CDI-I

1^a Ficha-1^a Aula Prática

0. Desigualdades e Módulos

1.

1.1 $\frac{x-1}{x+3} < 2 \Longleftrightarrow (x > -3 \wedge x - 1 < 2\,(x+3)) \vee (x < -3 \wedge x - 1 > 2\,(x+3)) \Longleftrightarrow$
$\Longleftrightarrow (x > -3 \wedge x - 1 < 2x + 6) \vee (x < -3 \wedge x - 1 > 2x + 6) \Longleftrightarrow$
$\Longleftrightarrow (x > -3 \wedge -x < 7) \vee (x < -3 \wedge -x > 7) \Longleftrightarrow$
$\Longleftrightarrow (x > -3 \wedge x > -7) \vee (x < -3 \wedge x < -7) \Longleftrightarrow$
$\Longleftrightarrow x > -3 \vee x < -7 \Longleftrightarrow x \in \left]-\infty, -7\right[\cup \left]-3, +\infty\right[.$
Logo,

$$\left\{x \in \mathbb{R} : \frac{x-1}{x+3} < 2\right\} = \left]-\infty, -7\right[\cup \left]-3, +\infty\right[.$$

1.2 $\left\{x \in \mathbb{R} : \frac{x-1}{x+3} \geq 2\right\}$ é o conjunto complementar, em $\mathbb{R}\backslash\{-3\}$, de $\left\{x \in \mathbb{R} : \frac{x-1}{x+3} < 2\right\}$
Logo (ver **1.1**),

$$\left\{x \in \mathbb{R} : \frac{x-1}{x+3} \geq 2\right\} = \left[-7, -3\right[.$$

O estudo da resolução do exercício **1.13** deve ser precedido da leitura do **Apêndice I**.

1.13 $\frac{x-2}{x+3} < 2x \Longleftrightarrow (x > -3 \wedge x - 2 < 2x\,(x+3)) \vee (x < -3 \wedge x - 2 > 2x\,(x+3)) \Longleftrightarrow$
$\Longleftrightarrow \left(x > -3 \wedge x - 2 < 2x^2 + 6x\right) \vee \left(x < -3 \wedge x - 2 > 2x^2 + 6x\right) \Longleftrightarrow$
$\Longleftrightarrow \left(x > -3 \wedge 0 < 2x^2 + 5x + 2\right) \vee \left(x < -3 \wedge 0 > 2x^2 + 5x + 2\right).$
Ora,
$2x^2 + 5x + 2 = 0 \Longleftrightarrow x = \frac{-5-\sqrt{25-16}}{4} \vee x = \frac{-5+\sqrt{25-16}}{4} \Longleftrightarrow$
$\Longleftrightarrow x = -2 \vee x = -\frac{1}{2}.$
Sendo assim,
$\left(x > -3 \wedge 0 < 2x^2 + 5x + 2\right) \vee \left(x < -3 \wedge 0 > 2x^2 + 5x + 2\right) \Longleftrightarrow$
$\Longleftrightarrow \left(x > -3 \wedge x \in \left]-\infty, -2\right[\cup \left]-\frac{1}{2}, +\infty\right[\right) \vee \left(x < -3 \wedge x \in \left]-2, -\frac{1}{2}\right[\right) \Longleftrightarrow$
$\Longleftrightarrow x \in \left]-3, -2\right[\cup \left]-\frac{1}{2}, +\infty\right[.$
Logo,

$$\left\{x \in \mathbb{R} : \frac{x-2}{x+3} < 2x\right\} = \left]-3, -2\right[\cup \left]-\frac{1}{2}, +\infty\right[.$$

1.14 $\left\{x \in \mathbb{R} : \frac{x-2}{x+3} \geq 2x\right\}$ é o conjunto complementar, em $\mathbb{R}\backslash\{-3\}$, de $\left\{x \in \mathbb{R} : \frac{x-2}{x+3} < 2x\right\}$

Logo (ver **1.13**),

$$\left\{x \in \mathbb{R} : \frac{x-2}{x+3} \geq 2x\right\} =]-\infty, -3[\cup \left[-2, -\frac{1}{2}\right].$$

2.

O estudo dos exercícios seguintes deve ser precedido da leitura dos **Apêndices II** e **III**.

2.2 $|x+2| \leq 1 \Longleftrightarrow |x-(-2)| \leq 1 \Longleftrightarrow -2-1 \leq x \leq -2+1 \Longleftrightarrow$
$\Longleftrightarrow x \in [-3, -1]$
Logo,

$$\{x \in \mathbb{R} : |x+2| \leq 1\} = [-3, -1].$$

2.5 $3 < 2\,|x-1| \leq 5 \Longleftrightarrow \frac{3}{2} < |x-1| \leq \frac{5}{2} \Longleftrightarrow$

$\Longleftrightarrow \left(x < 1 - \frac{3}{2} \vee x > 1 + \frac{3}{2}\right) \wedge \left(1 - \frac{5}{2} \leq x \leq 1 + \frac{5}{2}\right) \Longleftrightarrow$

$\Longleftrightarrow x \in \left]-\infty, -\frac{1}{2}\right[\cup \left]\frac{5}{2}, +\infty\right[\wedge x \in \left[-\frac{3}{2}, \frac{7}{2}\right] \Longleftrightarrow$

$\Longleftrightarrow x \in \left(\left]-\infty, -\frac{1}{2}\right[\cup \left]\frac{5}{2}, +\infty\right[\right) \cap \left[-\frac{3}{2}, \frac{7}{2}\right] \Longleftrightarrow$

$\Longleftrightarrow x \in \left(\left]-\infty, -\frac{1}{2}\right[\cap \left[-\frac{3}{2}, \frac{7}{2}\right]\right) \cup \left(\left]\frac{5}{2}, +\infty\right[\cap \left[-\frac{3}{2}, \frac{7}{2}\right]\right) \Longleftrightarrow$

$\Longleftrightarrow x \in \left[-\frac{3}{2}, -\frac{1}{2}\right[\cup \left]\frac{5}{2}, \frac{7}{2}\right]$.
Logo,

$$\{x \in \mathbb{R} : 3 < 2\,|x-1| \leq 5\} = \left[-\frac{3}{2}, -\frac{1}{2}\right[\cup \left]\frac{5}{2}, \frac{7}{2}\right].$$

3.

3.5 $|6x-5| < |1-8x| \Longleftrightarrow |6x-5|^2 < |1-8x|^2 \Longleftrightarrow$
$\Longleftrightarrow (6x-5)^2 < (1-8x)^2 \Longleftrightarrow 36x^2 - 60x + 25 < 1 - 16x + 64x^2 \Longleftrightarrow$
$\Longleftrightarrow 0 < 28x^2 + 44x - 24 \Longleftrightarrow 0 < 7x^2 + 11x - 6$.
Ora,
$7x^2 + 11x - 6 = 0 \Longleftrightarrow x = \frac{-11-\sqrt{121+168}}{14} \vee x = \frac{-11+\sqrt{121+168}}{14} \Longleftrightarrow$
$\Longleftrightarrow x = \frac{-11-17}{14} \vee x = \frac{-11+17}{14} \Longleftrightarrow x = -2 \vee x = \frac{3}{7}$.
Sendo assim,
$0 < 7x^2 + 11x - 6 \Longleftrightarrow x \in]-\infty, -2[\cup \left]\frac{3}{7}, +\infty\right[$.
Logo,

$$\{x \in \mathbb{R} : |6x-5| < |1-8x|\} =]-\infty, -2[\cup \left]\frac{3}{7}, +\infty\right[.$$

3.20 $3|x-2| > |x| \iff 9|x-2|^2 > |x|^2 \iff$
$\iff 9(x-2)^2 > x^2 \iff 9x^2 - 36x + 36 > x^2 \iff$
$\iff 8x^2 - 36x + 36 > 0$.
Ora,
$8x^2 - 36x + 36 = 0 \iff 2x^2 - 9x + 9 = 0 \iff$
$\iff x = \frac{9-\sqrt{81-72}}{4} \vee x = \frac{9+\sqrt{81-72}}{4} \iff$
$\iff x = \frac{9-3}{4} \vee x = \frac{9+3}{4} \iff x = \frac{3}{2} \vee x = 3$.
Sendo assim,
$8x^2 - 36x + 36 > 0 \iff x \in \left]-\infty, \frac{3}{2}\right[\cup]3, +\infty[$.
Logo,

$$\{x \in \mathbb{R} : 3|x-2| > |x|\} = \left]-\infty, \frac{3}{2}\right[\cup]3, +\infty[.$$

4.

4.2 $9 \leq (x-1)^2 < 25 \iff 9 \leq (x-1)^2 \wedge (x-1)^2 < 25 \iff$
$\iff 9 \leq x^2 - 2x + 1 \wedge x^2 - 2x + 1 < 25 \iff$
$\iff 0 \leq x^2 - 2x - 8 \wedge x^2 - 2x - 24 < 0$.
Ora,
$x^2 - 2x - 8 = 0 \iff x = \frac{2-\sqrt{4+32}}{2} \vee x = \frac{2+\sqrt{4+32}}{2} \iff$
$\iff x = \frac{2-6}{2} \vee x = \frac{2+6}{2} \iff x = -2 \vee x = 4$.
$x^2 - 2x - 24 = 0 \iff x = \frac{2-\sqrt{4+96}}{2} \vee x = \frac{2+\sqrt{4+96}}{2} \iff$
$\iff x = \frac{2-10}{2} \vee x = \frac{2+10}{2} \iff x = -4 \vee x = 6$.
Sendo assim,
$0 \leq x^2 - 2x - 8 \wedge x^2 - 2x - 24 < 0 \iff$
$\iff (x \in]-\infty, -2] \cup [4, +\infty[) \wedge (x \in]-4, 6[) \iff$
$\iff x \in (]-\infty, -2] \cup [4, +\infty[) \cap]-4, 6[\iff$
$\iff x \in (]-\infty, -2] \cap]-4, 6[) \cup ([4, +\infty[\cap]-4, 6[) \iff$
$\iff x \in]-4, -2] \cup [4, 6[$.
Logo,

$$\left\{x \in \mathbb{R} : 9 \leq (x-1)^2 < 25\right\} =]-4, -2] \cup [4, 6[.$$

4.7 $\left|x^2 - 2\right| \leq 1 \iff -1 \leq x^2 - 2 \leq 1 \iff$
$\iff -1 \leq x^2 - 2 \wedge x^2 - 2 \leq 1 \iff$
$\iff 0 \leq x^2 - 1 \wedge x^2 - 3 \leq 0$.
Ora,
$x^2 - 1 = 0 \iff x^2 = 1 \iff x = -1 \vee x = 1$.
$x^2 - 3 = 0 \iff x^2 = 3 \iff x = -\sqrt{3} \vee x = \sqrt{3}$.
Sendo assim,
$0 \leq x^2 - 1 \wedge x^2 - 3 \leq 0 \iff$
$\iff (x \in]-\infty, -1] \cup [1, +\infty[) \wedge \left(x \in \left[-\sqrt{3}, \sqrt{3}\right]\right) \iff$
$\iff x \in (]-\infty, -1] \cup [1, +\infty[) \cap \left[-\sqrt{3}, \sqrt{3}\right] \iff$
$\iff x \in \left(]-\infty, -1] \cap \left[-\sqrt{3}, \sqrt{3}\right]\right) \cup \left([1, +\infty[\cap \left[-\sqrt{3}, \sqrt{3}\right]\right) \iff$
$\iff x \in \left[-\sqrt{3}, -1\right] \cup \left[1, \sqrt{3}\right]$.

Logo,

$$\left\{x \in \mathbb{R} : \left|x^2 - 2\right| \leq 1\right\} = \left[-\sqrt{3}, -1\right] \cup \left[1, \sqrt{3}\right].$$

5.

5.2 $1 - 3x \geq 0 \Longleftrightarrow x \leq \frac{1}{3}$.
$1 - 3x < 0 \Longleftrightarrow x > \frac{1}{3}$.
Sendo assim,
$|x\,(x-3)| > |1-3x| \Longleftrightarrow \left(x \leq \frac{1}{3} \wedge |x\,(x-3)| > 1 - 3x\right) \vee$
$\vee \left(x > \frac{1}{3} \wedge |x\,(x-3)| > -1 + 3x\right) \Longleftrightarrow$
$\Longleftrightarrow \left(x \leq \frac{1}{3} \wedge (x\,(x-3) < -1 + 3x \vee x\,(x-3) > 1 - 3x)\right) \vee$
$\vee \left(x > \frac{1}{3} \wedge (x\,(x-3) < 1 - 3x \vee x\,(x-3) > -1 + 3x)\right) \Longleftrightarrow$
$\Longleftrightarrow \left(x \leq \frac{1}{3} \wedge \left(x^2 - 6x + 1 < 0 \vee x^2 - 1 > 0\right)\right) \vee$
$\vee \left(x > \frac{1}{3} \wedge \left(x^2 - 1 < 0 \vee x^2 - 6x + 1 > 0\right)\right)$.
Ora,
$x^2 - 6x + 1 = 0 \Longleftrightarrow x = \frac{6 - \sqrt{36-4}}{2} \vee x = \frac{6 + \sqrt{36-4}}{2} \Longleftrightarrow$
$\Longleftrightarrow x = \frac{6-\sqrt{32}}{2} \vee x = \frac{6+\sqrt{32}}{2} \Longleftrightarrow x = \frac{6-4\sqrt{2}}{2} \vee x = \frac{6+4\sqrt{2}}{2} \Longleftrightarrow$
$\Longleftrightarrow x = 3 - 2\sqrt{2} \vee x = x = 3 + 2\sqrt{2}$.
$x^2 - 1 = 0 \Longleftrightarrow x^2 = 1 \Longleftrightarrow x = -1 \vee x = 1$.
Sendo assim,
$\left(x \leq \frac{1}{3} \wedge \left(x^2 - 6x + 1 < 0 \vee x^2 - 1 > 0\right)\right) \vee$
$\vee \left(x > \frac{1}{3} \wedge \left(x^2 - 1 < 0 \vee x^2 - 6x + 1 > 0\right)\right) \Longleftrightarrow$
$\Longleftrightarrow \left(x \in \left]-\infty, \frac{1}{3}\right] \cap \left(\left]3 - 2\sqrt{2}, 3 + 2\sqrt{2}\right[\cup (]-\infty, -1[\cup]1, +\infty[)\right)\right) \vee$
$\vee \left(x \in \left]\frac{1}{3}, +\infty\right[\cap \left(]-1, 1[\cup \left(]-\infty, 3 - 2\sqrt{2}\right[\cup]3 + 2\sqrt{2}, +\infty[)\right)\right) \Longleftrightarrow$
$\Longleftrightarrow \left(x \in \left]3 - 2\sqrt{2}, \frac{1}{3}\right] \cup]-\infty, -1[\right) \vee$
$\vee \left(x \in \left]\frac{1}{3}, 1\right[\cup \left]3 + 2\sqrt{2}, +\infty\right[\right) \Longleftrightarrow$
$\Longleftrightarrow x \in \left]3 - 2\sqrt{2}, \frac{1}{3}\right] \cup]-\infty, -1[\cup \left]\frac{1}{3}, 1\right[\cup \left]3 + 2\sqrt{2}, +\infty\right[\Longleftrightarrow$
$\Longleftrightarrow x \in \left]3 - 2\sqrt{2}, 1\right[\cup]-\infty, -1[\cup \left]3 + 2\sqrt{2}, +\infty\right[$.
Logo,

$$\{x \in \mathbb{R} : |x\,(x-3)| > |1 - 3x|\} =]-\infty, -1[\cup \left]3 - 2\sqrt{2}, 1\right[\cup \left]3 + 2\sqrt{2}, +\infty\right[.$$