Course: FOURIER-MUKAI AND BGG TRANSFORMS
IN THE COHOMOLOGICAL STUDY OF
PROJECTIVE VARIETIES

Lecturer : Mihnea Popa

Abstract: The Fourier-Mukai transform is an equivalence between the derived categories of coherent sheaves on an abelian variety, and on its dual variety. Introduced by Mukai, it has found numerous applications and has become a fundamental tool in the study of abelian varieties and, more generally, of irregular varieties. Furthermore, similar "integral" transforms play a major role in studying the birational geometry of algebraic varieties via derived categories (as for example in the work of Bondal-Orlov, Bridgeland and Kawamata). The Bernstein-Gel'fand-Gel'fand (BGG) correspondence is an equivalence between the derived category of modules over the exterior algebra of a vector space and that of linear complexes of modules over the symmetric algebra of the dual vector space. It has been recently further developed in work of Eisenbud-Floystad-Schreyer.

In these lectures I will focus on applications of these two types of equivalences to the cohomological study of irregular varieties (or compact Kaehler manifolds). I will explain how they can be used to extend the Generic Vanishing theorems of Green-Lazarsfeld, and to bound the holomorphic Euler characteristic and the Hodge numbers of varieties without higher irrational pencils. I will also describe a surprisingly natural structure that the cohomology of the canonical bundle acquires as a module over the exterior algebra. Plenty of concrete applications will be provided.

Course: GENERIC VANISHING AND CONTINUOUS GLOBAL GENERATION ON IRREGULAR AND ABELIAN VARIETIES

Lecturer : Giuseppe Pareschi

Abstract: Two basic tools provided by the theory of coherent sheaves on projective varieties are vanishing of the cohomology and global generation. On irregular varieties there are wonderful natural weakenings of such notions: generic vanishing and continuous global generation. They go together with the systematic use of the Fourier-Mukai transform. These ideas have their roots in three important bodies of work:

- Mukai's theory of the Fourier-Mukai trasform;

- Green-Lazarsfeld's generic vanishing theorems, and their applications to the geometry of irregular varieties due to Ein, Lazarsfeld, Hacon and others;

- Kempf's work on theta-functions.

In my lectures I will focus on such concepts. Moreover I will describe several concrete applications to geometric and algebraic problems, as syzygies of abelian varieties, special subvarieties of abelian varieties, invariants and pluricanonical maps of irregular varieties of maximal Albanese dimension.