COMBINATÓRIA E TEORIA DE CÓDIGOS

Homework 6 (deadline 24/5/2013, in class)

1. Let $C = \text{Ham}(3, 2)$ be the binary Hamming code with redundancy 3 and generator polynomial $g(t) = 1 + t + t^3$.

   (a) Find the parameters $[n, k, d]$ of the interleaved code $C^{(3)}$.

   (b) Find the generator and the check polynomials of $C^{(3)}$.

   (c) Show that $C^{(3)}$ corrects all burst-m errors with $m \leq 3$, but does not correct all burst errors with length 4.

   (d) Using the Burst Error Trapping Algorithm, decode the following received vector

   $$y(t) = t + t^3 + t^4 + t^9 + t^{13} \ .$$

2. Let C be the binary linear code with the following parity-check matrix

   $$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} .$$

   Find the minimum distance $\mathrm{d}(C)$, and determine the code capacity for detecting and correcting random errors. Show also that C detects all burst-m errors with $m \leq 3$.

3. Let $C = \langle (0, \alpha, \alpha^2, 1), (1, 1, 1, 1) \rangle \subset \mathbb{F}_4^4$, where $\mathbb{F}_4 = \mathbb{F}_2[\alpha]$ with $\alpha^2 = 1 + \alpha$.

   (a) Determine a generator matrix and the parameters of the concatenation code $C^* = \phi^*(C)$, where $\phi : \mathbb{F}_4 \longrightarrow \mathbb{F}_2^2$ is the $\mathbb{F}_2$-linear aplication defined by $\phi(1) = 10$ and $\phi(\alpha) = 01$.

   (b) Justify that the code C^* is equivalent to $\widehat{\text{Ham}}(3, 2)^{\perp}$.

4. Let C be a q-ary MDS code with parameters $[n, k]$, where $k < n$.

   (a) Show that there is a q-ary MDS code with lenght n and dimention $n - k$.

   (b) Show that there is a q-ary MDS code with lenght $n - 1$ and dimention k.

5. Let C be the linear code over $\mathbb{F}_7$, with generator matrix

   $$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 3^2 & 3^3 & 3^4 & 3^5 \end{bmatrix}$$

   (a) Show that C is a cyclic code.

   (b) Find the generator polynomial of C.

   (c) Justify that C is a Reed-Solomon code and find its parameters $[n, k, d]$.