. It is easy to see that the above structure provides a fixed vocabulary and a set of non-literal grammatical translations that characterizes a large adequate PTS for **CPL**, alternative to the more usual set of bivaluations presented in the last subsection. Such PTS is not that terribly interesting, but it does provide a characterization for **CPL** in terms of a factor that contains no atomic sentences, so that the talk about 'propositions' in classical logic turns to be just a *façon de parler*, nothing deeper than that. The above structure also exemplifies the kind of construction that stems from the general adequacy results from the first paper of this chapter.

More interestingly, in WeakPTS (**Chapter 2.2**), a few large PTS with fixed vocabularies and based on a collection of 'informative' literal and grammatical translations were shown to adequately *split* a number of non-finitely-valued paraconsistent logics into 3-valued scenarios. Many more illustrations of that same phenomenon were exhibited in the last few years, as applied to much more complicated paraconsistent logics (cf. [23, 13]). But there is more. As it was shown in [23, 16], our PTS can also be used, for instance, to *splice* a new logic as the deductive limit of a sequence of other logics (for that effect we might let the vocabulary vary over the sequence and take the identity mappings as translations). Such a 'limit logic' might in fact happen to be quite strange (not compact, for instance) and difficult to characterize by other means than a PTS. As pointed out in the **Errata** to the previous chapter of this thesis, in the case of the original sequence of daCostian logics, C_n, $1 \leq n < \omega$, its deductive limit C_{Lim} provided us in fact with an example of an **LFI** that is not a **C**-system, given that consistency is not characterizable in this logic by a single unary connective, nor by any finite set of unary connectives.

If we recall that we are talking here about a certain way of *combining logics*, the immediate question as to which properties *transfer* from the factors into the logic defined by the PTR/PTS can be raised (cf. [29]). That is not easy to answer, though, if you consider the generality of our definitions (and they can be made more general, for instance, if you just change the underlying formalism or if you start considering logics as richer structures such as Pi-institutions instead of those simpler structures based on arbitrary sets of formulas and consequence relations of a certain sort). Given our current definition of a translation, at least soundness is sure to be guaranteed for the source logic of a PTR/PTS. Some other transference results can be investigated, in particular cases. For example, in the specific case of PTS based on finite-valued factors and recursive translations that produce a limited number of possible interpretations for each formula, the resulting structures were shown in the above subsection to preserve decidability. On the other hand, even for the case of identity translations, a non-fixed vocabulary of non-finitely-valued logics was able to produce, as we saw just above, a counter-example to the preservation of compacity. There is a whole area or research here still wide open for exploration.

But there *is* more (and this point is really worth emphasizing, in case one might still retain the wrong idea): Despite the circumstantial fact that PTS have been used most of the time up to now to provide adequate semantics for some rather recalcitrant logics, the scope of application of this tool is surely not limited to paraconsistent logics plus some not very informative examples such as the one of **CPL** at the beginning of the present subsection. In [16], for instance, the construction was dualized so as to apply to paracomplete logics as well. In [23, 22] similar characterizations were used for providing non-simple non-literal PTS for splitting many-valued logics of all sorts as combinations of 2-valued factors. Many other applications however can be imagined, for less exotic logics, and they *should* be investigated by competent researchers. In [20] the idea of using a sort of PTR for producing answers to 'general set-representation problems for algebras' is put forward. Indeed, if a logic has no usual algebraization in the sense of Blok-Pigozzi (cf. [7]), can we use an adequate PTR so as to split its algebraization problem in terms of the algebraization problems of simpler factors of it? Some initial investigations in that direction were reported in [8]. Will this alternative generalized style of algebraization help us solve interesting problems and prove some interesting new bridge theorems between logic and algebra? This line seems worth investigating. And similarly for proof-theoretic presentations: Will our PTRs help providing interesting new hypersequent-style or labelled tableaux for otherwise unruly logics? Some results in that direction were already reported in [2] for a particular class of PTS called 'non-deterministic semantics'. This line of investigation also definitively seems worth pursuing. As a matter of fact, the present version of WeakPTS does already hint at a procedure according to which any non-deterministic semantics could be recast in terms of a possible-translations semantics. In a recent paper, [1], Avron shows how our logics **bC** and **Ci** can be endowed with 3-valued non-deterministic semantics, and how the logic **Cila** (da Costa's C_1) cannot be given a finite-valued non-deterministic semantics, but only an infinite-valued one —with a 'locality property' that guarantees that it has an appropriate associated decision procedure. This clearly contrasts with the situation in possible-translations semantics, where 3-valued factors are known to be enough for characterizing **Cila** (cf. [23]). The relations between these two genres of semantics should still be studied with due care.

Brief history

The paper PTSurvey started as a research note (cf. [26]) on May 2003. Though the term 'possible-translations semantics' (PTS) had appeared as early as 1990 (cf. [12]), and my Master's Thesis was dedicated to the theme in 1999, none of the papers published on the topic so far had offered a clear-cut general definition of the term, and no real study had been made of the very scope of application of the PTS. My note aimed at filling those blanks, for the benefit of newcomers to the scene, whose first question would invariably be: But what *is* a PTS, after all?

On what concerns the general definition, I had been trying my hands on it since 1998, and I gave some talks on my evolving view of that topic at a few venues, on invitation: at the State University of Campinas (BR) in March 1999, at the occasion of a scientific visit of our ProBrAl German partners to our group; at the Rand Africaans University, in Johannesburg (ZA) in December 1999; at the Max Planck Institute in Saarbrücken (DE) in October 2000; at the University of Ghent (BE) in March 2001; at the University of Łódz (PL) in September 2002. On what concerns its scope of application, by the beginning of 2003 I was convinced that multiple-conclusion consequence relations and abstract logics should be used as the underlying framework for that study, for the sake of generality and symmetry. I had been working on multiple-conclusion consequence relations since 2002, and my interest on abstract logics had been on the increase since much longer —furthermore, I had just participated at the end of 2002 (while I was living in Brazil for a while, working under a CNPq doctoral grant) of the research and writing of the papers [10, 9], which expanded on an earlier draft of mine (cf. [22]) and relied strongly on that abstract sort of approach. As it was revealed in the paper [17] and as it had been explored in categorial terms by Carnielli & Coniglio in [14], our intuition was that the notion of a PTS could be seen as a way of combining logics —or, even better, a way of 'discombining' them. To describe that opposition we had coined in 1999 the terms 'splicing logics' and 'splitting logics'.

Portugal was by then probably the best place in the world for combining-logicians. After two short scientific visits to the Center for Logic and Computation (CLC) of the IST in January 2001 and January 2002, I was invited to come and work there as a student member of the CLC for a while, and that's what I did, from March 2003 on, under an FCT doctoral grant. A workshop on combination of logics (CombLog'04) was being organized in Lisbon for July 2004, thus I decided to submit an improved version of my PTS-note to it, and this contribution would in fact be accepted and published there as an extended abstract (cf. [24]) a few months later. The present version of the material, in this thesis, is a variation of that extended abstract, after the correction of a few inaccuracies and the addition of the proofs of all main claims.

The research report WeakPTS was written in Portugal around October 2003. We had all these new paraconsistent logics sprouting from **Chapter 1.0**, and we had the intuition that they could in general be endowed with adequate PTS with 3-valued factors, as it had been done earlier with some stronger samples among these logics, in [23, 13, 16]. We were also badly in need of characterizing beyond any doubt, syntactically and semantically, the weaker Logics of Formal of Inconsistency that we had created, such as **mbC**, **mCi**, **bC** and **Ci**. Both tasks were elegantly accomplished in the above mentioned report, whose results were heavily used in [15]. Those results were polished and corrected along the subsequent months until the present version of the research report, from November 2004. In helping me spot the mistakes I am much obliged to the unfailing interventions of Marcelo Coniglio.

The continuous support I received from Walter Carnielli and Carlos Caleiro, and the heated debates (plus the pingponging) I had with Arnon Avron at the Dagstuhl Castle (DE) in June 2003 and at the IST (PT) in March 2004 were also nothing less than essential to the design of the current version of this chapter.

Bibliography

[1] Arnon Avron. Non-deterministic semantics for paraconsistent **C**-systems. Submitted for publication, 2004.
`http://antares.math.tau.ac.il/∼aa/articles/c-systems.pdf`.

[2] Arnon Avron and Beata Konikowska. Proof systems for logics based on non-deterministic multiple-valued structures. Submitted for publication, 2004.
`http://antares.math.tau.ac.il/∼aa/articles/proof-nmatrices.pdf`.

[3] Jean-Yves Béziau. Calcul des séquents pour logique non-alèthique. *Logique et Analyse (N.S.)*, 32(125/126):143–155, 1989.

[4] Jean-Yves Béziau. Logiques construites suivant les méthodes de da Costa. I. Logiques paraconsistantes, paracompletes, non-alèthiques construites suivant la première méthode de da Costa. *Logique et Analyse (N.S.)*, 33(131/132):259–272, 1990.

[5] Jean-Yves Béziau. Nouveaux résultats et nouveau regard sur la logique paraconsistante C_1. *Logique et Analyse (N.S.)*, 36(141/142):45–58, 1993.

[6] Jean-Yves Béziau. Sequents and bivaluations. *Logique et Analyse (N.S.)*, 44(176):373–394, 2001.

[7] Willem J. Blok and Don Pigozzi. Algebraizable Logics. *Memoirs of the American Mathematical Society*, 396, 1989.

[8] Juliana Bueno, Marcelo E. Coniglio, and Walter A. Carnielli. Finite algebraizability via possible-translations semantics. In Carnielli et al. [11], pages 79–85.
`http://wslc.math.ist.utl.pt/comblog04/abstracts/bueno.pdf`.

[9] Carlos Caleiro, Walter A. Carnielli, Marcelo E. Coniglio, and João Marcos. Dyadic semantics for many-valued logics. Research report, CLC, Department of Mathematics, Instituto Superior Técnico, 1049-001 Lisbon, PT, 2003. Presented at the III World Congress on Paraconsistency, Toulouse, FR, July 28–31, 2003.
`http://wslc.math.ist.utl.pt/ftp/pub/CaleiroC/03-CCCM-dyadic2.pdf`.

[10] Carlos Caleiro, Walter A. Carnielli, Marcelo E. Coniglio, and João Marcos. Suszko's Thesis and dyadic semantics. Research report, CLC, Department of Mathematics, Instituto Superior Técnico, 1049-001 Lisbon, PT, 2003. Presented at the III World Congress on Paraconsistency, Toulouse, FR, July 28–31, 2003.
`http://wslc.math.ist.utl.pt/ftp/pub/CaleiroC/03-CCCM-dyadic1.pdf`.

[11] W. A. Carnielli, F. M. Dionísio, and P. Mateus, editors. *Proceedings of the Workshop on Combination of Logics: Theory and applications* (CombLog'04), held in Lisbon, PT, 28–30 July 2004. Departamento de Matemática, Instituto Superior Técnico, 2004.

[12] Walter Carnielli. Many-valued logics and plausible reasoning. In *Proceedings of the* XX *International Congress on Many-Valued Logics,* held at the University of Charlotte / NC, US, 1990, pages 328–335. IEEE Computer Society, 1990.

[13] Walter A. Carnielli. Possible-translations semantics for paraconsistent logics. In D. Batens, C. Mortensen, G. Priest, and J. P. Van Bendegem, editors, *Frontiers of Paraconsistent Logic,* Proceedings of the I World Congress on Paraconsistency, held in Ghent, BE, July 29–August 3, 1997, pages 149–163. Research Studies Press, Baldock, 2000.

[14] Walter A. Carnielli and Marcelo E. Coniglio. A categorial approach to the combination of logics. *Manuscrito—Revista Internacional de Filosofia*, XXII(2):69–94, 1999.

[15] Walter A. Carnielli, Marcelo E. Coniglio, and João Marcos. Logics of Formal Inconsistency. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic*, volume 14. Kluwer Academic Publishers, 2nd edition, 2004. In print. Preprint available at:
`http://wslc.math.ist.utl.pt/ftp/pub/MarcosJ/03-CCM-lfi.pdf`.

[16] Walter A. Carnielli and João Marcos. Limits for paraconsistent calculi. *Notre Dame Journal of Formal Logic*, 40(3):375–390, 1999.
`http://projecteuclid.org/Dienst/UI/1.0/Display/euclid.ndjfl/1022615617`.

[17] Walter A. Carnielli and João Marcos. *Ex contradictione non sequitur quodlibet.* In R. L. Epstein, editor, *Proceedings of the* II *Annual Conference on Reasoning and Logic*, held in Bucharest, RO, July 2000, volume 1, pages 89–109. Advanced Reasoning Forum, 2001.
`http://www.advancedreasoningforum.org/Journal-BARK/V1TOC/v1toc.html`.

[18] Walter A. Carnielli and João Marcos. A taxonomy of **C**-systems. In W. A. Carnielli, M. E. Coniglio, and I. M. L. D'Ottaviano, editors, *Paraconsistency: The Logical Way to the Inconsistent,* Proceedings of the II World Congress on Paraconsistency, held in Juquehy, BR, May 8–12, 2000, volume 228 of *Lecture Notes in Pure and Applied Mathematics*, pages 1–94. Marcel Dekker, 2002. Preprint available at:
`http://www.cle.unicamp.br/e-prints/abstract_5.htm`.

[19] Richard L. Epstein. *Propositional Logics: The semantic foundations of logic.* Wadsworth-Thomson Learning, 2000.

[20] Dov M. Gabbay. What is a logical system? In D. M. Gabbay, editor, *What is a Logical System?*, volume 4 of *Studies in Logic and Computation*, pages 179–216. Oxford University Press, New York, 1994.

[21] Douglas R. Hofstadter. *Le Ton Beau de Marot: In praise of the music of language.* Basic Books, 1998.

[22] João Marcos. Many values, many semantics. Draft, 2000.

[23] João Marcos. Possible-Translations Semantics (in Portuguese). Master's thesis, State University of Campinas, BR, 1999.
`http://www.cle.unicamp.br/students/J.Marcos/index.htm`.

[24] João Marcos. Possible-translations semantics. In Carnielli et al. [11], pages 119–128. Extended version available at:
`http://wslc.math.ist.utl.pt/ftp/pub/MarcosJ/04-M-pts.pdf`.

[25] João Marcos. Possible-translations semantics for some weak classically-based paraconsistent logics. Research report, CLC, Department of Mathematics, Instituto Superior Técnico, 1049-001 Lisbon, PT, 2004.
`http://wslc.math.ist.utl.pt/ftp/pub/MarcosJ/04-M-PTS4swcbPL.pdf`.

[26] João Marcos. Every tarskian logic has a possible-translations semantics (plus some sufficient conditions for the validity of the converse). Technical report, CLC / IST, May 23, 2003.

[27] Edgar Allan Poe. *O Corvo e suas Traduções*. Lacerda Editores, Rio de Janeiro, 2000.

[28] Andrés R. Raggio. Propositional sequence-calculi for inconsistent systems. *Notre Dame Journal of Formal Logic*, 9:359–366, 1968.

[29] Amílcar Sernadas and Cristina Sernadas. Combining logic systems: Why, how, what for? *CIM Bulletin*, 15:9–14, December 2003.
`http://wslc.math.ist.utl.pt/ftp/pub/SernadasA/03-SS-fiblog22.pdf`.

[30] D. J. Shoesmith and Timothy J. Smiley. Deducibility and many-valuedness. *The Journal of Symbolic Logic*, 36(4):610–622, 1971.

[31] Roman Suszko. The Fregean axiom and Polish mathematical logic in the 1920's. *Studia Logica*, 36:373–380, 1977.

[32] Ryszard Wójcicki. *Theory of Logical Calculi*. Kluwer, Dordrecht, 1988.

[33] Jan Zygmunt. *An Essay in Matrix Semantics for Consequence Relations*. Wydawnictwo Uniwersytetu Wrocławskiego, Wrocław, 1984.