CDI-I-2^a Ficha de Avaliação

02/11/2012-MEFT

Versão 2

1)

a) f é prolongável por continuidade ao ponto $0 \Longleftrightarrow$

$$\Longleftrightarrow \lim_{x\longrightarrow 0^-} f(x) = \lim_{x\longrightarrow 0^+} f(x) \Longleftrightarrow \lim_{x\longrightarrow 0^-} (-x(2+x)) = \lim_{x\longrightarrow 0^+} \log\left(\frac{k}{2+x^2}\right) \Longleftrightarrow$$

$$\Longleftrightarrow 0 = \log\left(\frac{k}{2}\right) \Longleftrightarrow \frac{k}{2} = 1 \Longleftrightarrow k = 2 \ .$$

b) Sendo F o prolongamento por continuidade de f ao ponto 0 , tem-se:

$$F(x) = \begin{cases} \log\left(\frac{2}{2+x^2}\right), \ x > 0 \\ 0, \ x = 0 \\ -x(2+x), \ x < 0 \end{cases}$$

Determinemos o contradomínio de F , CD_F .
Ora,
$x \in \left]0, +\infty\right[\Longrightarrow 0 < \frac{2}{2+x^2} < 1 \Longrightarrow F(x) = \log\left(\frac{2}{2+x^2}\right) \in \left]-\infty, 0\right[$,

$\lim_{x\longrightarrow 0^+} F(x) = \lim_{x\longrightarrow 0^+} \log\left(\frac{2}{2+x^2}\right) = \log 1 = 0$,

$\lim_{x\longrightarrow +\infty} F(x) = \lim_{x\longrightarrow +\infty} \log\left(\frac{2}{2+x^2}\right) \underset{\lim_{x\longrightarrow+\infty} \frac{2}{2+x^2} = 0^+}{=} \lim_{y\longrightarrow 0^+} \log y = -\infty$.

Logo, pelo Teorema de Bolzano:
F assume todos os valores do intervalo $\left]-\infty, 0\right[$, quando $x \in \left]0, +\infty\right[$.
$F(0) = 0$.
Por outro lado,
$-x(2+x) = 0 \Longleftrightarrow x = -2 \vee x = 0$,

$F\left(\frac{-2+0}{2}\right) = F(-1) = 1(2-1) = 1$,

$\lim_{x\longrightarrow -\infty} F(x) = \lim_{x\longrightarrow -\infty} (-x(2+x)) = \lim_{x\longrightarrow -\infty} \left(-x^2 - 2x\right) = -\infty$.

Logo, pelo Teorema de Bolzano:
F assume todos os valores do intervalo $\left]-\infty, 1\right]$, quando $x \in \left]-\infty, 0\right[$.
Sendo assim,
$CD_F = \left]-\infty, 1\right] \cup \{0\} \cup \left]-\infty, 0\right[= \left]-\infty, 1\right]$.

2)

(a) $f'(x)=\left(\frac{\sqrt{x}}{1+x}\right)'=\left(\frac{x^{\frac{1}{2}}}{1+x}\right)'=\frac{\frac{1}{2}x^{\frac{1}{2}-1}(1+x)-x^{\frac{1}{2}}}{(1+x)^2}=$

$=\frac{\frac{1}{2}x^{-\frac{1}{2}}(1+x)-x^{\frac{1}{2}}}{(1+x)^2}=\frac{\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}}{(1+x)^2}=\frac{\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}}{(1+x)^2}$.

(b) $f'(x)=\left(e^{\sin^2 x}\right)'=\left(\sin^2 x\right)' e^{\sin^2 x}=(2\sin x\cos x)\, e^{\sin^2 x}=(\sin 2x)\, e^{\sin^2 x}$.

3) $\lim\limits_{x\longrightarrow 0}(1-\cos x)^x=\lim\limits_{x\longrightarrow 0}e^{x\log(1-\cos x)}=\lim\limits_{x\longrightarrow 0}e^{\frac{\log(1-\cos x)}{\frac{1}{x}}}$.

Ora,

$\lim\limits_{x\longrightarrow 0}\frac{\log(1-\cos x)}{\frac{1}{x}}\underset{\text{Regra de Cauchy}}{=}\lim\limits_{x\longrightarrow 0}\frac{\frac{\sin x}{1-\cos x}}{-\frac{1}{x^2}}=$

$=-\lim\limits_{x\longrightarrow 0}\frac{x^2\sin x}{1-\cos x}\underset{\text{Regra de Cauchy}}{=}-\lim\limits_{x\longrightarrow 0}\frac{2x\sin x+x^2\cos x}{\sin x}=$

$=-\left(\lim\limits_{x\longrightarrow 0}2x\right)-\left(\lim\limits_{x\longrightarrow 0}\frac{x^2\cos x}{\sin x}\right)=-\left(\lim\limits_{x\longrightarrow 0}2x\right)-\left(\lim\limits_{x\longrightarrow 0}\frac{x\cos x}{\frac{\sin x}{x}}\right)=$

$=-0-\frac{0\cos 0}{1}=0$.

Logo,

$\lim\limits_{x\longrightarrow 0}e^{\frac{\log(1-\cos x)}{\frac{1}{x}}}=\lim\limits_{y\longrightarrow 0}e^y=e^0=1$.

4) Seja $\varepsilon=\frac{1}{2}$.

Então,

$$\exists a_1,a_2\in A:\inf A\le a_1<\inf A+\varepsilon\wedge\sup A-\varepsilon<a_2\le\sup A\ .$$

Sendo assim, para tais $a_1,a_2\in A$, tem-se:

$\sup A-\varepsilon-(\inf A+\varepsilon)<a_2-a_1\le\sup A-\inf A\Longleftrightarrow$

$\Longleftrightarrow\sup A-\inf A-2\varepsilon<a_2-a_1\le\sup A-\inf A\Longleftrightarrow$

$\Longleftrightarrow 2-2\varepsilon<a_2-a_1\le 2\Longleftrightarrow 1<a_2-a_1\le 2$.