Geometric Mechanics

Homework 9

Due on November 23

1. Consider the symplectic structure on

\[ S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\} \]

determined by the usual volume form. Compute the Hamiltonian flow generated by the function \( H(x, y, z) = z \).

2. Let \((M, \omega)\) be a symplectic manifold. Show that:

(a) \( \omega = \sum_{i=1}^{n} dp_i \wedge dx^i \) if and only if \( \{x^i, x^j\} = \{p_i, p_j\} = 0 \) and \( \{p_i, x^j\} = \delta_{ij} \) for \( i, j = 1, \ldots, n \);
(b) \( M \) is orientable;
(c) If \( M \) is compact then \( \omega \) cannot be exact;
(d) The only sphere that admits a symplectic structure is \( S^2 \).