CDI-I-2^a Ficha de Avaliação

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Versão 1

1)

a) f é prolongável por continuidade ao ponto $0 \Longleftrightarrow$

$$\Longleftrightarrow \lim_{x \longrightarrow 0^-} f(x) = \lim_{x \longrightarrow 0^+} f(x) \Longleftrightarrow \lim_{x \longrightarrow 0^-} (k-x)(2+x) = \lim_{x \longrightarrow 0^+} 2\cos\left(\frac{\pi}{2+x^2}\right) \Longleftrightarrow$$

$$\Longleftrightarrow 2k = 2\cos\frac{\pi}{2} \Longleftrightarrow k = 0 \ .$$

b) Sendo F o prolongamento por continuidade de f ao ponto 0 , tem-se:

$$F(x) = \begin{cases} 2\cos\left(\frac{\pi}{2+x^2}\right), \ x > 0 \\ 0, \ x = 0 \\ -x(2+x), \ x < 0 \end{cases}$$

Determinemos o contradomínio de F , CD_F .
Ora,
$x \in \,]0, +\infty[\Longrightarrow 0 < \frac{\pi}{2+x^2} < \frac{\pi}{2} \Longrightarrow F(x) = 2\cos\left(\frac{\pi}{2+x^2}\right) \in \,]0, 2[$,

$\lim_{x \longrightarrow 0^+} F(x) = \lim_{x \longrightarrow 0^+} 2\cos\left(\frac{\pi}{2+x^2}\right) = 2\cos\frac{\pi}{2} = 0$,

$\lim_{x \longrightarrow +\infty} F(x) = \lim_{x \longrightarrow +\infty} 2\cos\left(\frac{\pi}{2+x^2}\right) \underset{\lim_{x \longrightarrow +\infty} \frac{\pi}{2+x^2} = 0}{=} 2\cos 0 = 2$.

Logo, pelo Teorema de Bolzano:
F assume todos os valores do intervalo $]0, 2[$, quando $x \in \,]0, +\infty[$.
$F(0) = 0$.
Por outro lado,
$-x(2+x) = 0 \Longleftrightarrow x = -2 \vee x = 0$,

$F\left(\frac{-2+0}{2}\right) = F(-1) = 1(2-1) = 1$,

$\lim_{x \longrightarrow -\infty} F(x) = \lim_{x \longrightarrow -\infty} (-x(2+x)) = \lim_{x \longrightarrow -\infty} \left(-x^2 - 2x\right) = -\infty$.

Logo, pelo Teorema de Bolzano:
F assume todos os valores do intervalo $]-\infty, 1]$, quando $x \in \,]-\infty, 0[$.
Sendo assim,
$CD_F = \,]-\infty, 1] \cup \{0\} \cup \,]0, 2[\, = \,]-\infty, 1[\cup [0, 2[\, = \,]-\infty, 2[$.

2)

(a) $f'(x) = \left(\frac{1}{1+\sqrt{x}}\right)' = \left(\frac{1}{1+x^{\frac{1}{2}}}\right)' = \frac{-\frac{1}{2}x^{\frac{1}{2}-1}}{\left(1+x^{\frac{1}{2}}\right)^2} =$

$= \frac{-\frac{1}{2}x^{-\frac{1}{2}}}{\left(1+x^{\frac{1}{2}}\right)^2} = -\frac{1}{2\sqrt{x}\left(1+\sqrt{x}\right)^2}$.

(b) $f'(x) = \left(xe^{x^2}\right)' = e^{x^2} + x.2xe^{x^2} = e^{x^2}\left(1+2x^2\right).$

3) $\lim\limits_{x\longrightarrow 0^+} (\sin x)^{\frac{1}{\log x}} = \lim\limits_{x\longrightarrow 0^+} e^{\frac{1}{\log x}\log(\sin x)} = \lim\limits_{x\longrightarrow 0^+} e^{\frac{\log(\sin x)}{\log x}}$.

Ora,

$\lim\limits_{x\longrightarrow 0^+} \frac{\log(\sin x)}{\log x} \underset{\text{Regra de Cauchy}}{=} \lim\limits_{x\longrightarrow 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{1}{x}} = \lim\limits_{x\longrightarrow 0^+} \frac{\cos x}{\frac{\sin x}{x}} =$

$= \frac{\cos 0}{1} = 1$.

Logo,

$\lim\limits_{x\longrightarrow 0^+} e^{\frac{\log(\sin x)}{\log x}} = \lim\limits_{y\longrightarrow 1} e^y = e$.

4) Seja $\varepsilon = \frac{1}{2}$.

Então,

$$\exists a_1, a_2 \in A : \inf A \leq a_1 < \inf A + \varepsilon \wedge \sup A - \varepsilon < a_2 \leq \sup A \ .$$

Sendo assim, para tais $a_1, a_2 \in A$, tem-se:

$\sup A - \varepsilon - (\inf A + \varepsilon) < a_2 - a_1 \leq \sup A - \inf A \Longleftrightarrow$

$\Longleftrightarrow \sup A - \inf A - 2\varepsilon < a_2 - a_1 \leq \sup A - \inf A \Longleftrightarrow$

$\Longleftrightarrow 2 - 2\varepsilon < a_2 - a_1 \leq 2 \Longleftrightarrow 1 < a_2 - a_1 \leq 2$.