1^a Ficha de Avaliação de CDI-I; LMAC, MEBiom e MEFT

12/Out/2012-MEFT-v.2

Resolução

1) $|7-2x| > |x+1| \Longleftrightarrow |7-2x|^2 > |x+1|^2 \Longleftrightarrow$
$\Longleftrightarrow (7-2x)^2 > (x+1)^2 \Longleftrightarrow 49 - 28x + 4x^2 > x^2 + 2x + 1 \Longleftrightarrow$
$\Longleftrightarrow 3x^2 - 30x + 48 > 0 \Longleftrightarrow x^2 - 10x + 16 > 0$.

Ora,

$x^2 - 10x + 16 = 0 \Longleftrightarrow x = 2 \vee x = 8$.

Sendo assim,

$x^2 - 10x + 16 > 0 \Longleftrightarrow x \in \left]-\infty, 2\right[\cup \left]8, +\infty\right[$.

Logo,

$$A = \left]-\infty, 2\right[\cup \left]8, +\infty\right[\ .$$

2) Pretende-se mostrar, por Indução Matemática, que:

$$\forall n \in \mathbb{N} : \sum_{k=1}^{n} \frac{k}{2^{k+1}} = 1 - \frac{n+2}{2^{n+1}} \ .$$

$$n = 1 :$$

$\sum_{k=1}^{n} \frac{k}{2^{k+1}} = \sum_{k=1}^{1} \frac{k}{2^{k+1}} = \frac{1}{2^2} = \frac{1}{4}$,

$1 - \frac{n+2}{2^{n+1}} = 1 - \frac{3}{2^2} = \frac{1}{4}$.

Logo, $P(n)$ é satisfeita para $n = 1$.

Hipótese de Indução (H.I.): $\underbrace{\sum_{k=1}^{n} \frac{k}{2^{k+1}} = 1 - \frac{n+2}{2^{n+1}}}_{P(n)}$.

Tese de Indução: $\underbrace{\sum_{k=1}^{n+1} \frac{k}{2^{k+1}} = 1 - \frac{n+3}{2^{n+2}}}_{P(n+1)}$.

Ora,

$\sum_{k=1}^{n+1} \frac{k}{2^{k+1}} = \sum_{k=1}^{n} \frac{k}{2^{k+1}} + \frac{n+1}{2^{n+2}} \underset{\textbf{H.I.}}{=} 1 - \frac{n+2}{2^{n+1}} + \frac{n+1}{2^{n+2}} =$
$= 1 - \frac{2(n+2)-(n+1)}{2^{n+2}} = 1 - \frac{2n+4-n-1}{2^{n+2}} = 1 - \frac{n+3}{2^{n+2}}$.

3) $\lim\limits_{x\longrightarrow 0^-} \sinh\left(\frac{1}{x^2}\right) = \lim\limits_{x\longrightarrow 0^-} \frac{e^{\frac{1}{x^2}} - e^{-\frac{1}{x^2}}}{2} \underset{\lim\limits_{x\longrightarrow 0^-} \frac{1}{x^2} = +\infty}{=}$

$= \frac{1}{2}\left(\lim\limits_{y\longrightarrow +\infty} e^y - \lim\limits_{y\longrightarrow +\infty} e^{-y}\right) = \frac{1}{2}\left(+\infty - 0\right) = +\infty$.