1^a Ficha de Avaliação de CDI-I; LMAC, MEBiom e MEFT

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Resolução

1) $|9-4x|<|6-x| \Longleftrightarrow |9-4x|^2<|6-x|^2 \Longleftrightarrow$
$\Longleftrightarrow (9-4x)^2<(6-x)^2 \Longleftrightarrow 81-72x+16x^2<36-12x+x^2 \Longleftrightarrow$
$\Longleftrightarrow 15x^2-60x+45<0 \Longleftrightarrow x^2-4x+3<0$.

Ora,
$x^2-4x+3=0 \Longleftrightarrow x=1 \vee x=3$.
Sendo assim,
$x^2-4x+3<0 \Longleftrightarrow x\in\,]1,3[$.
Logo,

$$A=\,]1,3[\ .$$

2) Pretende-se mostrar, por Indução Matemática, que:

$$\forall n\in\mathbb{N}:\sum_{k=1}^{n}(k+1)\,2^{k-1}=n2^n\ .$$

$$n=1:$$

$\displaystyle\sum_{k=1}^{n}(k+1)\,2^{k-1}=\sum_{k=1}^{1}(k+1)\,2^{k-1}=(1+1)\,2^{1-1}=2,$
$n2^n=2$.
Logo, $P(n)$ é satisfeita para $n=1$.

Hipótese de Indução (H.I.): $\displaystyle\underbrace{\sum_{k=1}^{n}(k+1)\,2^{k-1}=n2^n}_{P(n)}$.

Tese de Indução: $\displaystyle\underbrace{\sum_{k=1}^{n+1}(k+1)\,2^{k-1}=(n+1)\,2^{n+1}}_{P(n+1)}$.

Ora,
$\displaystyle\sum_{k=1}^{n+1}(k+1)\,2^{k-1}=\sum_{k=1}^{n}(k+1)\,2^{k-1}+(n+2)\,2^n\underset{\textbf{H.I.}}{=}n2^n+(n+2)\,2^n=$
$=n2^n+n2^n+2^{n+1}=2n2^n+2^{n+1}=n2^{n+1}+2^{n+1}=(n+1)\,2^{n+1}$.

3) $\displaystyle\lim_{x\longrightarrow0^-}\cosh\left(\frac{1}{x}\right)=\lim_{x\longrightarrow0^-}\frac{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}{2}\underset{\lim_{x\longrightarrow0^-}\frac{1}{x}=-\infty}{=}$
$\displaystyle=\frac{1}{2}\left(\lim_{y\longrightarrow-\infty}e^y+\lim_{y\longrightarrow-\infty}e^{-y}\right)=\frac{1}{2}\left(0+(+\infty)\right)=+\infty$.