1. Let \((M, \langle \cdot, \cdot \rangle)\) be a Riemannian manifold with Levi-Civita connection \(\nabla\) and let \(\langle \langle \cdot, \cdot \rangle \rangle = e^{2\rho} \langle \cdot, \cdot \rangle\) be a metric conformally related to \(\langle \cdot, \cdot \rangle\) (where \(\rho \in C^\infty(M)\)). Show that the Levi-Civita connection \(\tilde{\nabla}\) of \(\langle \langle \cdot, \cdot \rangle \rangle\) is given by
\[
\tilde{\nabla}_X Y = \nabla_X Y + d\rho(X)Y + d\rho(Y)X - \langle X, Y \rangle \text{grad } \rho
\]
for all \(X, Y \in \mathfrak{X}(M)\), where the gradient is taken with respect to \(\langle \cdot, \cdot \rangle\). (Hint: Use the Koszul formula).

2. Prove that a curve \(c : I \subset \mathbb{R} \to M\) is a reparameterized geodesic of a Riemannian manifold \((M, \langle \cdot, \cdot \rangle)\) if and only if it satisfies
\[
\frac{D\dot{c}}{dt} = f(t) \dot{c}
\]
for some differentiable function \(f : I \to \mathbb{R}\).

3. Recall that the hyperbolic plane is the upper half plane
\[
H = \{ (x, y) \in \mathbb{R}^2 \mid y > 0 \}
\]
with the Riemannian metric
\[
\langle \cdot, \cdot \rangle = \frac{1}{y^2} (dx \otimes dx + dy \otimes dy)
\]
Use the local coordinate expression of Newton’s equation to compute the Christoffel symbols for the Levi-Civita connection of \((H, \langle \cdot, \cdot \rangle)\) in the coordinates \((x, y)\).