Considere a especificação algébrica $nat = \langle \Sigma, X, \Gamma \rangle$ onde

- $E = \{nat\}$
  $z :\rightarrow nat$
  $s : nat \rightarrow nat$
  $som : nat\ nat \rightarrow nat$
- $X_{nat} = \{n, m, n_1, m_1, \ldots\}$
- Axiomas
  - $som(n, z) = n$
  - $som(n, s(m)) = s(som(n, m))$

Seguem-se alguns modelos de *nat*.

1. $A_1 = \langle |A_1|, -_{A_1} \rangle$ tal que

   $|A_1|_{nat} = \mathbb{N}_0$
   $\underline{z}_{A_1} = 0$
   $\underline{s}_{A_1} = \lambda a.a + 1$
   $\underline{som}_{A_1} = \lambda a_1 a_2.a_1 + a_2$

2. $A_2 = \langle |A_2|, -_{A_2} \rangle$ tal que

   $|A_2|_{nat} = \{0, 2, 4, 6, \ldots\}$
   $\underline{z}_{A_2} = 0$
   $\underline{s}_{A_2} = \lambda a.a + 2$
   $\underline{som}_{A_2} = \lambda a_1 a_2.a_1 + a_2$

3. $A_3 = \langle |A_3|, -_{A_3} \rangle$ tal que

   $|A_3|_{nat} = \{0, 1\}$
   $\underline{z}_{A_3} = 0$
   $\underline{s}_{A_3} = \lambda a.1 - a$
   $\underline{som}_{A_3} = \lambda a_1 a_2.(1 \text{ se } a_1 \neq a_2 \text{ e } 0 \text{ se } a_1 = a_2)$

4. $A_4 = \langle |A_4|, -_{A_4} \rangle$ tal que

   $|A_3|_{nat} = \{0\}$
   $\underline{z}_{A_4} = 0$
   $\underline{s}_{A_4} = \lambda a.0$
   $\underline{som}_{A_4} = \lambda a_1 a_2.a_1 + a_2$

5. $A_5 = \langle |A_5|, -_{A_5} \rangle$ tal que

   $|A_5|_{nat} = \mathbb{Z}$
   $\underline{z}_{A_5} = 0$
   $\underline{s}_{A_5} = \lambda a.a + 1$
   $\underline{som}_{A_5} = \lambda a_1 a_2.a_1 + a_2$

6. $A_6 = \langle |A_6|, -_{A_6} \rangle$ tal que

   $|A_6|_{nat} = \mathbb{N}$
   $\underline{z}_{A_6} = 1$
   $\underline{s}_{A_6} = \lambda a.a + a$
   $\underline{som}_{A_6} = \lambda a_1 a_2 . a_1 \times a_2$

7. $A_7 = \langle |A_7|, -_{A_7} \rangle$ tal que

   $|A_7|_{nat} = \{2^i : i \in \mathbb{N}_0\}$
   $\underline{z}_{A_7} = 1$
   $\underline{s}_{A_7} = \lambda a.a + a$
   $\underline{som}_{A_7} = \lambda a_1 a_2 . a_1 \times a_2$

Exercício:

1. Verifique que as álgebras acima referidas são de facto modelos de *nat*.
2. Para cada par de álgebras identifique, se possível, um homomorfismo entre elas.
3. Quais das álgebras anteriores são isomorfas a A_1?