Ficha #12

12.

b)

$$\lim_{x\longrightarrow 0^+} \frac{\displaystyle\int_0^{x^2} te^{\sqrt{t}}dt}{\displaystyle\int_0^{x^3} \left(e^{\sqrt[3]{t}}-1\right)dt} \underset{\text{Regra de Cauchy}}{=} \lim_{x\longrightarrow 0^+} \frac{\left(\displaystyle\int_0^{x^2} te^{\sqrt{t}}dt\right)'}{\left(\displaystyle\int_0^{x^3} \left(e^{\sqrt[3]{t}}-1\right)dt\right)'} =$$

$$\underset{\substack{\text{Teorema Fundamental do Cálculo e}\\ \text{Teorema de Derivação da Função Composta}\\ \text{(ou, \textbf{apenas}, Regra de Leibniz)}}}{=} \lim_{x\longrightarrow 0^+} \frac{x^2e^x.2x}{(e^x-1).3x^2} =$$

$$= \lim_{x\longrightarrow 0^+} \frac{2x^3e^x}{3x^2(e^x-1)} = \lim_{x\longrightarrow 0^+} \frac{2xe^x}{3(e^x-1)} = \frac{2}{3}\lim_{x\longrightarrow 0^+} \frac{xe^x}{e^x-1} \underset{\text{Regra de Cauchy}}{=} \frac{2}{3}\lim_{x\longrightarrow 0^+} \frac{e^x+xe^x}{e^x} =$$

$$= \frac{2}{3}\lim_{x\longrightarrow 0^+} (1+x) = \frac{2}{3}\ .$$