**CDI-I-2$^a$ Ficha de Avaliação**

**02/11/2012-MEBiom**

**Versão 1**

**1)**

**a)** $f$ é prolongável por continuidade ao ponto $0 \Longleftrightarrow$
$\Longleftrightarrow \lim\limits_{x \longrightarrow 0^-} f(x) = \lim\limits_{x \longrightarrow 0^+} f(x) \Longleftrightarrow \lim\limits_{x \longrightarrow 0^-} (k-x)(2+x) = \lim\limits_{x \longrightarrow 0^+} 2\cos\left(\frac{\pi}{2+x^2}\right) \Longleftrightarrow$
$\Longleftrightarrow 2k = 2\cos\frac{\pi}{2} \Longleftrightarrow k = 0$ .

**b)** Sendo $F$ o prolongamento por continuidade de $f$ ao ponto $0$ , tem-se:

$$F(x) = \begin{cases} 2\cos\left(\frac{\pi}{2+x^2}\right), \ x > 0 \\ 0, \ x = 0 \\ -x(2+x), \ x < 0 \end{cases}$$

Determinemos o contradomínio de $F$ , $CD_F$ .
Ora,
$x \in ]0, +\infty[ \Longrightarrow 0 < \frac{\pi}{2+x^2} < \frac{\pi}{2} \Longrightarrow F(x) = 2\cos\left(\frac{\pi}{2+x^2}\right) \in ]0, 2[$ ,

$\lim\limits_{x \longrightarrow 0^+} F(x) = \lim\limits_{x \longrightarrow 0^+} 2\cos\left(\frac{\pi}{2+x^2}\right) = 2\cos\frac{\pi}{2} = 0$ ,

$\lim\limits_{x \longrightarrow +\infty} F(x) = \lim\limits_{x \longrightarrow +\infty} 2\cos\left(\frac{\pi}{2+x^2}\right) \underset{\lim\limits_{x \longrightarrow +\infty} \frac{\pi}{2+x^2} = 0}{=} 2\cos 0 = 2$ .

Logo, pelo Teorema de Bolzano:
$F$ assume todos os valores do intervalo $]0, 2[$ , quando $x \in ]0, +\infty[$ .
$F(0) = 0$ .
Por outro lado,
$-x(2+x) = 0 \Longleftrightarrow x = -2 \vee x = 0$ ,

$F\left(\frac{-2+0}{2}\right) = F(-1) = 1(2-1) = 1$ ,

$\lim\limits_{x \longrightarrow -\infty} F(x) = \lim\limits_{x \longrightarrow -\infty} (-x(2+x)) = \lim\limits_{x \longrightarrow -\infty} \left(-x^2 - 2x\right) = -\infty$ .

Logo, pelo Teorema de Bolzano:
$F$ assume todos os valores do intervalo $]-\infty, 1]$ , quando $x \in ]-\infty, 0[$ .
Sendo assim,
$CD_F = ]-\infty, 1] \cup \{0\} \cup ]0, 2[ = ]-\infty, 1[ \cup [0, 2[ = ]-\infty, 2[$ .

**2)**

**(a)** $f'(x) = \left(\frac{\sqrt{x}}{1+x}\right)' = \left(\frac{x^{\frac{1}{2}}}{1+x}\right)' = \frac{\frac{1}{2}x^{\frac{1}{2}-1}(1+x)-x^{\frac{1}{2}}}{(1+x)^2} =$

$= \frac{\frac{1}{2}x^{-\frac{1}{2}}(1+x)-x^{\frac{1}{2}}}{(1+x)^2} = \frac{\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}}{(1+x)^2} = \frac{\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}}{(1+x)^2}$ .

**(b)** $f'(x) = \left(e^{\sin^2 x}\right)' = \left(\sin^2 x\right)' e^{\sin^2 x} = (2\sin x\cos x)\, e^{\sin^2 x} = (\sin 2x)\, e^{\sin^2 x}$.

**3)** $\lim_{x\longrightarrow+\infty} \left(2+x^3\right)^{\frac{1}{\log x}} = \lim_{x\longrightarrow+\infty} e^{\frac{1}{\log x}\log\left(2+x^3\right)} = \lim_{x\longrightarrow+\infty} e^{\frac{\log\left(2+x^3\right)}{\log x}}$ .

Ora,

$\lim_{x\longrightarrow+\infty} \frac{\log\left(2+x^3\right)}{\log x} \underset{\text{Regra de Cauchy}}{=} \lim_{x\longrightarrow+\infty} \frac{\frac{3x^2}{2+x^3}}{\frac{1}{x}} = \lim_{x\longrightarrow+\infty} \frac{3x^3}{2+x^3} =$

$= \lim_{x\longrightarrow+\infty} \frac{3}{\frac{2}{x^3}+1} = 3$ .

Logo,

$\lim_{x\longrightarrow+\infty} e^{\frac{\log\left(2+x^3\right)}{\log x}} = \lim_{y\longrightarrow 3} e^y = e^3$ .

**4)** $f$ é contínua no intervalo limitado e fechado $[0,b]$ ; logo, pelo Teorema de Weierstrass, $f$ tem máximo, quando $x \in [0,b]$ .

Seja

$$M = \max_{x\in[0,b]} f(x) \ .$$

Ora,

$$\forall x \in \,]b,+\infty[\, : f(x) < f(0) \leq M \ .$$

Logo,

$$M = \max_{x\in[0,+\infty[} f(x) \ .$$