CDI-I

2^a Ficha-6^a Aula Prática

IV. Cálculo de Derivadas de Funções

Convém ler, primeiro, o Apêndice X.

1)

(h) $f'(x) = \left(\frac{x\sin x}{1+x^2}\right)' = \frac{\sin x + x\cos x + x^2\sin x + x^3\cos x - 2x^2\sin x}{(1+x^2)^2} = \frac{\sin x + x\cos x - x^2\sin x + x^3\cos x}{(1+x^2)^2}$

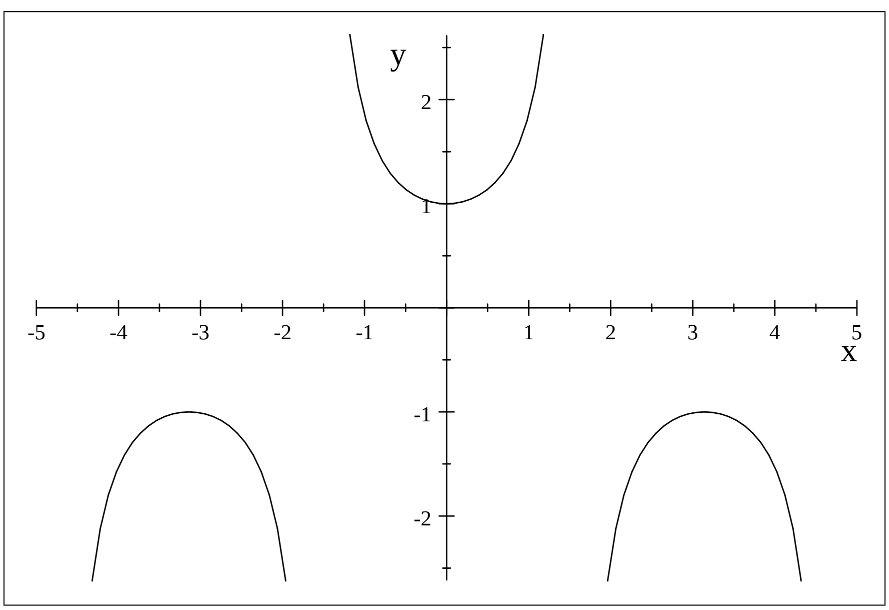
3)

(e) $f'(x) = \left(x^{\frac{1}{3}} + x^{-\frac{1}{4}}\right)' = \left(x^{\frac{1}{3}}\right)' + \left(x^{-\frac{1}{4}}\right)' = \frac{1}{3}x^{\frac{1}{3}-1} - \frac{1}{4}x^{-\frac{1}{4}-1} =$
$= \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{4}x^{-\frac{5}{4}}$.

4)

(f) $f'(x) = \left(\tan^2 x\right)' = 2\tan x . (\tan x)' = 2\tan x . \sec^2 x$.

Relembre-se que a função **secante** é definida por $\sec x = \frac{1}{\cos x}$ e o seu gráfico é o seguinte:



É claro que a função tem período 2π e assimptotas verticais nos pontos $x = k\pi + \frac{\pi}{2}$; $k \in \mathbb{Z}$.

Note-se, ainda, que

$$|\sec x| \geq 1 \ .$$

e, como se deduz facilmente da relação fundamental da trigonometria:

$$\tan^2 x + 1 = \sec^2 x \ .$$

10) Sejam $g : \mathbb{R} \longrightarrow \mathbb{R}$ uma função duas vezes diferenciável e $\phi :]0, +\infty[\longrightarrow \mathbb{R}$ a função definida por $\phi(x) = e^{g(\log x)}$. Pretende-se determinar $\phi'(1)$ e $\phi''(e)$.

Ora,

$\phi'(x) = \left(e^{g(\log x)}\right)' = (g(\log x))' \, e^{g(\log x)} = g'(\log x) \, . \, (\log x)' \, . e^{g(\log x)} =$
$= g'(\log x) \, . \frac{1}{x} . e^{g(\log x)} = \frac{g'(\log x)}{x} e^{g(\log x)}$.

$\phi''(x) = \left(\frac{g'(\log x)}{x} e^{g(\log x)}\right)' = \left(\frac{g'(\log x)}{x}\right)' e^{g(\log x)} + \frac{g'(\log x)}{x} \left(e^{g(\log x)}\right)' =$
$= \frac{g''(\log x) . \frac{1}{x} . x - g'(\log x)}{x^2} e^{g(\log x)} + \frac{g'(\log x)}{x} g'(\log x) \, . \frac{1}{x} . e^{g(\log x)} =$
$= \frac{g''(\log x) - g'(\log x)}{x^2} e^{g(\log x)} + \frac{\left(g'(\log x)\right)^2}{x^2} e^{g(\log x)} =$
$= \frac{g''(\log x) - g'(\log x) + \left(g'(\log x)\right)^2}{x^2} e^{g(\log x)}$.

Logo,

$\phi'(1) = \frac{g'(\log 1)}{1} e^{g(\log 1)} = g'(0) \, e^{g(0)}$.

$\phi''(e) = \frac{g''(\log e) - g'(\log e) + \left(g'(\log e)\right)^2}{e^2} e^{g(\log e)} =$
$= \frac{g''(1) - g'(1) + \left(g'(1)\right)^2}{e^2} e^{g(1)}$.

11)

(a) $f'(x) = \left(\log\left(1 + x^2\right)\right)' = \frac{2x}{1+x^2}$.

(p) $f'(x) = \left(2^{\sqrt{x}}\right)' = 2^{\sqrt{x}} . \log 2 . \left(\sqrt{x}\right)' =$
$= 2^{\sqrt{x}} . \log 2 . \left(x^{\frac{1}{2}}\right)' = 2^{\sqrt{x}} . \log 2 . \frac{1}{2} x^{\frac{1}{2} - 1} =$
$= 2^{\sqrt{x} - 1} x^{-\frac{1}{2}} \log 2$.

(w) $f'(x) = (x^x)' = x x^{x-1} + x^x \log x =$
$= x^x (1 + \log x)$.

V. Teoremas de Rolle, Lagrange e Cauchy

1)

(a) Convém ler, primeiro, o Apêndice XI.

Ora,

$$\lim_{x\longrightarrow 0}\frac{\sinh x-\sin x}{x^3}\underset{\text{Regra de Cauchy}}{=}\lim_{x\longrightarrow 0}\frac{\cosh x-\cos x}{3x^2}=$$

$$\underset{\text{Regra de Cauchy}}{=}\lim_{x\longrightarrow 0}\frac{\sinh x+\sin x}{6x}\underset{\text{Regra de Cauchy}}{=}\lim_{x\longrightarrow 0}\frac{\cosh x+\cos x}{6}=\frac{2}{6}=\frac{1}{3}\ .$$

(o) $$\lim_{x\longrightarrow +\infty}\sin\left(\frac{1}{x}\right)e^x=\lim_{x\longrightarrow +\infty}\frac{\sin\left(\frac{1}{x}\right)}{e^{-x}}\underset{\text{Regra de Cauchy}}{=}\lim_{x\longrightarrow +\infty}\frac{-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)}{-e^{-x}}=$$

$$=\lim_{x\longrightarrow +\infty}\left(\frac{\frac{1}{x^2}}{e^{-x}}\cos\left(\frac{1}{x}\right)\right)=\lim_{x\longrightarrow +\infty}\left(\frac{e^x}{x^2}\cos\left(\frac{1}{x}\right)\right)=\left(\lim_{x\longrightarrow +\infty}\frac{e^x}{x^2}\right)\left(\lim_{x\longrightarrow +\infty}\cos\left(\frac{1}{x}\right)\right)=$$

$$=\left(\lim_{x\longrightarrow +\infty}\frac{e^x}{x^2}\right)\cos(0)=\lim_{x\longrightarrow +\infty}\frac{e^x}{x^2}\underset{\text{Regra de Cauchy}}{=}\lim_{x\longrightarrow +\infty}\frac{e^x}{2x}\underset{\text{Regra de Cauchy}}{=}$$

$$=\lim_{x\longrightarrow +\infty}\frac{e^x}{2}=+\infty\ .$$