IST Courses on Algebraic Geometry. III

Course: INTRODUCTION TO CLASSICAL CREMONA TRANSFORMATIONS I

Lecturer: S. Verra

Abstract: A Cremona transformation is a birational automorphism of a complex projective space $\mathbb{P}^r$. The study of such transformations and the group which they generate was a popular subject of classical algebraic geometry flourishing more than one century ago. Although much progress has been made in the two-dimensional case, in spite of the efforts of many classical and modern algebraic geometers, most of the fundamental problems in the higher-dimensional case remain unsolved. The aim of these lectures is to report on the classical techniques for studying Cremona transformations and specifically on the rich legacy of classical examples. The plan is to present a series of examples in modern terms and to use these examples to introduce basic constructions and techniques, as well as more recent results and open problems. The program will include some of the following topics:

-Base locus and numerical characters of Cremona transformations.

-Transformations defined by quadrics.

-The cubo-cubic transformation of $\mathbb{P}^3$ and related topics.

-Birational involutions of $\mathbb{P}^2$ and $\mathbb{P}^3$.

-Homaloidal linear system of surfaces with finite base locus.

-Cremona transformations with smooth and connected base locus.

-Classification problems.

Course: INTRODUCTION TO CLASSICAL CREMONA TRANSFORMATIONS II

Lecturer: I. Dolgachev

Abstract: The group structure of the group Cr(2) of Cremona transformations of projective plane is unlike any other familiar group structures. Although it is generated by projective transformations and a single non-projective transformation, its structure is very complicated. For example, the conjugacy classes of elements of given finite order are parametrized by

an algebraic variety with finitely many irreducible components of different dimension. This is very different from the case of Lie groups. One of the oldest conjectures is that the group is simple as an abstract group.

In these lectures we will briefly discuss the now completed classification of conjugacy classes of finite groups of Cr(2) which is equivalent to the classification of pairs (S,G), where S is a rational surface and G is a finite group of its automorphisms, up to equivariant birational maps.

We will also discuss known examples of infinite subgroups of Cr(2) which can be realized as automorphism groups of rational surfaces, and their relationship to complex dynamics of rational maps.