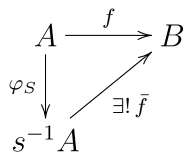
FUNDAMENTOS DE ÁLGEBRA

HWK 6 – bonus exercise

(deadline 29/10/2014, in class)

A. Prove the universal property of the ring $S^{-1}A$: Let A be a commutative ring and $S \subset A$ be a multiplicative subset. Given a commutative ring B and a ring homomorphism $f : A \to B$ such that $f(S) \subset B^\times$, then there is a unique homomorphism $\bar{f} : S^{-1}A \to B$ such that $\bar{f} \circ \varphi_S = f$, i.e., the following diagram commutes



where $\varphi_S : A \to S^{-1}A$, $\varphi_S(a) = \frac{a}{1}$.

B. Let A be a comutative ring and let S and T be multiplicative subsets of A such that $S \subset T$.

(a) Show that $\psi : S^{-1}A \to T^{-1}A$, given by $\psi(\frac{a}{s}) = \frac{a}{s}$, defines a ring homomorphism.

(b) If $T \setminus S \subset A^\times$, show that ψ is an isomorphism.