Lectures

Optimal design for a system of diffusion equations

Gregoire Allaire, Université Paris VI, France

Summary: In this talk we describe how the homogenization method can be used for optimizing the position of fuel assemblies in a nuclear reactor core. This is an optimal design problem for the neutronic diffusion equations. The goal is to maximize the reactivity (i.e. the inverse of the first eigenvalue) while keeping the neutron flux (i.e. the first eigenvector) as spatially uniform as possible. Although this is truly a discrete optimization problem, our strategy is to embed it in a continuous one which is solved by the homogenization method. Work developed in collaboration with Carlos Castro.

Line energies for gradient vector fields in the plane

Luigi Ambrosio, Scuola Normale Superiore di Pisa, Italy

Summary: We study the singular perturbation of $\int(1-|\nabla u|^2)^2$ by $\epsilon^2|\nabla^2 u|^2$. This problem, which could be thought as the natural second order version of the classical singular perturbation of the potential energy $\int(1-u^2)^2$ by $\epsilon^2|\nabla u|^2$, leads, as in the first order case, to energy concentration effects on hypersurfaces. In the two dimensional case we study the natural domain for the limiting energy and prove a compactness theorem in this class.

A new effective approach to the problem of the minimization of funtionals

Nikolay V. Azbelev, Perm State University, Russia

Summary: On the base of the "Theory of abstract differential equations" a group of mathematician at the town of Perm (Urals, Russia) has worked out some new methods for the problem of minimization.

Considering a particular case for lack of time, let $\mathbf{L}_2$ be a Hilbert space of $z:[a,b]\to\mathbb{R}$, $\mathbf{D}$ be a Banach space isomorphical to the direct product $\mathbf{L}_2\times\mathbb{R}^n$. There are proposed some theorems on the existence of the minimum in $\mathbf{D}$ of the functional

$$Jx=\int_a^b f(s,(T_1x)(s),\ldots,(T_mx)(s))\,ds$$

with constraints $\ell^i x=\alpha^i,\quad i=1,\ldots,n$. Here $T_j:\mathbf{D}\to\mathbf{L}_2$, $j=1,\ldots,m$, $\ell^i:\mathbf{D}\to\mathbb{R}$ are bounded linear. In the case of square functional J the operator $K:\mathbf{L}_2\to\mathbf{L}_2$ may be constructed in the explicit form such that the condition $\|K\|_{\mathbf{L}_2\to\mathbf{L}_2}<1$ is sufficient to the existence of a unique minimum in $\mathbf{D}$. If K is isotonic, the inequality $\|K\|_{\mathbf{L}_2\to\mathbf{L}_2}<1$ is also necessary.

There are given solutions to some problems discussed in the literature.

On the relaxation of multiwell problems

Kaushik Bhattacharya, California Institute of Technology, U.S.A.

Summary: Crystalline solids that undergo a martensitic phase transformation are often modelled by a variational problem of energy minimization where the energy density has multiple wells. The energy density is thus not quasiconvex, and the minimization problem often does not have classical solutions; instead minimizing sequences develop fine oscillations which may be interpreted as the fine phase microstructure that is observed in these materials. The macroscopic behavior of the crystal is then described by the relaxation, or quasiconvexification, of the multiwell energy density. An important feature of this relaxation is the quasiconvex hull of the wells. This talk, based on joint work with Georg Dolzmann, will describe a characterization of this hull, and construction of models for the relaxed energy in some two-dimensional examples.

Optimization problems for obstacles

Giuseppe Buttazzo, Università di Pisa, Italy

Summary: We consider optimization problems for obstacles of the form

$$\min\Big\{F(g) : g \in X(\Omega)\Big\}$$

where the class $X(\Omega)$ of admissible obstacles is given by

$$X(\Omega) = \Big\{g \leq \psi \text{ on } \Omega,\ \int g\,dx = c\Big\}$$

with the function $\psi \in W_0^{1,p}(\Omega)$ and the constant $c \in \mathbf{R}$ a priori given. It is well known that in general the problem does not admit any solution, and a relaxation procedure is needed. Here we prove that under suitable monotonicity and semicontinuity assumptions on the cost F the existence of a classical unrelaxed solution occurs.

References

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Characterization of optimal shapes and masses through Monge-Kantorovich equation

Guy Bouchitté, Université de Toulon et du Var, France

Summary: In this joined work with G. Buttazzo, we consider a *mass optimization problem* which consists in finding the best distribution of a given amount of elastic material, in order to achieve the minimal compliance. The unkown mass distribution is then a nonnegative measure which may vary in the class of admissible choices, with total mass prescribed, and support possibly constrained in a given *design region.* Dealing with general measures pushed us (see [2]) to develop a general framework of variational calculus on measures, based on a new notion of *tangent bundle* for a measure, which includes and unifies the classical cases of low dimensional manifolds (membranes, string, junctions, ...).

The phenomenon of appearance of low dimensional network structures was already remarked (see [1]) in the cases of optimal mixtures of two materials, when the percentage of the strong one tends to zero. Moreover, because of capacitary arguments, concentrated loads are forbidden in the classical framework, but they become

admissible as soon as we allow the distribution of material to be singular, or more generally a measure. Then in the framework of our mass optimization problems, we are allowed to consider the general case when for a load we take a given measure.

A first result is that we obtain the existence of an optimal mass distribution for which the elastic compliance is minimal. This optimal measure may present the interesting feature to be composed by terms of different dimensions. Moreover, we characterize these optimal solutions by means of a generalized version ([3]) of the Monge-Kantorovich partial differential equation which describes the mass transfer problem (see [4]).

In the scalar case (heat equation), our approach applies directly to shape optimization and different explicit examples are presented. In the vector case of elasticity, we make the connection with the problem of fictitious materials and, in the 2D-case, with the Michell trusses. We conclude by some numerical results.

References

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[3] G. Bouchitté, G. Buttazzo, P. Seppecher: Optimal design for minimum compliance via solving a multi-dimensional Monge Kantorovich equation. CRAS, t.324, Serie I, p. 1185-1191, 1997.

[4] L. C. Evans, W. Gangbo: Differential equations methods for the Monge-Kantorovich mass transfer problem, memoirs of AMS, 1997.

Some properties of convex hulls and applications to partial differential equations

Bernard Dacorogna, Ecole Polytéchnique Fédérale de Lausanne, Switzerland

Summary: I will define and give some properties of convex, polyconvex, quasiconvex and rank one convex hulls of a given set of matrices. I will then discuss applications of these results to existence of solutions of some partial differential equations.

The calibration method for the Mumford-Shah functional

Gianni Dal Maso, SISSA, Trieste, Italy

Summary: The calibration method is adapted to the Mumford-Shah functional, and provides an efficient technique to prove that some functions are energy minimizers. In particular the case of triple junctions is considered. Applications to the evolution equation for the Mumford-Shah functional are given (work in progress with Alberti and Bouchitté). The minimality of the solutions to the Euler equation in sufficiently small domains is discussed (work in progress with Mora and Morini).

Soft ferromagnetic films

Antonio De Simone, Max-Plank Institute, Leipzig, Germany

Summary: Soft ferromagnetic films provide a representative example for the behavior of materials whose response to external actions involves several interacting length scales. Typically, the size of a sample, of its domains, of the domain walls, and of the singular objects they may contain (vortices, cross-ties, etc.) are well separated, and orders of magnitude apart. The macroscopic response of a specimen to applied magnetic fields is due to the cooperative evolution of its magnetic domains. Domain evolution, however, is strongly dependent on the mobility of the walls, which is in turn affected by interactions occurring at the smaller length scales on which singularities live.

Direct numerical simulations are based on a well established continuum model (Micromagnetics), but are often impractical, because of the necessity of resolving many length scales simultaneously. We discuss a simple two-dimensional variational model which seems to adequately capture the salient features of domain response to applied fields in (magnetically) soft films.

Second order singular perturbation models for phase transitions

Irene Fonseca, Carnegie Mellon University, Pittsburgh, U.S.A.

Summary: Singular perturbation models involving a penalization of the first order derivatives have provided a new insight into the role played by surface energies in the study of phase transition problems. It is known that if $W : \mathbb{R}^d \to [0, +\infty)$ grows at least linearly at infinity and it has exactly two potential wells of level zero at $a, b \in \mathbb{R}^d$, then the $\Gamma(L^1)$-limit of the family of functionals

$$\mathcal{F}_\varepsilon(u) := \begin{cases} \int_\Omega \left(\frac{W(u)}{\varepsilon} + \varepsilon |\nabla u|^2 \right) dx & \text{if } u \in W^{1,2}(\Omega; \mathbb{R}^d), \\ +\infty & \text{if } u \in L^1(\Omega; \mathbb{R}^d) \setminus W^{1,2}(\Omega; \mathbb{R}^d), \end{cases}$$

where Ω is a bounded, open set in $\mathbb{R}^N$, is given by

$$\mathcal{F}(u) := \begin{cases} \mathbf{m} \, \mathrm{Per}_\Omega(\{u = a\}) & \text{if } u \in BV(\Omega; \{a, b\}), \\ +\infty & \text{otherwise}, \end{cases}$$

for a suitable constant $\mathbf{m}$ depending on the energy density W. In 1989 the study of a singular perturbation model for phase transitions of nonlinear elastic materials was initiated in collaboration with Luc Tartar. Here u above satisfies the additional constraint $\mathrm{curl}\, u = 0$ and W has two potential wells invariant under $SO(N)$. The case where W has one spherical well, and therefore admissible limiting fields satisfy the eikonal equation $|u| = 1, \mathrm{curl}\, u = 0$, has recently been the focus of much attention, motivated in part by its relevance in the analysis of thin film blisters and related phenomena.

In this talk the $\Gamma(L^1)$-limit is obtained in the case where in $\mathcal{F}_\varepsilon(u)$ the penalization term $\varepsilon|\nabla u|^2$ is replaced by $\varepsilon^3 |\nabla^2 u|^2$, for $u \in W^{2,2}(\Omega; \mathbb{R}^d)$. The resulting functional is of the same form as $\mathcal{F}(u)$ above.

This is joint work with Carlo Mantegazza.

A variational approach to brittle fracture

Gilles Francfort, Université Paris Nord, France

Summary: This is a collaborative effort with J.-J. Marigo, B. Bourdin, and, more recently, A. Chambolle.

First, I will briefly mention the difficulties encountered by Griffith's theory of brittle fracture and then proceed to describe the model that we suggest as a possible remedy. I will then stress the obstacles – whether real or imaginary – that the model is in turn confronted with.

In a second part, I will evoke the mathematical formulation of the model as based on the work of E. De Giorgi et al. for the D. Mumford–J. Shah image segmentation functional.

In a third part, I will describe the numerical implementation of the model which constituted the bulk of B. Bourdin's doctoral thesis at the Université Paris-Nord. The ensuing numerics will illustrate the ability of the model at predicting crack initiation and crack path on examples that are, I believe, beyond the reach of the usual methods of classical fracture mechanics.

Existence theorems for quantum many-body systems via variational methods

Gero Friesecke, University of Oxford, U.K.

Summary: The nonlinear simplifications of "exact" N-body quantum mechanics (density functional theory, the Hartree-Fock equations, the Configuration- Interaction equations) have an enormous physics and chemistry literature, but few rigorous results. Recent progress has relied on the systematic application of variational methods and includes a proof of existence of solutions and integer quantization of charge for the CI equations of atoms and molecules. (This had not previously been shown even for the Helium atom.) As special cases one recovers the basic existence theorems of Zhislin in quantum mechanics (infinite rank CI) and Lieb-Simon in Hartree-Fock theory (rank-N CI) in a unified and natural way.

Duality involving polyconvex integrands

Wilfrid Gangbo, Georgia Institute of Technology, U.S.A.

Summary: We show that given a probability measure μ on $\mathbf{R}^d$, with finite first moments, and given a bounded open set $\Lambda \subset \mathbf{R}^d$, the variational problems $\inf_{(\psi,\phi)\in\mathcal{A}}\{\int_{\mathbf{R}^d}\psi(z)dz+\int_\Lambda \phi(y)dy\}$ and $\sup_{\gamma\in\Gamma(\mu)}\{\int_C(y\cdot z-h(\alpha))d\gamma(\alpha,y,z)\}$ are dual to each other. Here $\mathcal{A}$ is the class of all pairs (ψ,ϕ) such that $\psi(z)+\alpha\phi(y)\geq y\cdot z-h(\alpha)$ and $\Gamma(\mu)$ is the set of all measures on $C := (0,\infty)\times\mathbf{R}^{2d}$ such that $\int_{\mathbf{R}^d} f(z)d\mu(z) = \int_C f(z)d\gamma(\alpha,y,z)$ and $\int_\Lambda f(y)dy = \int_C \alpha f(y)d\gamma(\alpha,y,z)$ for all $f\in C_o(\mathbf{R}^d)$. As a consequence we identify a problem dual to $\inf_{\mathbf{u}} E[\mathbf{u}]$, where the infimum is performed over the set of all deformations $\mathbf{u}:\Omega\to\Lambda$. Here $E[\mathbf{u}] := \int_\Omega(h(detD\mathbf{u})-\mathbf{F}\cdot\mathbf{u})d\mathbf{x}$ is the energy occuring in solid crystals.

Remarks about the analysis and simulation of grain boundary systems

David Kinderlehrer, Carnegie Mellon University, Pittsburgh, U.S.A.

Summary: Among the most important features of the recrystallization process is curvature-driven evolution of an immense system of grain boundaries and associated triple lines, where these boundaries intersect. Understanding the excess free energy and mobilities of grain boundaries and the way in which they effect the coarsening process is a goal of our analysis and simulation. We report on our progress. To motivate our discussion we begin by explaining the Herring Relation as a criterion for stability of system evolution and proceed with a description of how its adoption can assist in the simulation of huge grain boundary systems. We shall then describe an analytical result about evolution to equilibrium and provide some simulations. This introduces new challenges about the simulation of large metastable systems which can be approached only with innovative new methods. This is joint work with Chun Liu, Florin Manolache, and Shlomo Ta'asan.

Mazur's lemma for quasiconvexity

Jan Kristensen, University of Oxford, UK

Summary: Variational problems for multiple integrals on Sobolev spaces of vector-valued mappings are intrinsically connected to Morrey's notion of quasiconvexity. In this talk we discuss a version of Mazur's Lemma, which is valid in the context of quasiconvex functionals defined by means of the Lebesgue-Serrin extension procedure. As an easy application of the result we derive a new existence theorem for minimisers of quasiconvex integrals.

Weak limits of Jacobians

Jan Malý, Charles University, Praha, Czech Republic

Summary: The topic of the talk is a joint work with Irene Fonseca. We consider a sequence of functions $u_n : \Omega \to \mathbb{R}^N$, where $\Omega \subset \mathbb{R}^N$ is an open set. If $u_n \rightharpoonup u$ in the Sobolev space $W^{1,N}(\Omega)$, then by Reshetnyak's theorem $\det \nabla u_n \overset{*}{\rightharpoonup} \det \nabla u$ in (Radon) measures on Ω. We study situations when the weak convergence assumption is weakened. As a positive result, we prove that if $u_n \in W^{1,N}$, $u_n \rightharpoonup u$ weakly in $W^{1,N-1}$ and $\det \nabla u_n \overset{*}{\rightharpoonup} f$ in measures, then the absolutely continuous part of f is $\det \nabla u$. A nontrivial singular part of f may exist. If the convergence $u_n \rightharpoonup u$ is only in $W^{1,N-1-\epsilon}$, then, roughly speaking, for any prescribed pair (u, f) we may find a sequence u_n such that $u_n \rightharpoonup u$ and $\det \nabla u_n \overset{*}{\rightharpoonup} f$. The same situation occurs already for the convergence in $W^{1,N-\epsilon}$ if the assumption $u_n \in W^{1,N}$ is violated.

Attainment results for some nonconvex variational problems

Paolo Marcellini, Università di Firenze, Italy

Summary: A main motivation for studying first order *implicit* partial differential equations comes from the calculus of variations. This kind of “implicit” PDE have been studied in detail in the forthcoming book by B. Dacorogna and P. Marcellini, *Birkhäuser*, Boston, 1999.

We will explain the link between the existence of minimizers of some nonconvex integrals of the calculus of variations and first (and second) order implicit partial differential equations. We will present some new results and we will describe some work actually in progress in this field, joint with I. Fonseca and N. Fusco.

From a nonlinear, nonconvex problem to a linear, convex formulation

Pablo Pedregal, Universidad de Castilla-La Mancha, Spain

Summary: By means of a Parseval-type identity involving functions and Young measures, we can formally consider a nonlinear, nonconvex variational problem as a new optimization problem with a linear objective function under a convex set of constraints. This formalism can lead in some cases to explicit solutions or to standard algorithms for numerical approximation for nonconvex problems. We will illustrate both possibilities carrying out this programme for some explicit examples of scalar, one-dimensional variational problems where the dependence of the integrand upon the derivative variable is a polynomial of degree four.

On the differences between Homogenization and Γ-convergence

Luc Tartar, Carnegie Mellon University, Pittsburgh, U.S.A.

Summary: In the late 60s, Sergio Spagnolo introduced a notion which he called G-convergence, the prefix G recalling that it is related to the convergence of Green kernels. In the early 70s, he collaborated with Ennio De Giorgi, who had already been influential in the initiation of the theory, and when I first heard Ennio De Giorgi talk about Γ-convergence at the seminar of Jacques-Louis Lions in 1977, I understood it as a natural generalization of that earlier joint work on G-convergence, and of a generalization which Umberto Mosco had obtained. I had been impressed by the fact that energy localized on a surface could appear as the Γ-limit of three-dimensional problems (in a particular result of Luciano Modica), and I immediately thought that surface tension in liquids should be determined from three-dimensional laws; in order to study that question one obviously needs to extend the idea of Γ-convergence to evolution problems, because liquid flows are obviously not restricted to stationary questions and (as far as one understands it now) are not related to minimizing any reasonable functional either.

In the Spring 1977, I gave a set of six Peccot lectures at Collège de France, entitled "Homogénéisation dans les équations aux dérivées partielles", and I described a generalization of the work of Sergio Spagnolo that I had obtained with François Murat in the early 70s, presenting my method of oscillating test functions which extends easily to all kind of linear variational problems, not necessarily related to any minimization question (the same method was obtained independently later by Leon Simon). Although our framework, like that of Sergio Spagnolo, was certainly not restricted to questions with periodic structures, I had adopted the term Homogenization which had been used by Ivo Babuška for periodic problems occuring in Engineering applications, similar to those which Henri Sanchez-Palencia had also studied in the early 70s. Our approach to Homogenization was then more general but nevertheless quite adapted to questions in Continuum Mechanics or Physics, and François Murat later coined the term H-convergence for describing it.

It seems that many people have still not yet understood the differences between Homogenization (as H-convergence) and Γ-convergence, and tend to use the term Homogenization in a way which is neither conform with Ivo Babuška's meaning nor with mine (which is perfectly fine but a source of confusion if one does not say which definitions are used). I will explain a theorem related to a question of Homogenization in linearized Elasticity due to Sergio Gutiérrez, and discuss the curious belief which some have expressed that it contradicts results of Γ-convergence (it is doubtful that one has at last found a proof of the contradiction of Mathematics). There is obviously no opposition between the theory of Γ-convergence and the theory of Homogenization according to our work, and none of these two theories generalize the other. I will explain a different fact, which is why many questions of Continuum Mechanics are not covered by the theory of Γ-convergence, at least in the way people use it now, i.e. by choosing the topologies that they like instead of identifying the adapted topologies as it is done in Homogenization theory, and Ennio De Giorgi certainly already knew that difference in 1977: he had not defined the theory with only strong topologies or weak topologies, because he knew that many other topologies must be considered.

An existence and uniqueness result for a nonlocal adaptive elasticity model

Luís Trabucho, University of Lisbon, Portugal

Summary: In some Biomechanics applications it is known that the dependence of the elastic constants on the density of the material (or on the volumetric fraction) may vary significantly with time. This is the case in bone remodeling, for instance, where an adaptive process is continuously under way. In order to be able to predict this behaviour, from a strict mechanical point of view, Cowin and Hegedus derived, in the framework of Continuum Mechanics, a model known as adaptive elasticity.

In this work we establish an existence and uniqueness result for a modified Cowin-Hegedus type model which includes most of the empiric and/or experimental existing models for bone remodeling.

Two-well structure and intrinsic mountain pass points

Kewei Zhang, Macquarie University, New South Wales, Australia

Summary: A class of quasiconvex functions with strongly quasimonotone gradients is introduced motivated by the variational approach to material microstructure. The functions are of quadratic growth with the 'two-well' structure. Under small dead-load perturbations, we show that the natural boundary value problem (Neumann problem) has at least three critical points: a global minimizer, a local minimizer and an unstable critical point (mountain pass point).

Short Talks

Numerical optimization of perforated materials

Cristian Barbăroşie, University of Lisbon, Lisboa

Summary: Consider an elastic material containing many small holes, arranged periodically. The overall behaviour of the perforated material will depend on the elastic properties of the matrix material, on the volume fraction of the holes, but also on the shape of each hole. Homogenization theory is a tool for describing effective properties of such perforated materials, as well as of mixtures of two or more materials.

We address the following problem: given the properties of the base material and a certain volume fraction for the holes, optimize the shape of the holes in order to achieve certain properties for the composite. This problem has already been studied by several authors (N. Kikuchi, J. Guedes, H. Rodrigues, P. Fernandes, M. Neves). Our approach is different from theirs, being similar to the one used by Dvorak and Haslinger. We construct a finite element mesh on the periodicity cell; fill some of the elements with the base material (the remaining are void); solve the celular problems and compute the homogenized coefficients. Then we compute the derivatives of the coefficients with respect to the position of the vertices on the boundary of the hole and based on these derivatives we deform the mesh accordingly. In this way we minimize a functional, which gives us the desired effective properties for the perforated material.

Multiple integrals of product type

Pietro Celada, Università di Trieste, Italy

Summary: We consider the problem of minimizing multiple integrals of product type, i.e.,

$$(\mathcal{P}) \qquad \min\left\{\int_\Omega g(u(x))f(\nabla u(x))\,dx : \quad u \in u_0 + W_0^{1,p}(\Omega)\right\}$$

where Ω is a bounded open set in $\mathbb{R}^N$, $f : \mathbb{R}^N \to [0,\infty)$ is a possibly nonconvex, lower semicontinuous function with p-growth at infinity for some $1 < p < \infty$ and the boundary datum u_0 is in $W^{1,p}(\Omega) \cap L^\infty(\Omega)$ (or simply in $W^{1,p}(\Omega)$ if $N < p < \infty$).

Assuming that the convex envelope f^{**} of f is affine on each connected component of the set $\{f^{**} < f\}$, we prove attainment for $(\mathcal{P})$ for every continuous, positively bounded below function g such that

a) for every $t \in \mathbb{R}$, there is $\delta > 0$ such that g is either non increasing or non decreasing on both intervals $[t-\delta, t]$ and $[t, t+\delta]$;

b) g has no strict relative minima.

This shows in particular that the class of coefficents g that yield existence to $(\mathcal{P})$ is dense in the space of continuous, positive functions on $\mathbb{R}$. We present examples that show that such conditions for attainment are essentially sharp.

The proof consists of the following two steps.

Step 1. We prove that every solution v of the relaxed problem

$$(\mathcal{P}^{**}) \qquad \min\left\{\int_\Omega g(v(x))f^{**}(\nabla v(x))\,dx : \, v \in u_0 + W_0^{1,p}(\Omega)\right\}$$

is continuous and almost everywhere differentiable on Ω. We remark that this property does not follow from standard regularity results for variational integrals.

Step 2. Relying on Step 1, we show that every solution v to $(\mathcal{P}^{**})$ can be modified so as to find a new solution u to $(\mathcal{P}^{**})$ satisfying

$$f^{**}(\nabla u) = f(\nabla u) \qquad \text{a.e. on } \Omega.$$

This obviuosly implies that u is a solution to $(\mathcal{P})$ as well.

Hölder continuity of local minimizers

Giovanni Cupini, Università di Firenze, Italy

Summary: Let Ω be an open set in $\mathbb{R}^N$, Q the unit cube $(0,1)^N$, $p > 1$, $L > 1$, $\mu \geq 0$. In the paper "Hölder continuity of local minimizers", to appear on "Journal of Mathematical Analysis and Applications", the authors Giovanni Cupini, Nicola Fusco and Raffaella Petti (University of Florence, Italy) study the regularity of local minimizers of integral functionals of the type

$$\mathcal{F}(v;\Omega) := \int_\Omega F(x, v(x), Dv(x))\, dx\,, \tag{1}$$

where $F : \Omega\times\mathbb{R}\times\mathbb{R}^N \to \mathbb{R}$ is a continuous function satisfying the following assumptions: for any $x, y \in \Omega$, $u, v \in \mathbb{R}$, $z \in \mathbb{R}^N$

(F_1) $(\mu^2 + |z|^2)^{p/2} \leq F(x,u,z) \leq L(\mu^2 + |z|^2)^{p/2}$,

(F_2) $|F(x,u,z) - F(y,v,z)| \leq \omega(|x-y| + |u-v|)(\mu^2 + |z|^2)^{p/2}$,
where $\omega : [0,+\infty) \to [0,+\infty)$ is a continuous, not decreasing, bounded function with $\omega(0) = 0$.

We do not make any differentiability assumption on F and in particular we do not require an ellipticity condition of the type

$$\sum_{i,j=1}^N D_{ij}F(x,u,z)\xi_i\xi_j \geq \nu(\mu^2 + |z|^2)^{(p-2)/2}|\xi|^2 \qquad \forall \xi \in \mathbb{R}^N\,.$$

Instead, we assume that condition

(F_3) $$\int_Q F(x_0, u_0, z + D\varphi(x))\, dx \geq \int_Q \Big[F(x_0,u_0,z) + \nu(\mu^2 + |z|^2 + |D\varphi(x)|^2)^{(p-2)/2}|D\varphi(x)|^2\Big]\, dx\,,$$

holds (uniformly with respect to (x,u)), for any $x_0 \in \Omega$, $u_0 \in \mathbb{R}$ and $\varphi \in C_0^1(Q)$.

We notice that it can be proved that F satisfies the conditions (F_1), (F_2) and (F_3) if and only if a function $f : \Omega\times\mathbb{R}\times\mathbb{R}^N \to \mathbb{R}$ exists, such that f is convex in z,

$$F(x,u,z) = \nu(\mu^2 + |z|^2)^{p/2} + f(x,u,z), \qquad (\nu > 0)$$

$$0 \leq f(x,u,z) \leq L(\mu^2 + |z|^2)^{p/2},$$

and f satisfies (F_2).

Under these assumptions we prove the following theorem

Theorem 1 *Let $F : \Omega\times\mathbb{R}\times\mathbb{R}^N \to \mathbb{R}$ be a continuous function verifying assumptions (F_1), (F_2) and (F_3) above. If $u \in W^{1,p}_{\mathrm{loc}}(\Omega)$ is a local minimizer of the functional*

$$\mathcal{F}(w;\Omega) := \int_\Omega F\left(x, w(x), Dw(x)\right)\, dx$$

then $u \in C^{0,\alpha}_{\mathrm{loc}}(\Omega)$ for all $\alpha < 1$.

Moreover an example shows that we cannot expect minimizers to be locally Lipschitz continuous, that is $u \in C^{0,1}_{\mathrm{loc}}(\Omega)$.

A droplets model as a singular limit of a Cahn-Hilliard type model

Christophe Dubs, Université Toulon Var, France

Summary: Based on the Cahn-Hilliard model of two-phase fluids, we consider a class of smooth local non convex functionals defined on $W^{1,2}$, depending on a small parameter ε, and we prove that they converge, as ε tends to 0, to a functional defined on measures. This provides a new model describing the dependance of the energy of a droplet of liquid (seen as a Dirac mass) in relation to its mass.

Convergence of Sobolev spaces on varying manifolds

Ilaria Fragalà, Università di Pisa, Italy

Summary: We present a new definition of tangent bundle and of mean curvature for a general positive measure μ on $\mathbb{R}$. In particular, the classical notions are recovered when μ is the k-dimensional Hausdorff measure on a smooth k-manifold. Shape optimization problems, and in general optimal control problems where measures act as controls, can be considered in this new framework.

A volume constrained problem in SBV_2

José Matias, IST/Universidade Técnica de Lisboa, Portugal

Summary: We consider the problem of minimizing the energy

$$E(u) := \int_\Omega |\nabla u(x)|^2\, dx + \int_{S_u \cap \Omega} (1 + |[u(x)]|)\, dH^{N-1}(x)$$

among all functions $u \in SBV_2(\Omega)$ (i.e. $u \in SBV(\Omega)$ is such that $\nabla u \in L^2(\Omega, \mathbf{R}^N)$ and $H^{N-1}(S_u \cap \Omega) < +\infty$) whose level sets $\{u = l_i\}$ have prescribed Lebesgue measure α_i. Here α_i and l_i, $i = 0, \ldots, m$ are given real numbers satisfying $\alpha_i > 0$, $\sum_{i=0}^m \alpha_i \leq \mathcal{L}^N(\Omega)$ and $l_0 < l_1 < \ldots < l_m$. Subject to this volume constraint the existence of minimizers for $E(\cdot)$ is proved.

In the case where $m = 1$, i.e. when only two level sets are involved, the asymptotic behaviour of the solutions is also investigated.

Everywhere regularity for a class of elliptic systems with p, q growth conditions

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Summary: We consider integrals of the Calculus of Variations over an open bounded subset Ω of $\mathbf{R}^n$ and the related regularity problem: are the minimizers $u \in W^{1,p}\left(\Omega, \mathbf{R}^N\right)$ smooth functions, say for example Lipschitz continuous? Classically, the so-called *natural growth conditions* on the integrand have been the main sufficient assumptions for *everywhere regularity* ($C^{1,\alpha}\left(\Omega, \mathbf{R}^N\right)$) in the scalar case ($N = 1$) and *partial regularity* ($C^{1,\alpha}\left(\Omega_0, \mathbf{R}^N\right)$ with $\Omega_0 \subset \Omega$ and $|\Omega \backslash \Omega_0| = 0$) in the vectorial one ($N > 1$).

We deal with the vectorial case and we prove everywhere regularity for weak solutions of elliptic systems of the form

$$\sum \frac{\partial}{\partial x_i} a\left(x, |Du|\right) u^\alpha_{x_i} = 0 \tag{2}$$

under *general p, q growth conditions* and in particular for minimizers of a class of variational integrals, both degenerate and non degenerate ones, whose models are

$$\begin{aligned} I_1(u) &= \int_\Omega a(x)\,|Du|^{\alpha(x)}\,dx, \\ I_2(u) &= \int_\Omega a(x)\left(1+|Du|^2\right)^{\frac{\alpha(x)}{2}}dx. \end{aligned}$$

We consider integral of the Calculus of Variations

$$I(u) = \int_\Omega f(x, Du)\,dx, \qquad \text{with} \qquad f(x, Du) = g(x, |Du|) \tag{3}$$

where g is of class C^2 with g_t positive and increasing with respect to t.

We assume that there are two positive constants m and M such that for every $\lambda = (\lambda_i^\alpha)$ and $\xi = (\xi_i^\alpha) \in \mathbf{R}^{Nn}$ and for *a.e.* $x \in \Omega$ we have

$$m\left(\mu^2+|\xi|^2\right)^{\frac{p-2}{2}}|\lambda|^2 \leqslant \sum_{i,j,\alpha,\beta} f_{\xi_i^\alpha \xi_j^\beta}(x,\xi)\,\lambda_i^\alpha \lambda_j^\beta \leqslant M\left(\mu^2+|\xi|^2\right)^{\frac{q-2}{2}}|\lambda|^2, \tag{4}$$

$$\left|f_{\xi_i^\alpha x_s}(x,\xi)\right| \leqslant M\left(\mu^2+|\xi|^2\right)^{\frac{p+q-2}{4}}, \tag{5}$$

$$\text{for} \quad \mu = 0 \quad \text{or} \quad \mu = 1, \quad \forall \alpha = 1, 2, ..., N, \qquad \forall i,\ s = 1, 2, ..., n.$$

We give some *a priori* estimates when p and q satisfy

$$2 \leqslant p \leqslant q < \frac{n}{n-2}p \tag{6}$$

(simply $2 \leqslant p \leqslant q$ if $n = 2$), while we prove local Lipschitz continuity if

$$2 \leqslant p \leqslant q < \frac{n+2}{n}p. \tag{7}$$

Let us denote by B_ρ and B_R balls compactly contained in Ω of radii ρ and R respectively and with the same center. We prove the following theorem.

Theorem 2 *Under the assumptions* (4), (5), (6) *and* (7), *every weak solution* u *of the system* (2) *and every minimizer of the integral* (3) *is of class* $W^{1,\infty}_{loc}\left(\Omega, \mathbf{R}^N\right)$ *and, for every* ρ, R, *with* $0 < \rho \leqslant R < 1$, *there exists a constant* $c = c(\rho, R, n, N, p, q, m, M)$ *and an exponent* $\alpha = \alpha(p, q, n)$ *such that*

$$\|Du\|_{L^\infty(B_\rho, \mathbf{R}^{Nn})} \leqslant c\left\{\int_{B_R}\left[1 + f(x, |Du|)\right]dx\right\}^{\frac{\alpha}{p}}.$$

We make use of the two methods introduced by Marcellini in [5] and [6], combining them in order to handle the technical problems due to the x-dependence and the degenerate case $\mu = 0$ and we extend the classical regularity results known for the so-called *natural growth conditions* when $p = q$ (see for example Uhlenbeck [7], Giaquinta-Modica [4], Acerbi-Fusco [1]). In the context of integral functionals with variable growth exponent recent results are due to Chiadò Piat-Coscia [2] and Coscia-Mingione [3].

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On some necessary conditions for a nonlocal variational principle

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Summary: The aim of this work is to study the existence for a nonlocal variational problem through the use of optimality necessary conditions. We consider the relaxation of the problem in terms of Young measures. This relaxation is a new problem where we perform variations to deduce optimality conditions, and those conditions yield explicit information about minimizers in the homogeneous case. It also provides us a method able to find minimizers for some specific problems.

Anisotropic singular perturbation problems with lack of coercivity

Giuseppe Savaré, Università di Pavia, Italy

Summary: The aim of this talk is to present some results (obtained in collaboration with L. Ambrosio and P. Colli Franzone) on the Γ-convergence as $\varepsilon \downarrow 0$ of the family of integral functionals depending on $\boldsymbol{u} = (u_1, u_2) \in H^1(\Omega; \mathbb{R}^2)$

$$\mathcal{F}^\varepsilon(\boldsymbol{u}) := \varepsilon \int_\Omega q\big(x, \nabla \boldsymbol{u}(x)\big)\, dx + \frac{1}{\varepsilon} \int_\Omega W\big(u_1(x) - u_2(x)\big)\, dx.$$

Here $q : \overline{\Omega} \times \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}$ is a continuous family of quadratic forms satisfying the usual uniform ellipticity condition and $W : \mathbb{R} \to [0, +\infty)$ is a continuous double-well potential satisfying for some $p \geq 2$

$$W(v) = 0 \quad \Leftrightarrow \quad v \in \{-1, 1\}, \quad 0 < \liminf_{|v| \to +\infty} \frac{W(v)}{|v|^p} \leq \limsup_{|v| \to +\infty} \frac{W(v)}{|v|^p} < +\infty.$$

Functionals of this type naturally arise in the study of the cardiac excitation process and the associated electric potential distribution, by using the so called *bidomain* representation of the cardiac muscle. Since W depends only on the difference of the components of $\boldsymbol{u}$, this problem does not properly fit into the class of singular perturbation problems of anisotropic and vectorial type, which have already been studied in the literature: the main difference lies in the structure of the set of minimum points of the non-convex potential $W(u_1 - u_2)$, i.e. the union of two parallel lines $\big\{\boldsymbol{u} \in \mathbb{R}^2 : |u_1 - u_2| = 1\big\}$. This fact reflects that a general minimizing sequence $\boldsymbol{u}^\varepsilon$ for $\mathcal{F}^\varepsilon$ can be unbounded in $L^1(\Omega; \mathbb{R}^2)$, and *a priori* we can only hope some compactness properties for the difference $v^\varepsilon := u_1^\varepsilon - u_2^\varepsilon$.

We provide a variational characterization of the Γ-limit $\mathcal{F}$ and we show that $\mathcal{F}$ depends only on $v := u_1 - u_2$, it is finite if and only if v is a function of bounded variation in Ω assuming only the values $-1, 1$ a.e., and on this kind of function $\mathcal{F}$ can be represented as an integral functional on the discontinuity surface of v: the resulting integrand is a convex function of the approximate normal to the jump set of v and can be computed by solving a localized minimization problem.

Gradient regularity for minimizers of functionals under $p-q$ subquadratic growth

Francesco Siepe, Università di Firenze, Italy

Summary: Let us consider a functional of the calculus of variations of the type

$$\int_{\Omega} f(Du)dx,$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is an open bounded set, $u : \Omega \to \mathbb{R}^N$ $(N \geq 1)$ is a weakly differentiable function and, for every $z \in \mathbb{R}^{nN}$, we assume that f satisfy the $p-q$ growth conditions:

$$|z|^p \leq f(z) \leq c(1+|z|^q) \qquad p<q.$$

We study the higher integrability of local minimizers of such a functional, when $1<p<q\leq 2$.

The result is obtained by means of the method of *a-priori estimates:*
First one obtains a higher integrability result for the minimizers of a suitable perturbed functional, and then, by means of a double approximation argument, the same result is proved for the minimizers of the original functional.

In particular we find that for a general integrand function $f = f(z)$ it is needed that $p > \frac{2n}{n+2}$.
When we consider a particular class of integrands, i.e. $f(z) = g(|z|)$, with g a $N-$function of class Δ_2, then the same regularity result is proved without any further condition on p and q.

Existence and relaxation results in special classes of deformations

Mikhail Sytchev, Max-Plank Institute, Leipzig, Germany

Summary: In this talk we discuss existence and relaxation results for the problem

$$J(u) = \int_{\Omega} L(x, u(x), Du(x))dx \to \min,\ \Omega \subset \mathbf{R}^n,\ u \in W^{1,p}(\Omega),\ u\Big|_{\partial\Omega} = f.$$

We assume only that $L : \mathbf{R}^n \times \mathbf{R} \times \mathbf{R}^n \to \mathbf{R} \cup \{\infty\}$ is a Caratheodory function, which is locally bounded in the set where it assumes finite values, and $L \geq \alpha|Du|^p + \gamma$, $\alpha > 0$, $p > n$ since if the problem arizes as a minimization problem in the class of *suitable deformations* of elastic materials then L usually meets only the above assumptions.

It turns out that under these requirements the relaxation result holds, i.e., the lower semicontinuous extension of the integral functional in the weak topology is an integral functional with the integrand obtained by convexification of the original one with respect to Du. This results also shows that to establish attainment in such problems one still has to follow the standard scheme to find a solution of the relaxed problem along which the values of the original and the relaxed integrands coincide.

In the homogeneous case $L = L(Du)$ we show that the requirement *for each* $v \in \mathbf{R}^n$ *either* $\partial L(v) \neq \emptyset$ *or there exist* $v_1, \ldots, v_q \in \mathbf{R}^n$ *such that* v *belongs to the interior of the convex hull of* $\{v_1, \ldots, v_q\}$ *and* $\cap_{i=1}^q \partial L(v_i) \neq \emptyset$ is both necessary and sufficient for all boundary value minimization problems to have a solution provided $L \geq \alpha|Du|^p + \gamma$ with $\alpha > 0$, $p > n-1$ only.

A mathematical formulation of rate independent phase transformations using an extremum principle

Florian Theil, Max-Plank Institute, Leipzig, Germany

Summary: We study a mathematical model for the rate independent mesoscopic behavior of shape memory alloys which meets two basic requirements:

1. Nontrivial hysteresis loops occur, i.e., history dependence is reflected,
2. Three dimensional microstructures are taken into account.

Ingredients are a state space $P \subset X$ where X is a Banach space, a time dependent potential $I : [0,T] \times P \to \mathbb{R}^+$ and a dissipation functional $\Delta : X \to \mathbb{R}^+$, which is convex and homogeneous of degree 1. Admissible processes $c : [0,T] \to P$ are characterized by two inequalities

$$\begin{array}{ll} 0 \leq & \Delta(a - c(t)) + I(t,a) - I(t,c(t)), \\ 0 \leq & I(0,c(0)) - I(T,c(T)) - \int_0^T \{\Delta(\dot{c}) - \partial_t I(t,c(t))\}\,\mathrm{d}t, \end{array}$$

for $t \in [0,T]$. Qualitative properties, existence and uniqueness of admissible processes are discussed in simple cases.

In a second step energies describing materials which can undergo phase transformations are relaxed in such a way that they fit into the previously developed framework. Due to the generality of the formulation the existence of admissible processes can be expected in simple cases.

Scale convergence: a tool for problems related to the theory of homogenization

Anca-Maria Toader, Universidade de Lisboa, Portugal

Summary: We introduce a new concept, called scale convergence, that extends the two-scale convergence, in order to treat nonperiodic oscillations. The main idea is to consider, among all possible oscillations, only those that synchronize with the oscillations of the problem. The essential ingredients of the scale convergence are Young measures. We present examples and applications.

A relaxation theorem in the space of functions of bounded deformation

Rodica Toader, Universidade de Lisboa, Portugal

Summary: We obtain an integral representation for the relaxation, in the space of functions of bounded deformation, of the energy

$$\int_\Omega f(\mathcal{E}u(x))dx$$

with respect to L^1-convergence. Here $\mathcal{E}u$ represents the absolutely continuous part of the symmetrized distributional derivative Eu and the function f satisfies linear growth and coercivity conditions.

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