Licenciatura em Engenharia Informática e de Computadores - LEIC
Licenciatura em Engenharia de Redes de Comunicação e Informação - LERCI

Exercícios de Teoria da Computação

Lógica - Apêndice

Secção Ciência da Computação
Departamento de Matemática
Instituto Superior Técnico
2005/2006

Apêndice à secção 2

1. Sejam ψ_1, ψ_2 e ψ_3 símbolos proposicionais. Verifique se

   (a) $\vdash_{\mathcal{T}} ((\psi_1 \to \psi_2) \to \psi_1) \to \psi_1$

   (b) $\vdash_{\mathcal{T}} (\psi_1 \to \psi_2) \to (\psi_2 \to \psi_1)$

   (c) $\vdash_{\mathcal{T}} ((\psi_1 \to \psi_2) \wedge \psi_2) \to \psi_1$

   (d) $\vdash_{\mathcal{T}} (\psi_1 \to \psi_2) \vee (\psi_2 \to \psi_1)$

   (e) $\vdash_{\mathcal{T}} ((\psi_1 \wedge \psi_2) \vee \psi_3) \to ((\psi_1 \vee \psi_3) \wedge ((\psi_2 \vee \psi_3)))$

   (f) $\vdash_{\mathcal{T}} ((\psi_1 \vee \psi_3) \wedge (\psi_2 \vee \psi_3)) \to ((\psi_1 \wedge \psi_2) \vee \psi_3)$

   (g) $\vdash_{\mathcal{T}} ((\psi_1 \vee \psi_3) \wedge (\psi_2 \vee \psi_3)) \leftrightarrow ((\psi_1 \wedge \psi_2) \vee \psi_3)$

   (h) $\vdash_{\mathcal{T}} ((\psi_1 \vee \psi_2) \wedge \psi_3) \to ((\psi_1 \wedge \psi_3) \vee (\psi_2 \wedge \psi_3))$

   (i) $\vdash_{\mathcal{T}} ((\psi_1 \wedge \psi_3) \vee ((\psi_2 \wedge \psi_3))) \to ((\psi_1 \vee \psi_2) \wedge \psi_3)$

   (j) $\vdash_{\mathcal{T}} ((\psi_1 \wedge \psi_3) \vee ((\psi_2 \wedge \psi_3))) \leftrightarrow ((\psi_1 \vee \psi_2) \wedge \psi_3)$

   (k) $\vdash_{\mathcal{T}} (\psi_1 \wedge \psi_2) \wedge (\psi_1 \to \psi_3) \wedge (\psi_3 \to (\neg\psi_2))$

   (l) $\vdash_{\mathcal{T}} (\psi_1 \vee (\neg\psi_3)) \wedge (\psi_2 \to \psi_1) \wedge ((\neg\psi_3) \to \psi_2) \wedge (\neg\psi_1)$

   (m) $\vdash_{\mathcal{T}} ((\neg\psi_1) \vee \psi_2) \to (((\neg\psi_1) \to \psi_2) \vee (\psi_3 \to \psi_2))$

   (n) $\vdash_{\mathcal{T}} ((\psi_1 \vee \psi_2) \to (\neg\psi_3)) \to ((\psi_2 \to (\neg\psi_3)) \vee (\psi_1 \to (\neg\psi_3)))$

   (o) $\vdash_{\mathcal{T}} (\psi_1 \to (\psi_2 \to \psi_3)) \to ((\psi_1 \to \psi_2) \to (\psi_1 \to \psi_3))$

   (p) $\vdash_{\mathcal{T}} ((\psi_1 \to \psi_2) \wedge \to ((\neg\psi_1) \to (\neg\psi_2))) \to (\psi_1 \leftrightarrow \psi_2)$

2. Sejam ψ_1, ψ_2, ψ_3, ψ_4, ψ_5 e ψ_6 símbolos proposicionais. Verifique se

   (a) $\{\psi_1 \to (\psi_2 \wedge \psi_3)\} \vdash_{\mathcal{T}} \psi_1 \to \psi_3$

   (b) $\{\psi_1 \to (\psi_2 \vee \psi_3)\} \vdash_{\mathcal{T}} \psi_1 \to \psi_3$

   (c) $\{\psi_1 \to \psi_2\} \vdash_{\mathcal{T}} \psi_1 \to (\psi_2 \wedge \psi_3)$

   (d) $\{\psi_1 \to \psi_2\} \vdash_{\mathcal{T}} \psi_1 \to (\psi_2 \vee \psi_3)$

   (e) $\{(\psi_1 \wedge \psi_2) \to \psi_3\} \vdash_{\mathcal{T}} \psi_1 \to \psi_3$

   (f) $\{(\psi_1 \vee \psi_2) \to \psi_3\} \vdash_{\mathcal{T}} \psi_1 \to \psi_3$

   (g) $\{\psi_1 \to \psi_2, \ \psi_2 \to \psi_3\} \vdash_{\mathcal{T}} \psi_1 \to \psi_3$

   (h) $\{\psi_1 \to \psi_3, \ \psi_2 \to \psi_3\} \vdash_{\mathcal{T}} (\psi_1 \vee \psi_2) \to \psi_3$

   (i) $\{\psi_1 \to \psi_2, \ (\neg\psi_1) \to \psi_2\} \vdash_{\mathcal{T}} \psi_2$

   (j) $\{\psi_1 \vee (\psi_4 \wedge \psi_2)\} \vdash_{\mathcal{T}} ((\neg\psi_2) \to \psi_1) \wedge (\psi_1 \vee \psi_4)$

   (k) $\{(\psi_1 \to \psi_2) \vee \psi_4\} \vdash_{\mathcal{T}} ((\neg\psi_1) \to \psi_2) \vee (\psi_4 \to \psi_2)$

   (l) $\{\psi_3, \ \psi_2 \vee (\psi_3 \to \psi_4)\} \vdash_{\mathcal{T}} (\neg(\psi_3 \wedge \psi_4)) \to \psi_2$

   (m) $\{(\psi_1 \wedge \psi_2) \to \psi_3, \ \neg((\neg\psi_2) \vee \psi_3)\} \vdash_{\mathcal{T}} \neg\psi_1$

   (n) $\{(\psi_1 \to \psi_2) \vee (\psi_3 \to \psi_2)\} \vdash_{\mathcal{T}} (\neg\psi_1) \to \psi_2$

   (o) $\{\psi_1 \to (\psi_2 \vee \psi_3)\} \vdash_{\mathcal{T}} (\neg\psi_3) \to (\psi_2 \to (\neg\psi_1))$

   (p) $\{(\psi_3 \wedge \psi_4) \to \psi_1, \ \psi_2 \to \psi_3\} \vdash_{\mathcal{T}} \psi_4 \to (\psi_2 \to \psi_1)$

(q) $\{\psi_1 \to \psi_2,\ \ \psi_3 \to \psi_4,\ \ \psi_2 \vee \psi_4\} \vdash_{\mathcal{T}}\ \psi_1 \vee \psi_3$

(r) $\{\psi_1 \to (\psi_3 \wedge \psi_4),\ \ (\psi_2 \vee \psi_3) \to \psi_4\} \vdash_{\mathcal{T}}\ \psi_4 \to \psi_1$

(s) $\{(\psi_1 \to \psi_2) \to \psi_3,\ \ \psi_4 \to (\neg\psi_1)),\ \ \psi_5,\ \ \neg\psi_4,\ \ \psi_5 \to \psi_2\} \vdash_{\mathcal{T}}\ \psi_3$

(t) $\{\psi_1,\ \ \psi_2 \to \psi_3,\ \ \neg(\psi_1 \wedge \psi_3)\} \vdash_{\mathcal{T}}\ \neg\psi_2$

(u) $\{(\psi_1 \wedge \psi_2) \to \psi_3,\ \psi_2 \wedge (\neg\psi_4)\} \vdash_{\mathcal{T}}\ (\psi_3 \to \psi_4) \to (\neg\psi_1)$

(v) $\{\psi_1 \to (\neg\psi_2)\} \vdash_{\mathcal{T}}\ ((\neg\psi_3) \vee \psi_1) \to (\neg\psi_2)$

(w) $\{(\neg\psi_1) \to \psi_2,\ \psi_1 \to (\neg\psi_5),\ \psi_3 \to (\psi_4 \vee \psi_5),\ \psi_4 \to (\neg\psi_3)\}$
$\vdash_{\mathcal{T}}\ \psi_3 \to \psi_2$

(x) $\{(\psi_1 \wedge \psi_2) \to \psi_3,\ \psi_4 \wedge (\neg\psi_5),\ (\neg\psi_5) \wedge ((\neg(\psi_5 \vee \psi_1)) \to \psi_6),\ \psi_2 \wedge (\neg\psi_3)\}$
$\vdash_{\mathcal{T}}\ \psi_6$

(y) $\{\psi_1 \leftrightarrow \psi_2\} \vdash_{\mathcal{T}}\ (\psi_1 \wedge \psi_3) \leftrightarrow (\psi_2 \wedge \psi_3)$

(z) $\{\neg(\psi_2 \vee (\neg\psi_3)) \leftrightarrow (\neg\psi_4),\ \psi_4\} \vdash_{\mathcal{T}}\ \psi_3 \to (\psi_2 \vee \psi_1)$

3. Sejam P, Q, R e S símbolos de predicado, a um símbolo de constante, x e y variáveis e f um símbolo de função. Verifique se

(a) i. $\vdash_{\mathcal{T}} (\exists x P(x)) \to P(x)$
ii. $\vdash_{\mathcal{T}} (\exists x P(x)) \to P(y)$
iii. $\vdash_{\mathcal{T}} (\exists x P(x)) \to P(a)$
iv. $\vdash_{\mathcal{T}} P(x) \to (\exists x P(x))$
v. $\vdash_{\mathcal{T}} P(y) \to (\exists x P(x))$
vi. $\vdash_{\mathcal{T}} P(a) \to (\exists x P(x))$

(b) i. $\vdash_{\mathcal{T}} (\forall x P(x)) \to P(x)$
ii. $\vdash_{\mathcal{T}} (\forall x P(x)) \to P(y)$
iii. $\vdash_{\mathcal{T}} (\forall x P(x)) \to P(a)$
iv. $\vdash_{\mathcal{T}} P(x) \to (\forall x P(x))$
v. $\vdash_{\mathcal{T}} P(y) \to (\forall x P(x))$
vi. $\vdash_{\mathcal{T}} P(a) \to (\forall x P(x))$

(c) i. $\vdash_{\mathcal{T}} (\forall x\, P(x)) \to (\exists x\, P(x))$
ii. $\vdash_{\mathcal{T}} (\exists x\, P(x)) \to (\forall x\, P(x))$
iii. $\vdash_{\mathcal{T}} (\forall x\, P(x)) \to (\forall y\, P(y))$
iv. $\vdash_{\mathcal{T}} (\exists x\, P(x)) \to (\exists y\, P(y))$

(d) i. $\vdash_{\mathcal{T}} (\forall x\, P(x)) \to (\neg(\exists x\, (\neg P(x))))$
ii. $\vdash_{\mathcal{T}} (\neg(\exists x\, (\neg P(x)))) \to (\forall x\, P(x))$
iii. $\vdash_{\mathcal{T}} (\exists x\, P(x)) \to (\neg(\forall x\, (\neg P(x))))$
iv. $\vdash_{\mathcal{T}} (\neg(\forall x\, (\neg P(x)))) \to (\exists x\, P(x))$

(e) i. $\vdash_{\mathcal{T}} ((\forall x\, Q(x)) \to (\forall x\, R(x))) \to (\forall x\, (Q(x) \to R(x)))$

ii. $\vdash_{\mathcal{T}} (\forall x\, (Q(x) \to R(x))) \to ((\forall x\, Q(x)) \to (\forall x\, R(x)))$

iii. $\vdash_{\mathcal{T}} ((\exists x\, Q(x)) \to (\exists x\, R(x))) \to (\exists\, (Q(x) \to R(x)))$

iv. $\vdash_{\mathcal{T}} (\exists x\, (Q(x) \to R(x))) \to ((\exists x\, Q(x)) \to (\exists x\, R(x)))$

(f) i. $\vdash_{\mathcal{T}} ((\forall x\, Q(x)) \vee (\forall x\, R(x))) \to (\forall\, (Q(x) \vee R(x)))$

ii. $\vdash_{\mathcal{T}} (\forall x\, (Q(x) \vee R(x))) \to ((\forall x\, Q(x)) \vee (\forall x\, R(x)))$

iii. $\vdash_{\mathcal{T}} ((\exists x\, Q(x)) \vee (\exists x\, R(x))) \to (\exists\, (Q(x) \vee R(x)))$

iv. $\vdash_{\mathcal{T}} (\exists x\, (Q(x) \vee R(x))) \to ((\exists x\, Q(x)) \vee (\exists x\, R(x)))$

(g) i. $\vdash_{\mathcal{T}} ((\forall x\, Q(x)) \wedge (\forall x\, R(x))) \to (\forall\, (Q(x) \wedge R(x)))$

ii. $\vdash_{\mathcal{T}} (\forall x\, (Q(x) \wedge R(x))) \to ((\forall x\, Q(x)) \wedge (\forall x\, R(x)))$

iii. $\vdash_{\mathcal{T}} ((\exists x\, Q(x)) \wedge (\exists x\, R(x))) \to (\exists\, (Q(x) \wedge R(x)))$

iv. $\vdash_{\mathcal{T}} (\exists x\, (Q(x) \wedge R(x))) \to ((\exists x\, Q(x)) \wedge (\exists x\, R(x)))$

(h) i. $\vdash_{\mathcal{T}} (\forall x(\forall y S(x,y))) \to (\forall x(\forall y S(y,x)))$

ii. $\vdash_{\mathcal{T}} (\forall x(\forall y S(x,y))) \to (\forall y(\forall x S(x,y)))$

iii. $\vdash_{\mathcal{T}} (\forall x(\forall y S(x,y))) \to (\forall y(\forall x S(y,x)))$

iv. $\vdash_{\mathcal{T}} (\exists x(\exists y S(x,y))) \to (\exists x(\forall y S(y,x)))$

v. $\vdash_{\mathcal{T}} (\exists x(\exists y S(x,y))) \to (\exists y(\exists x S(x,y)))$

vi. $\vdash_{\mathcal{T}} (\exists x(\exists y S(x,y))) \to (\exists y(\exists x S(y,x)))$

vii. $\vdash_{\mathcal{T}} (\forall x(\exists y S(x,y))) \to (\exists x(\forall y S(x,y)))$

viii. $\vdash_{\mathcal{T}} (\exists x(\forall y S(x,y))) \to (\forall x(\exists y S(x,y)))$

(i) $\vdash_{\mathcal{T}} \exists x(P(x) \to (\forall x\, P(x)))$

(j) $\vdash_{\mathcal{T}} \forall x(P(x) \to (\forall x\, P(x)))$

(k) $\vdash_{\mathcal{T}} \exists y\, ((\forall x\, P(x)) \to P(y))$

(l) $\vdash_{\mathcal{T}} ((\exists x\, P(x)) \to (\forall x\, P(x))) \to (\forall x\, P(x))$

(m) $\vdash_{\mathcal{T}} (\forall x\ (P(x) \to P(f(x)))) \to (P(a) \to P(f(f(a))))$

4. Sejam P, Q e R símbolos de predicado, a um símbolo de constante, x e y variáveis e f um símbolo de função. Verifique se

(a) $\{Q(x)\} \vdash_{\mathcal{T}} \exists x(P(x) \to Q(x))$

(b) $\{P(x) \to Q(y)\} \vdash_{\mathcal{T}} (\exists x P(x)) \to (\exists x Q(x))$

(c) $\{\forall x(P(x) \to Q(x)),\ P(x)\} \vdash_{\mathcal{T}} Q(y)$

(d) $\{\forall x(P(x) \to Q(x)),\ \neg P(f(a))\} \vdash_{\mathcal{T}} \neg Q(f(a))$

(e) $\{\exists x(P(x) \to Q(x)),\ P(f(a))\} \vdash_{\mathcal{T}} Q(f(a))$

(f) $\{\exists x(P(x) \to Q(x)),\ P(x)\} \vdash_{\mathcal{T}} Q(y)$

(g) $\{\forall x(P(x) \to Q(x)),\ \exists y(\neg Q(y))\} \vdash_{\mathcal{T}} \exists z(\neg P(z))$

(h) $\{\exists x(P(x)\rightarrow Q(x)),\ \ \forall x P(x)\} \vdash_{\mathcal{T}}\ \exists x Q(x)$

(i) $\{\exists x(P(x)\vee Q(x)),\ \ \forall x(\neg P(x))\} \vdash_{\mathcal{T}}\ \exists x(\neg Q(x))$

(j) $\{\exists x(P(x)\vee Q(x)),\ \ \forall x(\neg P(x))\} \vdash_{\mathcal{T}}\ \forall x(\neg Q(x))$

(k) $\{\forall x(P(x)\vee Q(x)),\ \ \exists x(\neg P(x))\} \vdash_{\mathcal{T}}\ \exists x Q(x)$

(l) $\{\forall x(P(x)\vee Q(x)),\ \ \exists x(\neg P(x))\} \vdash_{\mathcal{T}}\ \forall x Q(x)$

(m) $\{\exists x(P(x)\rightarrow Q(x)))\} \vdash_{\mathcal{T}}\ (\forall x\ P(x))\rightarrow(\exists x\, Q(x))$

(n) $\{(\forall x\ P(x))\rightarrow(\exists x\, Q(x))\} \vdash_{\mathcal{T}}\ \exists x(P(x)\rightarrow Q(x))$

(o) $\{\forall x(P(x)\rightarrow Q(x))\} \vdash_{\mathcal{T}}\ (\exists x\ P(x))\rightarrow(\forall x\, Q(x))$

(p) $\{(\exists x\ P(x))\rightarrow(\forall x\, Q(x))\} \vdash_{\mathcal{T}}\ \forall x(P(x)\rightarrow Q(x))$

(q) $\{\exists x\, Q(x),\ \forall x\, R(x)\} \vdash_{\mathcal{T}}\ \forall x\,(R(x)\rightarrow Q(x))$

(r) $\{\forall x(P(x)\rightarrow(\neg(\exists y\ Q(y))))\} \vdash_{\mathcal{T}}\ \neg(\exists x(P(x)\wedge(\exists y\ Q(y))))$

(s) $\{\exists x(\neg(\exists y\, R(x,y)))\} \vdash_{\mathcal{T}}\ \neg\exists x\forall y(\neg R(x,y))$

(t) $\{\forall x(\forall y(R(y)\rightarrow Q(x)))\} \vdash_{\mathcal{T}}\ (\exists y R(y))\rightarrow(\forall x Q(x))$

(u) $\{\exists x\, P(x),\ \ \forall x(\forall y(P(x)\rightarrow Q(y)))\} \vdash_{\mathcal{T}}\ \forall y\, Q(y)$

(v) $\{\forall x(P(x)\rightarrow(\exists y R(x,y)))\} \vdash_{\mathcal{T}}\ \exists x\,(P(x)\wedge(\forall y\neg R(x,y)))$

(w) $\{\forall x(Q(x)\rightarrow(\forall y\, R(y)))\} \vdash_{\mathcal{T}}\ (\exists y(\neg R(y)))\rightarrow(\forall x\,(\neg Q(x)))$

(x) $\{\forall x((\exists y\, R(y))\rightarrow P(x))\} \vdash_{\mathcal{T}}\ \forall x((\forall y\,(\neg R(y)))\vee P(x))$

(y) $\{\forall x((\neg P(x)\wedge(\exists y\ Q(y)))\} \vdash_{\mathcal{T}}\ \forall x(\exists y((\neg P(x))\wedge Q(y)))$

(z) $\{\forall x(\exists y(P(x)\rightarrow R(x,y)))\} \vdash_{\mathcal{T}}\ \exists x\ (P(x)\wedge(\neg(\exists y\, R(x,y))))$

Apêndice à secção 3

1. Sejam P, Q, R, S e T símbolos proposicionais. Verifique se

   (a) $\{P\vee Q\vee R,\ \ (\neg R)\vee P,\ \ \neg P\} \vdash_{\mathcal{R}}\ Q$

   (b) $\{\neg P,\ \ (\neg Q)\vee R,\ \ Q,\ \ R,\ \ (\neg R)\vee S\} \vdash_{\mathcal{R}}\ S\vee(\neg P)$

   (c) $\{\neg R,\ \ (\neg R)\vee P,\ \ (\neg Q)\vee R,\ \ (\neg P)\vee Q\} \vdash_{\mathcal{R}}\ \neg P$

   (d) $\{(\neg Q)\vee P\vee R,\ \ Q\vee R,\ \ (\neg P)\vee Q,\ \ (\neg R)\vee Q\} \vdash_{\mathcal{R}}\ Q$

   (e) $\{(\neg Q)\vee R,\ \ Q\vee R,\ \ (\neg Q)\vee P,\ \ (\neg R)\vee Q,\ \ (\neg P)\vee Q,\ \ (\neg R)\vee(\neg Q)\} \vdash_{\mathcal{R}}\ \neg R$

   (f) $\{P\vee Q,\ \ (\neg P)\vee R\vee S,\ \ (\neg R)\vee S,\ \ (\neg R)\vee T,\ \ (\neg T)\vee S\vee(\neg R)\} \vdash_{\mathcal{R}}\ S\vee Q$